Multi-body Dynamics
Assignment 2

• Forward Kinematics:-

For given arrangement in Fig 1 write a code for any arbitrary values of the input parameters α, θ1, θ2 and d we get the position of the end effector.

1. Consider the planar chain of three links as shown in the figure below. The plane of motion is at an angle α. Link-1 is connected at point O through revolute joint. Link-2 is connected to Link-1 through revolute joint. Link-3 is connected to Link-2 through sliding joint.

   Take lengths $l_1 = l_2 = 3$

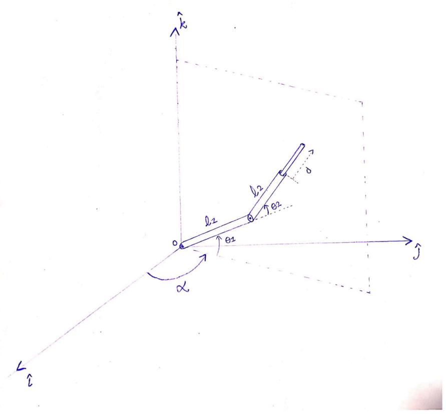

Fig.1 Configuration of arm

In mechanical engineering, the Denavit - Hartenberg parameters (also called DH parameters) are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain, or robot manipulator.

```
>> forward(60,55,35,3);
    0.8604

    1.4902

    8.4575
```

Fig.2 Result from Code

The following four transformation parameters are known as D–H parameters:

- d: offset along previous z to the common normal
- θ: angle about previous z, from old x to new x

α	Θ	D	R
-90	α	0	0
0	Θ1	0	L1
90	Θ2+90	0	0
0	0	L2+d	0

Table.1 DH-parameters

- r: length of the common normal (aka a, but if using this notation, do not confuse with α). Assuming a revolute joint, this is the radius about previous z.
- α: angle about common normal, from old z axis to new z axis

$$ {}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

Fig.2 Transformation Matrix or DH matrix

Using the DH-parameters we create a 4x4 matrix which is just x_new=R*x_old+c converted from 3x3 to 4x4 where the last column giving the position.

• Forces and Moments:-

To Compute the reaction forces and reaction moments at the joints required to produce the motion of the chain in which the magnitude of angular velocity of Link-1 with respect to fixed

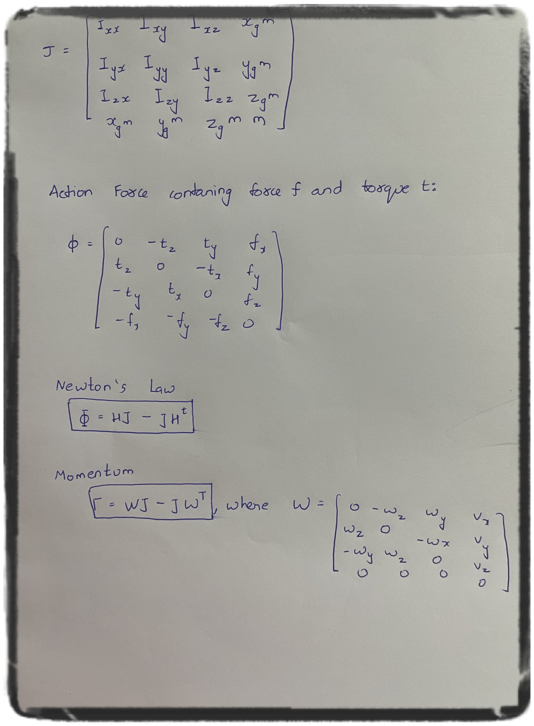

Fig.4 Calculations for Forces and Momentum

frame is ω1, angular velocity magnitude of Link-2 with respect to Link-1 is ω2 and velocity Link-3 with respect to Link-2 is v.
Compute the reaction forces and reaction moments at the joints required to produce the motion. For this case, take masses of link-1 and link-2 as m, link-3 to be mass-less, and α as 60◦ Assuming the cross section of the beam is circular.

For force we use Newton's laws the calculations are shown below to calculate the force and momentum

• Inverse Kinematics:-

Given position of end effector to calculate the theta1 and theta2 since we know the net transformation matrix we have equal no of variables and equations for this equation so we can solve it. But solving it is quite difficult as the equations are not linear so we have to use numerical method to solve it. We generally use the Newton Raphson method to solve non linear equations.

Suppose I want to find the root of a continuous, differentiable function f(x), and you know the root you are looking for is near the point x=x0. Then Newton's method tells us that a better approximation for the root is

x1 =x0 −f′(x0)f(x0)

This process may be repeated as many times as necessary to get the desired accuracy. In general, for any x value xn, the next value is given by

xn+1 =xn −f′(xn)/f(xn) .

Note: the term "near" is used loosely because it does not need a precise definition in this context. However x0 should be closer to the root you need than to any other root (if the function has multiple roots).

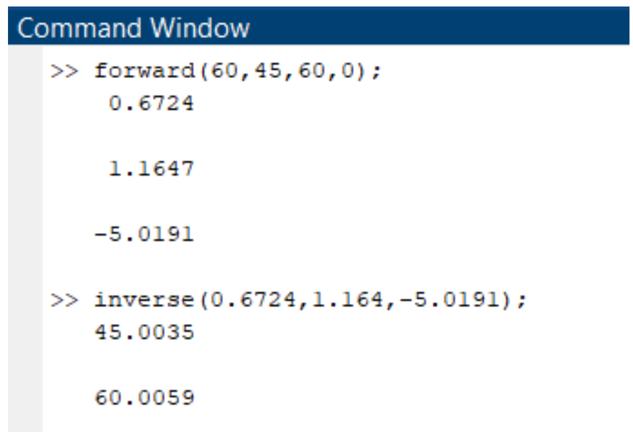

Fig.5 Cross-checking inverse kinematics code for the same input in forward kinematics

My code has been cross checked for inverse kinematics by checking the boundary conditions and with the forward kinematics.

• Conclusion:-

1. We can solve any robotic configurations using transformation matrices if we know the configuration, angles and lengths
2. We can find Force moment on each joint for the motor to provide by using newtons laws
3. If we know the trajectory then we need to find the angles this can be done by solving the equation in forward kinematics in reverse way
4. Since the equation in inverse kinematics is reverse