Fuzzy C-mean based brain MRI segmentation algorithms

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Objective

- Classify the voxels in an MRI Image of the brain into one of these 3 categories -
  - ➢ gray matter (GM),
  - ➢ white matter (WM)
  - ➢ cerebrospinal fluid (CSF)

- Implemented various extensions of FCM for Image Segmentation allowing pixels to be labeled by the influence of its neighborhood labels.
- The extensions are as follows -
  - FCM_S1 : Modification of objective function by incorporating a spatial penalty
  - EnFCM : The grey level histogram of pixel values is used to speed up the clustering process
  - FgFCM : Improves the robustness of the ENFCM Algorithm
  - FLICM : controls the balance between the image details and image noise
- Also implemented FCM Segmentation with bias field correction

FCM_S1

- Incorporates neighborhood information in the objective function
- Objective Function is defined as

$$\begin{cases} \min J_m(\mathbf{U}, \mathbf{V} : \mathbf{X}) = \sum_{i=1}^{c} \sum_{j=1}^{N} (u_{ij})^m \left\| x_j - v_i \right\|^2 + \alpha \sum_{i=1}^{c} \sum_{j=1}^{N} (u_{ij})^m \left\| \bar{x_j} - v_i \right\|^2 \\ \text{subject to} \sum_{i=1}^{c} u_{ij} = 1,\ 1 \le j \le N \end{cases}$$

- N is number of pixels , c is number of clusters
- where d is distance between data x_j and centre of the cluster i, v_i
- u is the fuzzy membership of data x_j to cluster with centre v_i.
- m specifies the degree of fuzziness in the clustering
- α is the weight of neighborhood information
- $\bar{x}_j$ is the average of the neighbors of pixel x_j , computed prior to the clustering process

Enhanced FCM (EnFCM)

- Firstly, a linearly-weighted image is obtained from the existing original image, which is defined in terms of local neighbors as:

$$\xi_j = \frac{1}{1+\alpha}\left(x_j + \frac{\alpha}{N_R}\sum_{r \in N_j} x_r\right), \quad 1 \le j \le N$$

- ξ_j represent the gray level value of jth pixel of the new image
- Nj represents the number of neighboring pixels around the centered pixel x_j , NR denotes the cardinality of Nj (4 - neighbourhood system is used here)
- α controls the tradeoff effects of the neighboring term.
- **The EnFCM algorithm is performed on the gray-level histogram of the image** ξ.

Enhanced FCM (contd)

- Objective function is defined as

$$\begin{cases} \min J_m(\mathbf{U}, \mathbf{V} : \boldsymbol{\xi}) = \sum_{i=1}^{c} \sum_{l=1}^{q} \gamma_l (u_{il})^m (\xi_l - v_i)^2 \\ \text{subject to} \sum_{i=1}^{c} u_{il} = 1, \quad 1 \le l \le q \end{cases}$$

- q denotes the number of gray levels of image
- $u_{il} (0 \le u_{il} \le 1)$ the membership degree of lth gray-level value to the ith cluster center
- The γ_l represents number of pixels with the gray-level value equal to l and it satisfies

$$\sum_{l=1}^{q} \gamma_l = N.$$

Fast generalized FCM (FgFCM)

- Exploits the local spatial information by introducing the local similarity measure S_ij , which is defined as:

$$S_{ij} = \begin{cases} \exp(-\max(|p_i - p_j|, |q_i - q_j|)/\lambda_s - \left\|x_i - x_j\right\|^2/(\lambda_g \sigma_j^2)) & i \neq j \\ 0 & i = j \end{cases}$$

- The terms (p_j , q_j) and x_j denote the two-dimensional spatial coordinate and gray-level value of the jth pixel respectively.
- The parameters λs (fixed to 3) and λg (fixed to 2) denote the scale factors of the spread of local spatial and gray-level relationship respectively.
- The parameter σj is calculated as

$$\sigma_j = \sqrt{\frac{\sum_{i \in N_j} \left\|x_i - x_j\right\|^2}{N_R}}, \quad 1 \leq j \leq N$$

- where Nj represents the number of neighboring pixels around the centered pixel x_j and NR denotes the cardinality of Nj

Fast generalized FCM (Contd)

- Using the factor S_ij , the newly generated image ξ is computed as:

$$\xi_j = \frac{\sum_{i \in N_j} S_{ij} x_i}{\sum_{i \in N_j} S_{ij}}, \quad 1 \leq j \leq N$$

- After computing ξ , rest is same as EnFCM .

Fuzzy local information c-means (FLICM)

- Includes a new factor G ij in its optimization function.
- This new factor considers the local spatial information and controls the balance between the image details and image noise, and it does not requires any parameter tuning.
- The O

$$\begin{cases} \min J_m(\mathbf{U}, \mathbf{V} : \mathbf{X}) = \sum_{i=1}^{c} \sum_{j=1}^{N} (u_{ij})^m \left\| x_j - v_i \right\|^2 + G_{ij} \\ \text{subject to} \sum_{i=1}^{c} u_{ij} = 1, \quad 1 \le j \le N \end{cases}$$

Fuzzy local information c-means (contd)

- The term G_ij is called as fuzzy factor, which is defined as:

$$G_{ij} = \sum_{\substack{k \in N_j \\ k \neq j}} \frac{1}{d_{jk}+1}(1-u_{ik})^m \left\| x_k - v_i \right\|^2, \quad 1 \leq i \leq c,\ 1 \leq j \leq N$$

- where kth pixel is neighboring pixels around the centered jth pixel
- N_j represents the number of neighboring pixels around the centered pixel x_j
- d_{jk} is the spatial Euclidean distance between jth and kth pixels

FCM Seg. + Bias-Field Correction

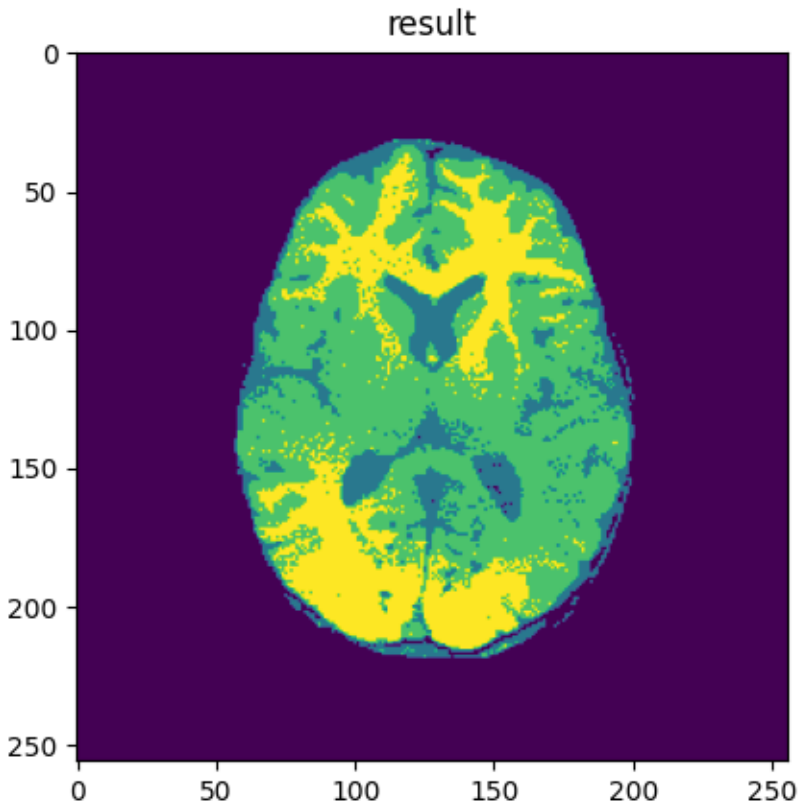
- Objective function is defined as follows

$$J := \sum_{i=1}^{N} J_i := \sum_{j=1}^{N} \sum_{k=1}^{K} u_{jk}^{q} \left(\sum_{i=1}^{N} w_{ij} (y_j - c_k b_i)^2 \right)$$

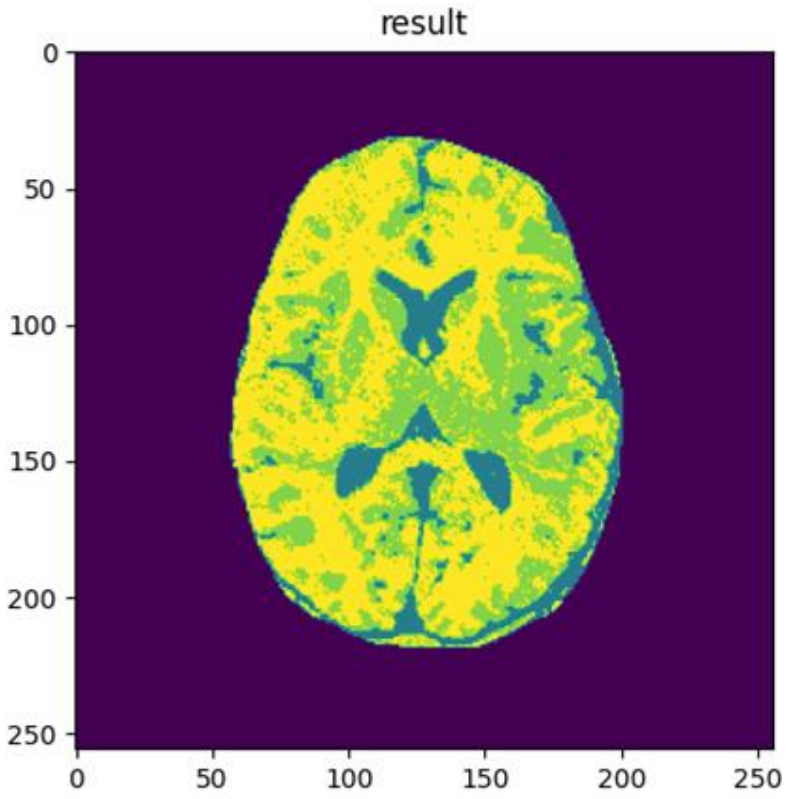
- N is number of pixels , K is number of clusters
- c_K is the centre of the cluster i
- b_i is the bias field at pixel i
- w_{ij} is a weight penalty based on distance between pixels i & j (gaussian mask)
- u is the fuzzy membership of data x_j to cluster with centre v_i.
- q specifies the degree of fuzziness in the clustering

Results

Performed standard FCM and FCM with bias correction on bias-corrupted and noise-corrupted MRI of a human brain



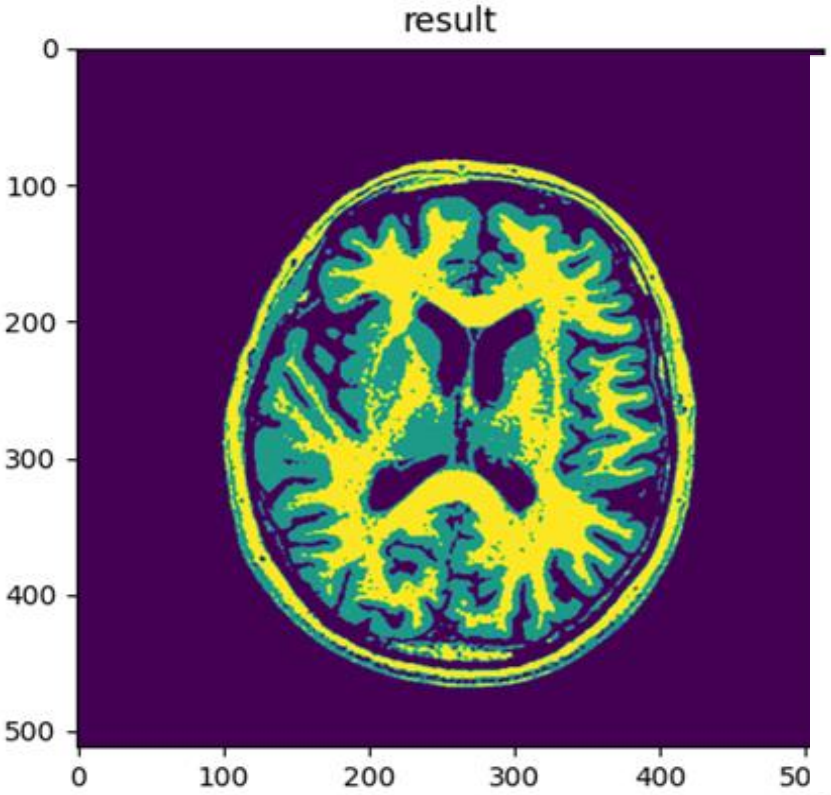
FCM

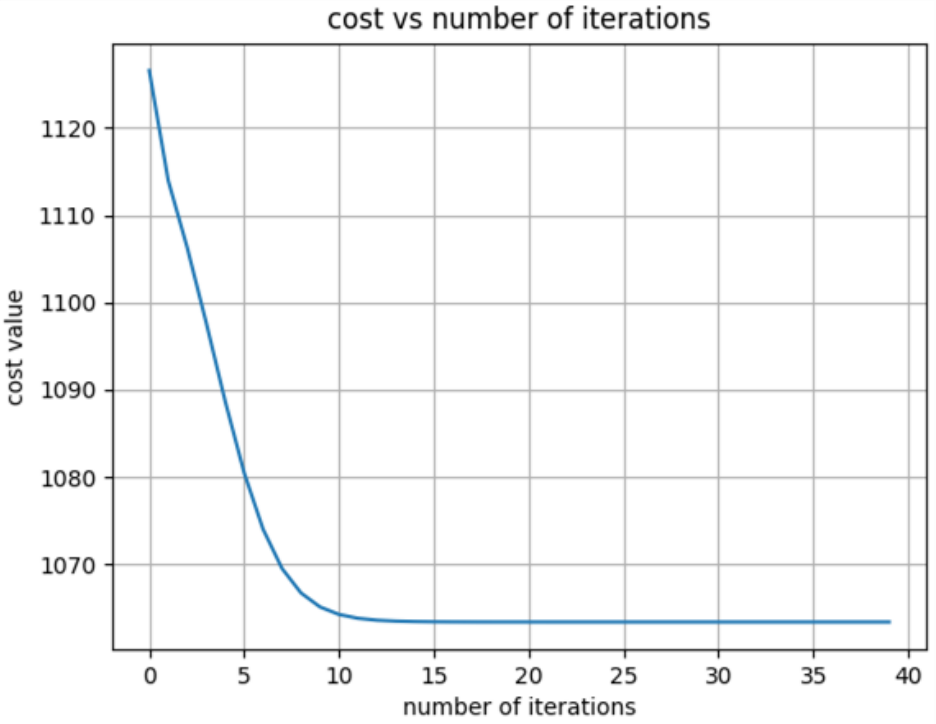


FCM with bias correction

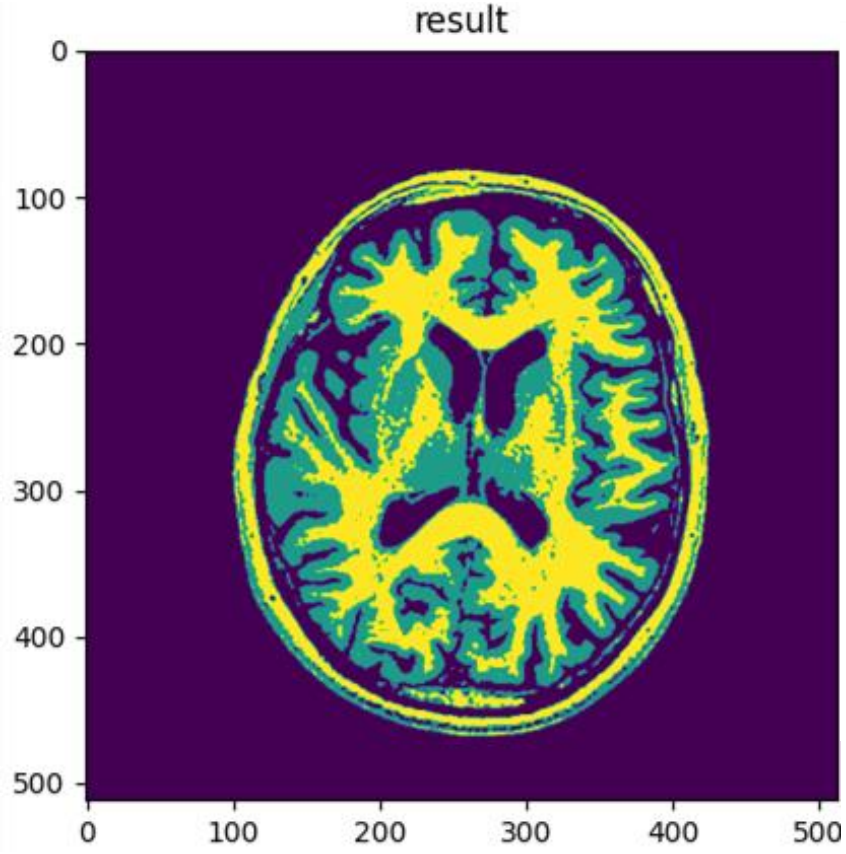
Outputs of various extension on clean MRI Image

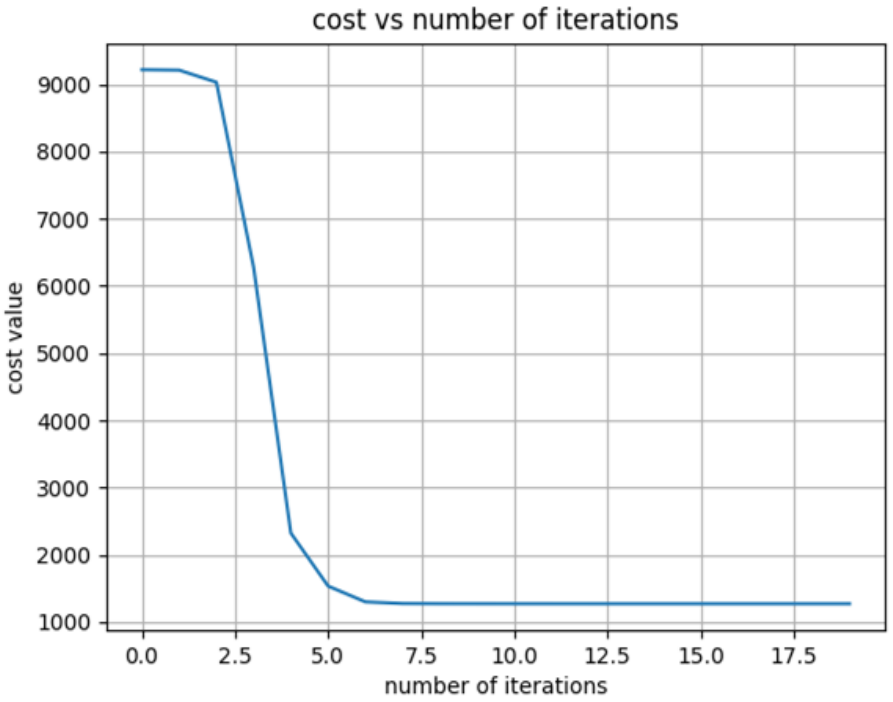
FCM


result
0
100
200
300
400
500
0
100
200
300
400
50

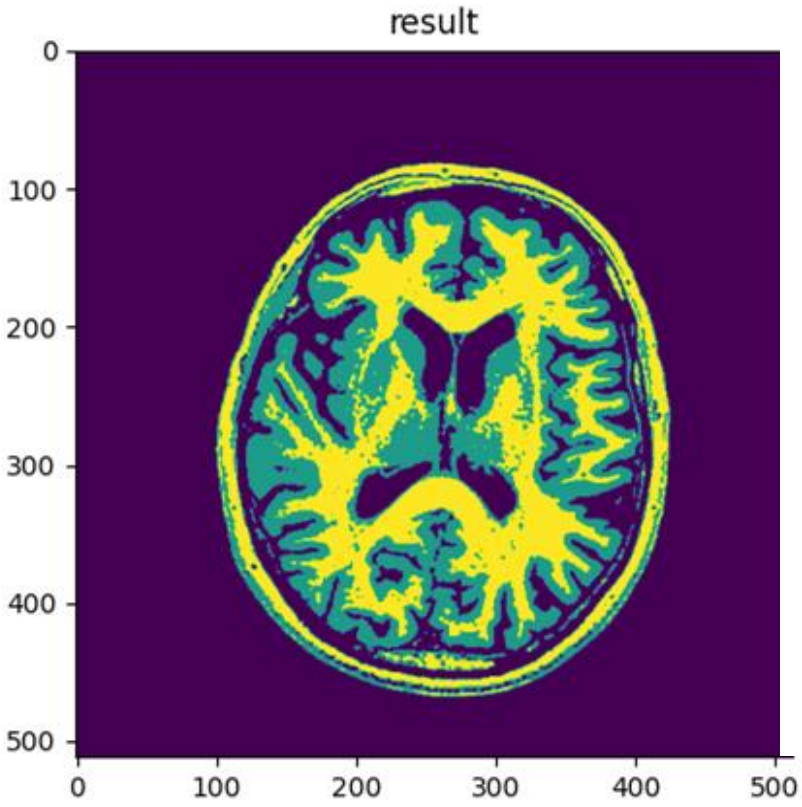

cost vs number of iterations
cost value
1120
1110
1100
1090
1080
1070
0
5
10
15
20
25
30
35
40
number of iterations

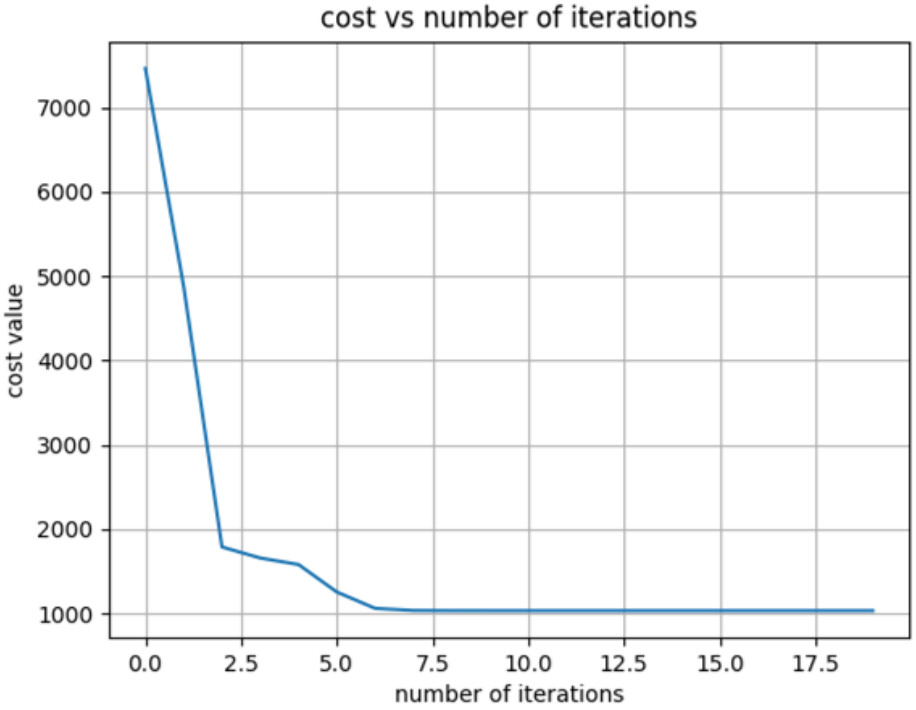
FCM_S1


result
0
100
200
300
400
500
0
100
200
300
400
500

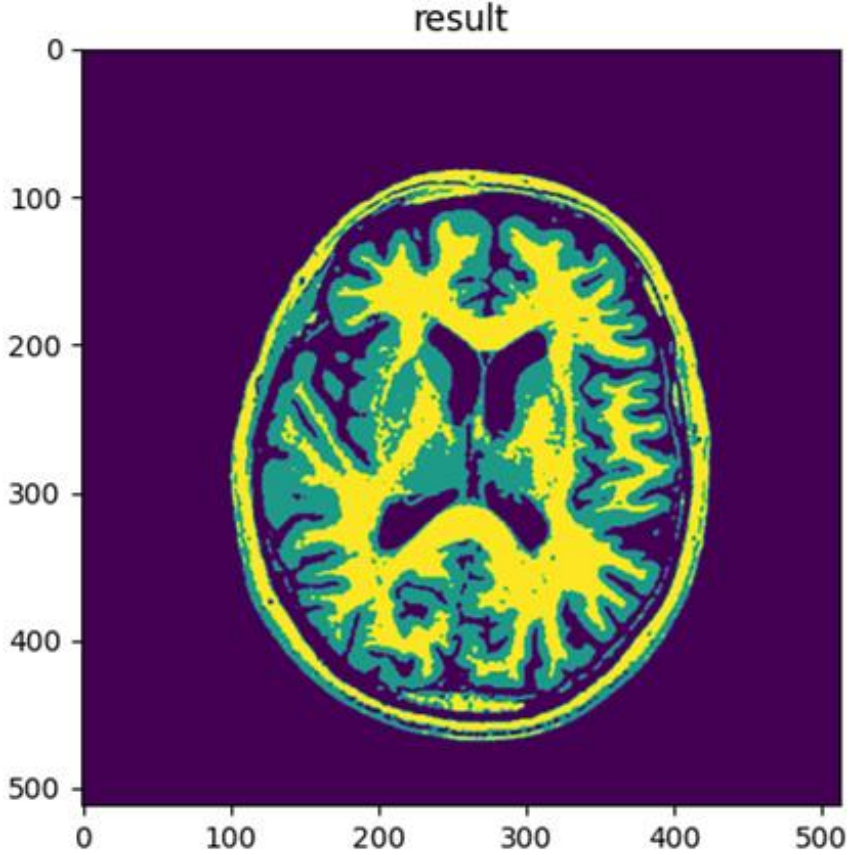

cost vs number of iterations
cost value
9000
8000
7000
6000
5000
4000
3000
2000
1000
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
number of iterations

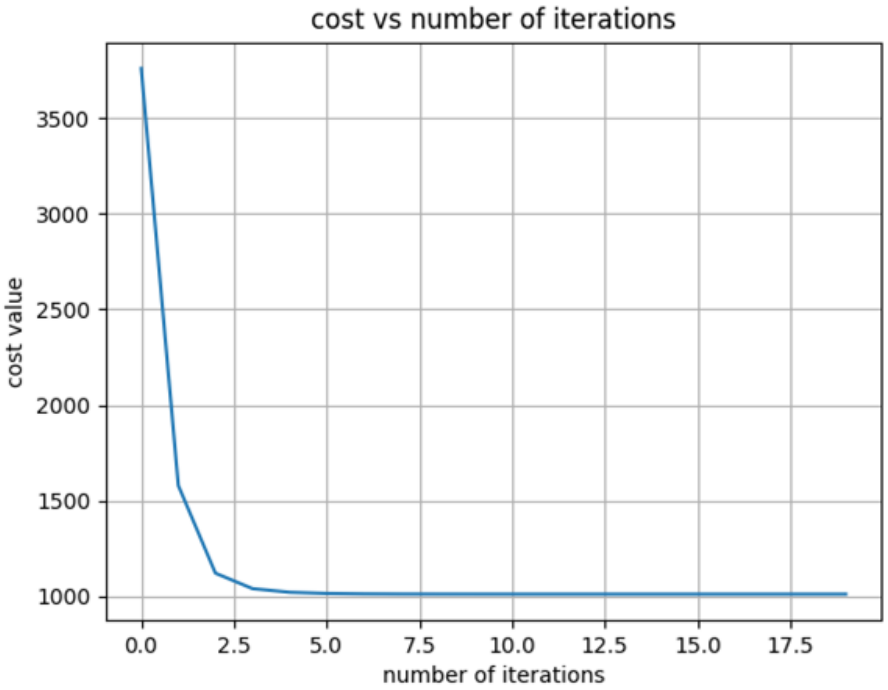
Enhanced FCM (EnFCM)



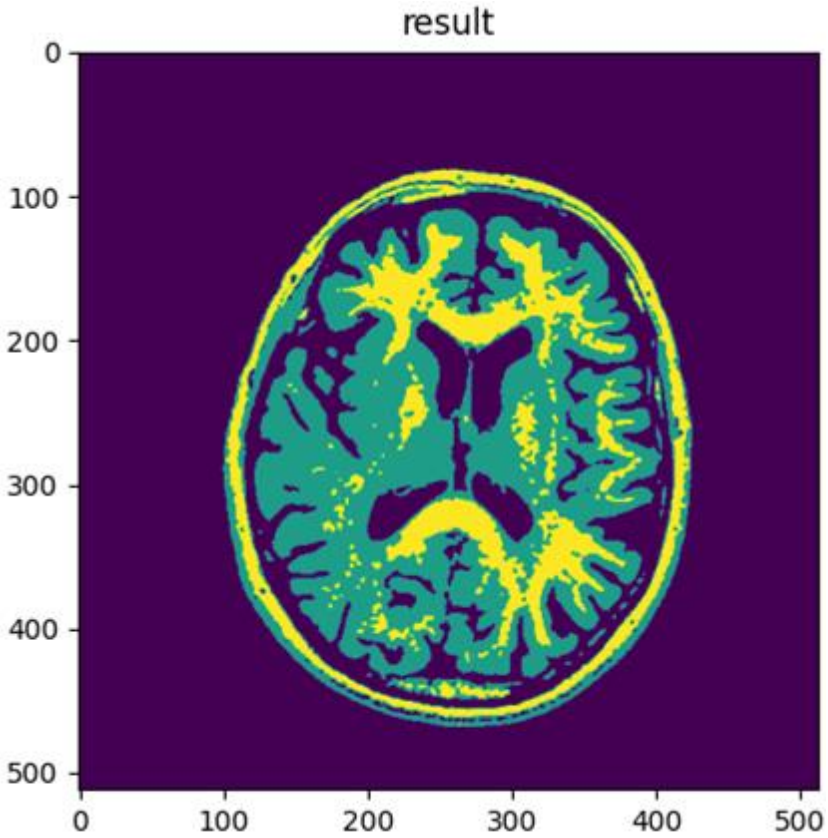


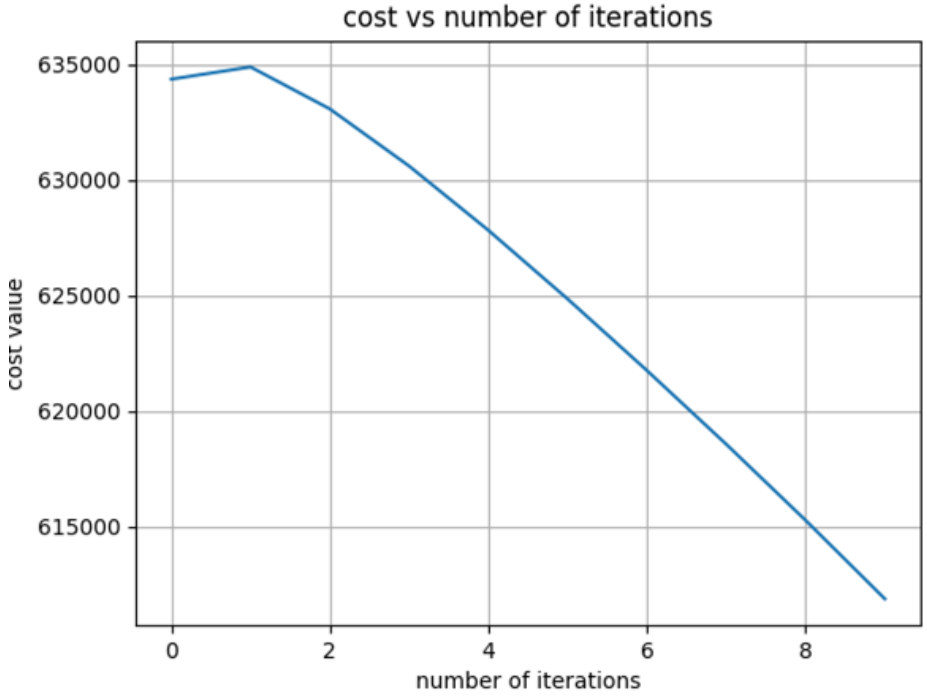
Fast Generalized FCM (FgFCM)



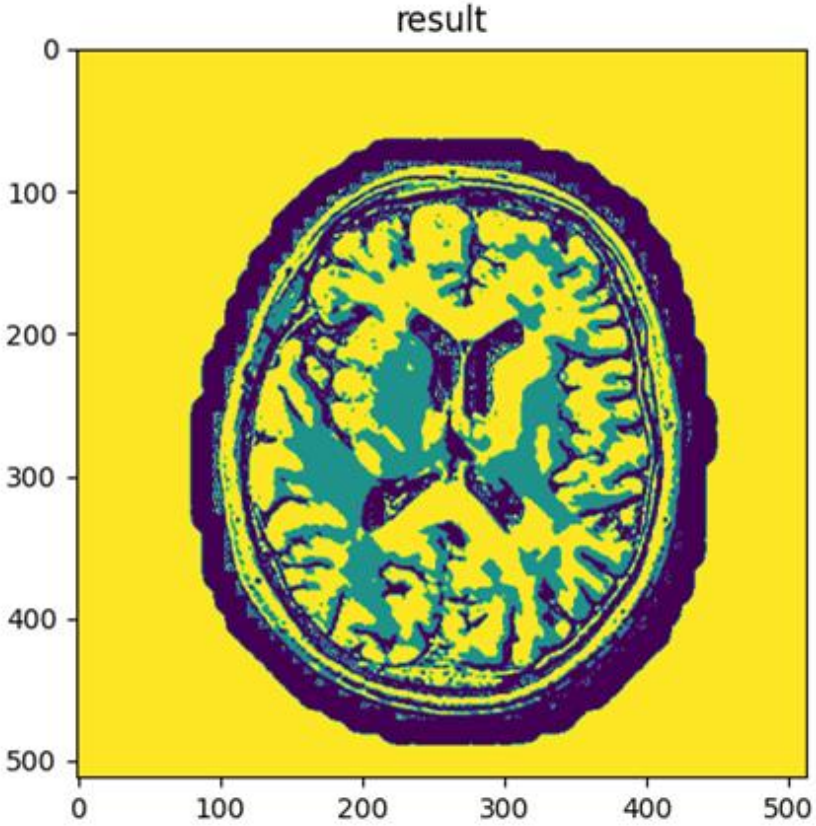


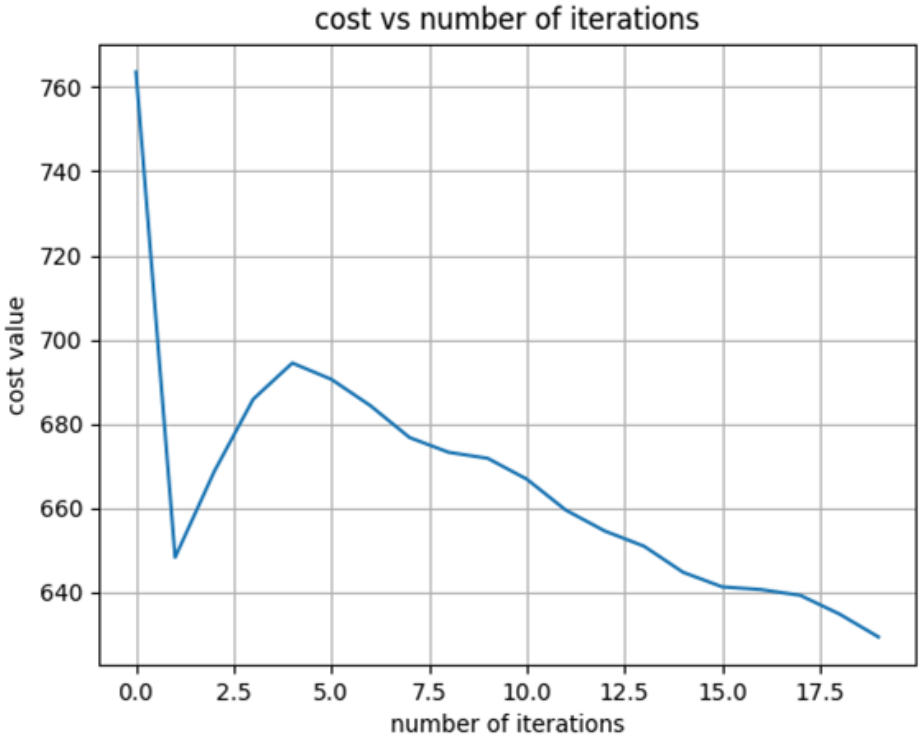
FCM - Local Information (FLICM)





FCM with bias correction





CONCLUSION

- Each flavor of fcm builds upon the last to optimize the run time or to provide bias correction
- FGFCM converges in about 3-4 iterations, in contrast to about 15 iterations in naive FCM and 6 iterations in ENFCM and FCM_S1
- Bias FCM requires many iterations but works well with biased images
- FCM with local information is quite slow as it requires a lot of computation, as the distance between each pair of neighbours need to be calculated, but provides a better correlation between the pixels of the image