Roll equations for high-powered rockets

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Nomenclature

Y_{MA}	Spanwise location of mean aerodynamic chord measured from the root chord
$(C_{N\alpha})_0$	Normal force coefficient derivative of a 2D airfoil
$(C_{N\alpha})_1$	Normal force coefficient derivative of one fin
δ	Fin cant angle
ω	Angular velocity
$\overline{q}$	Dynamic pressure
ρ	Ambient density
ξ	Distance to rotation axis
A_r	Reference area
C_r	Root chord
C_t	Tip Chord
$C_{ld\omega}$	Roll moment damping coefficient derivative
C_{ld}	Roll moment damping coefficient
$C_{lf\delta}$	Roll moment lift coefficient derivative
C_{lf}	Roll moment lift coefficient
F	Force
L_r	Reference length, rocket diameter
M_d	Roll damping moment
M_f	Roll forcing moment

M_{roll}	Roll moment
N	Number of fins
r_t	Reference radius at fins position
s	Span
v_0	Rocket speed in relation to the wind

1 Introduction

Calculating the rotational movement of a high powered rocket entails calculating the *Roll Moment*. Here all formulas and consideration for the implementation of the Roll Moment in *RocketPy* are shown and explained.

The main cause for a rocket roll movement is certain asymmetries in its construction. The most noteworthy of this possible asymmetries is the fin cant angle δ (Figure 2), which is considered for these calculations.

2 Coefficient derivatives

According to the equation formulated by Barrowman 1967, the rotational moment around the rockets axis is governed by two main forces: one that causes the rolling movement and one that damps the movement. Each of these forces generates its own moment, the forcing moment M_f and the damping moment M_d. The final roll moment can then be calculated with:

$$M_{roll} = M_{forcing} - M_{damping} \tag{1}$$

2.1 Roll Forcing

Roll forcing is the moment that causes the rocket to rotate around its axis. The calculations for this moment assumes $\omega = 0$ e $\delta \neq 0$.

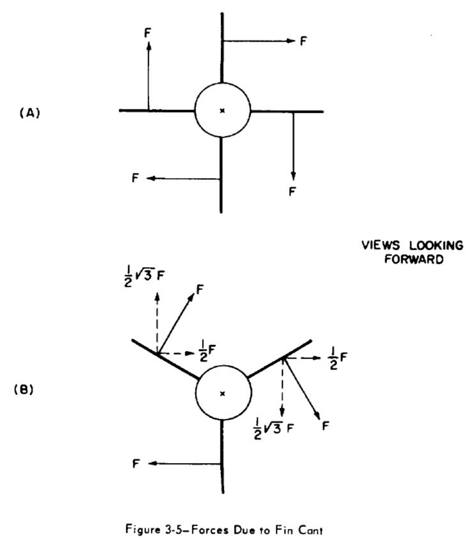

Figure 1: Forces due to fin cant. Source: Barrowman 1967

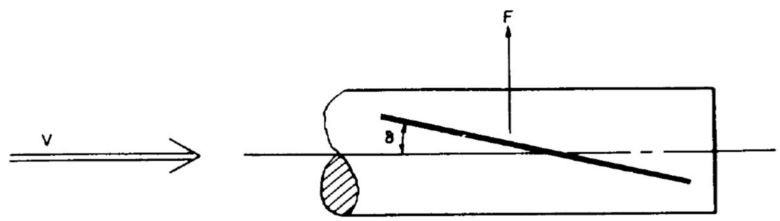

Figure 3-4–Fin Cant Angle

Figure 2: Fin cant angle. Source: Barrowman 1967

Due to the symmetry of the fins - as can be seen in Figure 1 - the forces cancel each other out so that the resulting force F_R is equal to zero. However, the resulting moment $M_R \neq 0$, that is, it constitutes a gyroscope binary.

According to Barrowman 1967, equation (3-31), the roll forcing moment is given by:

$$M_f = N(Y_{MA} + r_t)(C_{N\alpha})_1 \delta \overline{q} A_r \tag{2}$$

The author defines the roll forcing moment coefficient as:

$$C_{lf} = \frac{M_f}{\overline{q} A_r L_r} \tag{3}$$

The letter "*f*" has been added to the name to differentiate *Forcing* from *Damping*. Note the similarity with the definition of drag coefficient ($C_d = \frac{2F_{Drag}}{\rho V^2 A_{ref}}$). Finally, you can also calculate C_{lf} as:

$$C_{lf} = \frac{N(Y_{MA} + r_t)(C_{N\alpha})_1 \delta}{L_r} \tag{4}$$

And its derivative in relation to the cant angle as:

$$C_{lf\delta} = \frac{\partial C_{lf}}{\partial \delta} \tag{5}$$

$$C_{lf\delta} = \frac{N(Y_{MA} + r_t)(C_{N\alpha})_1}{L_r} \tag{6}$$

The forcing moment is then calculated with:

$$M_f = \overline{q}\ A_r L_r\ C_{lf\delta}\ \delta \tag{7}$$

or

$$M_f = \frac{1}{2}\rho v_0^2\ A_r L_r\ C_{lf\delta}\ \delta \tag{8}$$

2.2 Roll Damping

While roll forcing causes the rotation movement, the roll damping force is what counteracts this movement. It is a force that scales with angular velocity and acts in the opposite direction. $\omega \neq 0$ and $\delta = 0$ are assumed.

It is defined in the same way as roll forcing:

$$C_{ld} = \frac{M_d}{\overline{q} A_r L_r} \tag{9}$$

From Barrowman 1967, the roll damping moment is dependent on the angle of attack of the tangential velocity of the fin panel at a certain span wise position ξ (figure 3).

The damping moment at ξ is:

$$dM = \xi F(\xi) \tag{10}$$

Where $F(\xi)$ is the force generated at the span wise position ξ and its given by:

$$F(\xi) = -C_{N_{\alpha 0}}\, \overline{q}\, a(\xi)\, c(\xi)\, d\xi \tag{11}$$

$a(\xi)$ is the local angle of attack at ξ and is given by:

$$a(\xi) = \tan^{-1}(\frac{\omega\, \xi}{v_0}) \tag{12}$$

An approximation that is valid when $v_0 >> \omega\, \xi$ is made

$$a(\xi) = -\frac{\omega\, \xi}{v_0} \tag{13}$$

$c(\xi)$ is the cord length at the span wise ξ and is calculated differently for each fin shape. The damping moment can then be written as:

$$dM = -\frac{C_{N_{\alpha 0}}\, \overline{q}\, \omega}{v_0}\; c(\xi)\, \xi^2\, d\xi \tag{14}$$

From 9 we know:

$$dC_{ld} = \frac{C_{N_{\alpha 0}}\ \omega}{v_0\ A_r\, L_r}\; c(\xi)\, \xi^2\, d\xi \tag{15}$$

Integrating over the exposed fin geometry:

$$C_{ld} = \frac{C_{N_{\alpha 0}}\ \omega}{v_0\ A_r\, L_r}\; \int_{r_t}^{s+r_t} c(\xi)\, \xi^2\, d\xi \tag{16}$$

The initial hypothesis assumes that, for the roll damping calculation, the deflection is $\delta = 0$. This implies a larger cross-sectional area than is actually acting against the movement (analogous to flow passing through a surface). As a result, the term $\cos(\delta)$ was added to the original formulation:

$$C_{ld} = \frac{C_{N_{\alpha 0}}\ \omega}{v_0\ A_r\, L_r}\; \cos(\delta)\, \int_{r_t}^{s+r_t} c(\xi)\, \xi^2\, d\xi \tag{17}$$

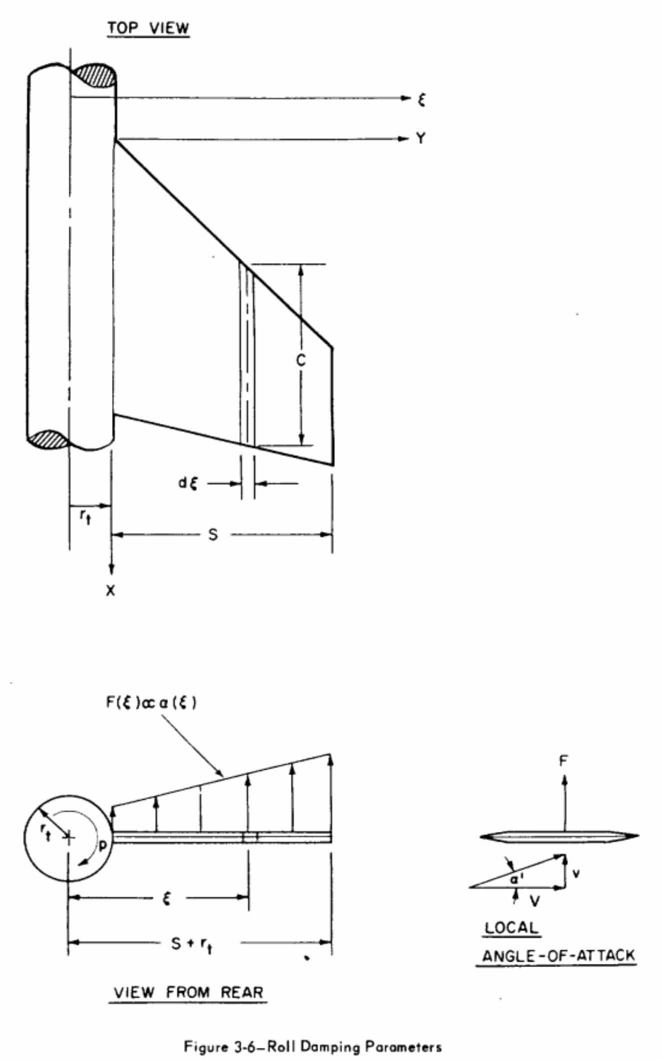

Figure 3: Fins parameters. Source: Barrowman 1967

The roll damping coefficient derivative can then be defined as:

$$C_{ld\omega} = \frac{\partial C_{ld}}{\partial(\frac{\omega L_r}{2v_0})} \tag{18}$$

$$C_{ld\omega} = \frac{2\ C_{N_{\alpha 0}}}{A_r\, L_r^2}\ \cos(\delta) \int_{r_t}^{s+r_t} c(\xi)\, \xi^2\, d\xi \tag{19}$$

Finally, $C_{N_{\alpha 0}}$ must be corrected for three dimensional effects:

$$C_{ld\omega} = \frac{2\ N\ (C_{N_\alpha})_1}{A_r\, L_r^2}\ \cos(\delta) \int_{r_t}^{s+r_t} c(\xi)\, \xi^2\, d\xi \tag{20}$$

The values of the definite integral can be calculated for each specific fin shape. For trapezoidal fins:

$$c(\xi) = C_r \left[1 - \frac{1 - \frac{C_t}{C_r}}{s} (\xi - r_t) \right] \tag{21}$$

$$\int_{r_t}^{s+r_t} c(\xi)\, \xi^2\, d\xi = \frac{s}{12} \left[(C_r + 3C_t)s^2 + 4(C_r + 2C_t)sr_t + 6(C_r + C_t)r_t^2 \right] \tag{22}$$

And for ellipsoidal fins:

$$c(\xi) = C_r \sqrt{1 - \left(\frac{\xi - r_t}{s} \right)^2} \tag{23}$$

$$\int_{r_t}^{s+r_t} c(\xi)\, \xi^2\, d\xi = C_r\, s\ \frac{\left(3\pi s^2 + 32 r_t s + 12\pi r_t^2\right)}{48} \tag{24}$$

The damping moment is finally:

$$M_d = \frac{1}{2} \rho v_0^2\ A_{ref}\, L_{ref}\ C_{ld\omega}\, \frac{\omega L_{ref}}{2 v_0} \tag{25}$$

3 Interference Coefficients

In Barrowman 1967 some fin-body interference factor are calculated. These factors are also implemented in the lift coefficient calculations.

For fins with canted angle:

$$\begin{aligned} K_f = \frac{1}{\pi^2} \Bigg[& \frac{\pi^2}{4} \frac{(\tau+1)^2}{\tau^2} + \frac{\pi \left(\tau^2+1\right)^2}{\tau^2(\tau-1)^2} \sin^{-1}\left(\frac{\tau^2-1}{\tau^2+1}\right) - \frac{2\pi(\tau+1)}{\tau(\tau-1)} \\ & + \frac{\left(\tau^2+1\right)^2}{\tau^2(\tau-1)^2} \left(\sin^{-1} \frac{\tau^2-1}{\tau^2+1}\right)^2 - \frac{4(\tau+1)}{\tau(\tau-1)} \sin^{-1}\left(\frac{\tau^2-1}{\tau^2+1}\right) \\ & + \frac{8}{(\tau-1)^2} \ln\left(\frac{\tau^2+1}{2\tau}\right) \Bigg] \end{aligned} \tag{26}$$

For the damping moment lift coefficient derivative:

$$K_d = 1 + \frac{\frac{\tau-\lambda}{\tau} - \frac{1-\lambda}{\tau-1} \ln \tau}{\frac{(\tau+1)(\tau-\lambda)}{2} - \frac{(1-\lambda)(\tau^3-1)}{3(\tau-1)}} \tag{27}$$

Where $\tau = \frac{s+r_t}{r_t}$ and $\lambda = \frac{C_t}{C_r}$.

The final lift coefficients are:

$$(C_{lf\delta})_{K\,f} = K_f \; C_{lf\delta} \tag{28}$$

$$(C_{ld\omega})_{K\,d} = K_d \; C_{ld\omega} \tag{29}$$

4 Comments

Roll moment is expected to increase linearly with velocity. This relationship can be verified in the rotation frequency equilibrium equation, described by Niskanen 2013 in equation (3.73), and again stated below:

$$f_{eq} = \frac{\omega}{2\pi} = \frac{A_{ref}\beta\overline{Y_t}(C_{N\alpha})_1}{4\pi^2 \sum_i c_i \xi^2 \Delta\xi} \, \delta V_0 \tag{30}$$

The auxiliary value β is defined as: $\beta = \sqrt{|1 - M|}$, where M is the speed of the rocket in Mach.

References

Barrowman, James S. (1967). “The practical calculation of the aerodynamic characteristics of slender finned vehicles”. In.

Niskanen, S. (2013). “OpenRocket technical documentation”. In: *Development of an Open Source model rocket simulation software.*