Fin Lift Coefficient Calculation

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December 2021

Nomenclature

$(C_{N\alpha})_0$ Normal force coefficient derivative of a 2D airfoil

$(C_{N\alpha})_1$ Normal force coefficient derivative of one fin

Γ_c Fin mid-chord sweep angle

ρ Air density

A_{ref} Reference area

AR Aspect ratio of a fin

C_N Normal force coefficient, Lift coefficient

C_r Root chord

C_t Tip Chord

C_{N_α} Normal force coefficient derivative

F_N Normal force, Lift force

l_m Fin mid-chord

n Number of fins

r_t Reference radius at fins position

s Span

V Free-stream velocity

1 Lift force

The calculations of the *Lift Force* - often called *Normal Force* - are made with Barrowman's estimations (Barrowman 1967). These estimations are derived on the basis of the following assumptions:

- The compressibility effect is negligible, i.e. $0.4 \leq M$.
- The rocket has a low Angle of attack (< 10).
- The lift on the rocket body tube is negligible
- The viscous forces can be neglected.
- The nose of the rocket is poised at a point.
- The rocket is relatively narrow in diameter.
- Steady air flow over the rocket without rapid changes.

The normal force is calculated by:

$$F_N = \frac{1}{2} \cdot \rho \cdot C_N \cdot V^2 \cdot A_{ref} \tag{1}$$

The *Normal Force Coefficient* C_N can be linearized at small angles of attack:

$$C_N \approx C_{N\alpha} \cdot \alpha \tag{2}$$

The lift force of the entire rocket can be calculated summing up the normal force coefficients of each of its relevant aerodynamic surfaces.

$$C_{N\alpha(R)} = \sum_{P \in R} C_{N\alpha(P)} \tag{3}$$

Where R represents the rocket and P each of the relevant parts.

2 Fins

According to Barrowman, a single fin the normal force coefficient at subsonic flow can be calculated as:

$$(C_{N_\alpha})_1 = \frac{C_{N_\alpha 0} F_D \left(\frac{A_{\text{fin}}}{A_{\text{ref}}}\right) \cos \Gamma_c}{2 + F_D \sqrt{1 + \frac{4}{F_D^2}}} \tag{4}$$

Where F_D is the Diederich's planform correlation parameter:

$$F_D = \frac{AR}{\frac{1}{2\pi} C_{N_{\alpha 0}} \cos \Gamma_c} \tag{5}$$

$C_{N_\alpha 0}$ represents the normal force coefficient derivative of a two dimensional airfoil, meaning the cross section of the fin. This value varies with the Mach number, thus being difficult to calculate. Simplified methods for both planar fins and any generic airfoil are used as a form of estimation.

3 Trapezoidal fins

For a fin set of 3 or 4 plane trapezoidal fins it is found in Tun et al. 2020 that the derivative of the normal force coefficient C_N in respect to the angle of attack α is given by:

$$C_{N\alpha} = K_{fb} \frac{4n\left(\frac{s}{2r_t}\right)^2}{1+\sqrt{1+\left(\frac{2l_m}{C_r+C_t}\right)^2}} \tag{6}$$

K_{fb} is a coefficient that takes into account an increase in the normal force due to interference effects between the fin and the body and is given by:

$$K_{fb} = 1 + \frac{\frac{2r_t}{2}}{\left(s+\frac{2r_t}{2}\right)} \tag{7}$$

The above calculation of C_N does not account for compressible flow. To extend the range of the aerodynamic coefficient into a compressible flow state the Prandtl-Glauert compressibility correction must be applied (Tun et al. 2020):

$$C'_{N\alpha(f)} = \begin{cases} C_{N\alpha(f)}\left(1-M^2\right)^{-1/2} & (M < 0.8) \\ C_{N\alpha(f)}\left(1-0.8^2\right)^{-1/2} & (0.8 \leq M \leq 1.1) \\ C_{N\alpha(f)}\left(M^2-1\right)^{-1/2} & (M > 1.1) \end{cases} \tag{8}$$

4 Airfoil and other shapes

The calculations involving airfoils revolve around *thin airfoil theory of potential flow corrected for compressible flow.* The material presented at Anderson n.d. gives a lot of the answers we need. It is important to note that even planar cross sections are considered thin airfoils.

Firstly, 3 theoretical results are established in chapter 4 for any thin symmetric airfoil:

1. $C_{N0} = 2\pi\alpha$
2. $C_{N\alpha 0} = 2\pi$
3. The center of pressure and the aerodynamic center are both located at the quarter-chord point.

This information is useful when combined with the *Prandtl-Glauert rule*:

$$C_p = \frac{C_{p,0}}{\beta} \tag{9}$$

Where

$$\beta = \begin{cases} \sqrt{1-M^2}, & M < 0.8 \\ \sqrt{M^2-1}, & M > 1.1 \\ \sqrt{1-0.8^2}, & 0.8 < M < 1.1 \end{cases} \tag{10}$$

This implies that, the compressible pressure distribution over an airfoil can be obtained with the incompressible pressure distribution over the same airfoil and the above equation.

Then, integrating the pressure coefficient over the body we get a valid expression for C_{N_α}:

$$C_{N_\alpha} = \frac{C_{N_\alpha 0}}{\beta} \tag{11}$$

Combining this with thin airfoil theory:

$$C_{N_\alpha} = \frac{2\pi}{\beta} \tag{12}$$

This result lets us finally calculate the lift coefficient. Simplifying:

$$FD = \frac{AR}{\frac{1}{2\pi} C_{N_\alpha 0} \cos \Gamma_c} \tag{13}$$

And substituting:

$$(C_{N_\alpha})_1 = \frac{2\pi \frac{s^2}{A_{\mathrm{ref}}}}{1 + \sqrt{1 + \left(\frac{\beta s^2}{A_{\mathrm{fin}} \cos \Gamma_c}\right)^2}} \tag{14}$$

K_{fb} must also be applied to this formula:

$$(C_{N_\alpha})_1 = K_{fb} \frac{2\pi \frac{s^2}{A_{\mathrm{ref}}}}{1 + \sqrt{1 + \left(\frac{\beta s^2}{A_{\mathrm{fin}} \cos \Gamma_c}\right)^2}} \tag{15}$$

This only applies for subsonic flow.

5 Number of fin correction

Niskanen 2013 goes into the effects of using different numbers of fins. The relevant results and equations used on *RocketPy* are as shown:

$$(C_{N_\alpha})_N = \begin{cases} \frac{N}{2} (C_{N_\alpha})_1 & N = 2, 3, 4 \\ 2.37 (C_{N_\alpha})_1 & N = 5 \\ 2.74 (C_{N_\alpha})_1 & N = 6 \\ 2.99 (C_{N_\alpha})_1 & N = 7 \\ 3.24 (C_{N_\alpha})_1 & N = 8 \end{cases} \tag{16}$$

For values higher than 8 fins the correcting factor used is $\frac{N}{2}$

6 Comments

At the current state of the code only the trapezoidal fin's lift calculations account for supersonic flight. Perhaps adjusting the other aerodynamic surfaces could be important for accuracy. Furthermore, drag coefficient is not currently added to RocketPy. When eventually added, it should account for the fin's airfoil shape.

Tun et al. 2020 shows a method to calculate slender body lift coefficient with Ericsson compressibility correction. This could be useful for when adding supersonic flight.

Niskanen 2013 shows a method to calculate the lift coefficient for generic cross section fins at supersonic speed.

Niskanen 2013 shows a way that the roll angle of the rocket affect the correction factor for the number of fins. This is not implemented.

With the lift calculation on 4, adding generic shaped fins to the code should be easy. All that is left to do is the center of pressure calculation for the possible different shapes.

References

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