Elliptical Fins Equations

Franz Masatoshi Yuri, Kaleb Ramos Wanderley Santos,
Mateus Stano Junqueira, Matheus Gonçalvez Doretto,

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Nomenclature

Y_{MA} Spanwise location of mean aerodynamic chord measured from the root chord

$(C_{N\alpha})_0$ Normal force coefficient derivative of a 2D airfoil

$(C_{N\alpha})_1$ Normal force coefficient derivative of one fin

δ Fin cant angle

ω Angular velocity

$\overline{q}$ Dynamic pressure

ρ Ambient density

ξ Distance to rotation axis

A_r Reference area

C_r Root chord

C_t Tip Chord

$C_{ld\omega}$ Roll moment damping coefficient derivative

C_{ld} Roll moment damping coefficient

$C_{lf\delta}$ Roll moment lift coefficient derivative

C_{lf} Roll moment lift coefficient

F Force

L_r Reference length, rocket diameter

M_d Roll damping moment

M_f	Roll forcing moment
M_{roll}	Roll moment
N	Number of fins
r_t	Reference radius at fins position
s	Span
v_0	Rocket speed in relation to the wind

1 Introduction

In order to calculate the effects of elliptical fins in RocketPy we need to calculate:

- Geometric parameters
- Center of pressure
- Interference Factor

All non proved equations where based on Barrowman 1967.

2 Geometrical Parameters

An elliptical fin can be defined with two basic parameters: the root chord (Cr) and the span (S). Through them, other geometrical properties are calculated, which are then used in the computations for the center of pressure and roll coefficients.

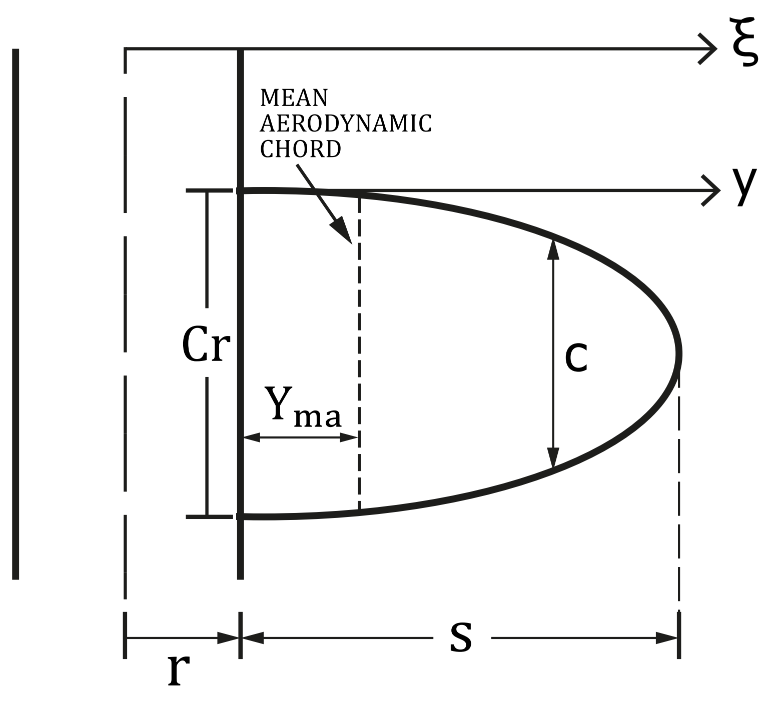

Figure 1: image of an elliptical fin.

2.1 Chord Length (c)

The chord length (c) at a spanwise position must be calculated through two axis: y, that begins at the fuselage wall, and ξ that begins in the fuselages center line (Figure 1).

First we calculate $c(y)$ through the following elliptical equation:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1 \tag{1}$$

Substituting variables for the elliptical fins parameters:

$$\frac{y^2}{S^2} + \frac{\left(\frac{c(y)}{2}\right)^2}{\left(\frac{Cr}{2}\right)^2} = 1 \tag{2}$$

Simplifying:

$$\frac{c(y)}{2} = \frac{Cr}{2}\sqrt{1 - \left(\frac{y}{S}\right)^2} \tag{3}$$

$$c(y) = Cr\sqrt{1 - \left(\frac{y}{S}\right)^2} \tag{4}$$

Transforming to the ξ axis:

$$c(\xi) = Cr\sqrt{1 - \left(\frac{\xi - r}{S}\right)^2} \tag{5}$$

2.2 Spanwise Position of the Mean Aerodynamic Chord (Y_{ma})

We can find the length of the *Mean Aerodynamic Chord* (Y_{ma}) using the known definition (Barrowman 1967):

$$c_{ma} = \frac{1}{A_f}\int_0^s c^2(y)\,dy \tag{6}$$

Where A_f is the area of the fin, in our case $A_f = \frac{\pi\,C_r\,S}{4}$

$$c_{ma} = \frac{4}{\pi\,C_r\,S}\int_0^s \left(Cr\sqrt{1 - \left(\frac{y}{S}\right)^2}\right)^2 dy \tag{7}$$

Solving the integral:

$$c_{ma} = \frac{8C_r}{3\pi} \tag{8}$$

Finally, the span wise position of the mean aerodynamic chord can be found by equating c_{ma} with $c(Y_{ma})$ and solving for Y_{ma}.

$$c_{ma} = c(Y_{ma}) \tag{9}$$

$$\frac{4C_r}{3\pi} = Cr\sqrt{1-\left(\frac{Y_{ma}}{S}\right)^2} \tag{10}$$

$$Y_{ma} = \frac{S}{3\pi}\sqrt{9\pi^2 - 64} \tag{11}$$

2.3 Roll Geometrical Constant (R_{cte})

For the calculation of roll moment induced by a cant angle in the fins, a geometrical constant that takes in regard the fin geometry is used in the computations.

The formula for the constant is as follows:

$$R_{cte} = \int_{r_t}^{s+r_t} c(\xi)\,\xi^2\,d\xi \tag{12}$$

$$R_{cte} = C_r\,S\;\frac{\left(3\pi S^2 + 32 r_t S + 12\pi r_t^2\right)}{48} \tag{13}$$

3 Center of Pressure

The position of center of pressure of a elliptical fin along the center line of a rocket can simply be calculated by the following equation, according to Barrowman 1970 on Model Rocketry:

$$\overline{X_f} = X_f + 0.288 \cdot S \tag{14}$$

4 Roll Damping Interference Factor

According to Barrowman 1967, the roll damping interference factor can be given by:

$$k_{R_D} = 1 + \frac{\displaystyle\int_r^{s+r} r^3\frac{c(\xi)}{\xi^2}\,d\xi}{\displaystyle\int_r^{s+r} \xi\,c(\xi)\,d\xi} \tag{15}$$

Solving the integrals for elliptical fin geometry, using equation (5):

$$k_{R_D} = 1 + \frac{r^2 \cdot \left(2r^2\sqrt{s^2-r^2}\ln\left(\frac{2s\sqrt{s^2-r^2}+2s^2}{r}\right) - 2r^2\sqrt{s^2-r^2}\ln(2s) + 2s^3 - \pi r s^2 - 2r^2 s + \pi r^3\right)}{2s^2\left(\frac{s}{3}+\frac{\pi r}{4}\right)\cdot\left(s^2-r^2\right)} \tag{16}$$

References

Barrowman, James S. (1967). “The practical calculation of the aerodynamic characteristics of slender finned vehicles”. In.

— (1970). “Model Rocketry Nov 1970”. In.