Modified Projected Landweber Method for Compressive-Sensing Reconstruction of Images with Non-Orthogonal Matrices

Vikramkumar Pudi, Anupam Chattopadhyay and Thambipillai Srikanthan
School of Computer Science & Engineering, Nanyang Technological University, Singapore

***Abstract*—Compressive Sensing (CS) reconstruction of images using the smoothed projected Landweber (SPL) method is known to achieve excellent performance. However, the complexity of SPL increases for a non-orthogonal measurement matrix. In this paper, we propose a variant of SPL for non-orthogonal matrices without increasing the complexity. In addition, we propose a uniform quantization method for compression of images so as to reduce the communication cost. The performance of the proposed method is comparable to the existing methods for lower measurement rates. This method works particularly well with non-orthogonal random matrices composed of only -1s and +1s.**

***Index Terms*—Compressive Sensing, Measurement Matrix, Projected Landweber, Quantization, Non-orthogonal matrices**

I. Introduction

Compressive Sensing (CS) is an emerging technique and it has attracted considerable interest in signal processing and wireless communication networks during the last decade. In compressive sensing, we recover a signal from far fewer samples than what is required by the classical Shannon Nyquist sampling theorem [2]. As a result, the compressive sensing technique reduces the communication cost and requires less resources for implementation. Compressive sensing uses a measurement matrix for compression of data. A random matrix made up of only +1 and -1 (entries) is preferred for the measurement matrix in resource constrained devices [3], [4], [13], [9].

Directly applying compressive sensing to 2D images increases the computational complexity of the reconstruction process in addition to increasing the memory required to store the random matrix operator. In [6], [10], the authors addressed this challenge using the block-based compressive sensing operation. Additionally in [6], [10], the authors proposed a smoothing operator along with iterative projected Landweber method for improving the quality of images by eliminating blocking artifacts.

In this paper, we adopt the block-based smoothed projected Landweber (BSC-SPL) method for compressive sensing-based reconstruction. Our contribution lies in the design of a less complex BSC-SPL method for non-orthogonal matrices compared to the one reported in [6], [10]. This is achieved by combining the compressive sensing reconstruction methods in [6] and [7]. The quality of the output is about the same as the existing methods [10] with lower measurement rates. It is worth noting that in real time applications, lower measurement rates are preferred. In addition, we propose uniform quantization values for the compression stage. This quantization does not require any additional circuit, because it removes the selected LSB bits and transmits only the remaining bits. The quantization is achieved by optimizing the reconstruction, but it is only applicable to the random measurement matrices composed of +1 and -1 values. We can further reduce the communication cost using the quantizations methods in [11], [8], [14]. We have compared our method with existing methods in [10] and the quality of our reconstructed images is comparable to [10]. The key advantages of our method are reduction of communication cost using uniform quantization and less computational complexity for reconstruction of images with non-orthogonal matrices.

II. Background

A. Compressive Sensing

Compressive sensing is a method of directly acquiring the signal at sub-Nyquist rate from sensor provided signal a $sparse$-signal. Compressive sensing acquires an M-samples signal Y from an N-samples X using the linear projection or measurement matrix Φ of size $M \times N$ provided $M << N$. This process is known as compression or CS-Encoding. The CS encoding equation is shown in Eq. (1). where X is a sparse signal.

$$Y_{M\times 1} = \Phi_{M\times N} X_{N\times 1} \qquad \text{where M} \ll \text{N} \tag{1}$$

We can recover the estimation of the original sparse signal X from Y using the the Basis-Pursuit(BP) or l_1-optimization method. l_1-optimization defined as

$$X = arg\ min \parallel X \parallel_1 subjected\ to\ Y = \Phi X \tag{2}$$

In general, signals generated by the sensors are not sparse signals. We can represent the sensors signals as sparse signals using a particular basis like Discrete Wavelet, Discrete Cosine etc. known as Ψ. If signal X is sparse in some basis Ψ, then we can directly compute Y using the Eq. (1) without transforming X. The signal recovery method in Eq. (2) is changed to Eq. (3)

$$X = arg\ min \parallel X \parallel_1 subjected\ to\ Y = \Phi\Psi X \tag{3}$$

where X is the original signal of size $N \times 1$, Φ is the CS matrix or measurement matrix (MM) of size $M \times N$, Ψ is the compression matrix (CM) of size $N \times N$, and Y is the compressed signal of size $M \times 1$ and $M << N$.

978-1-4673-9019-4/16/$31.00 ©2016 IEEE

B. Block-based Compressive Sensing

The CS encoding and reconstruction presented in Eq. (1) and (3) are used for compressing and recovering 1-D signals. Therefore we have to convert 2-D signals like images to 1-D signal before applying CS. Recovering 2-D signals using BP in Eq. (3) is computationally expensive because of large-size N. In [6], the authors proposed a method for compressing images using the CS. Instead of applying CS on the whole image, splitting the image into several blocks of size $B \times B$ and applying CS on each block, we can recover each block using the method in Eq. (3). This method of compressing and reconstructing signals is called as Block-based Compressive Sensing (BCS). BCS compression of images is written in Eq. (4).

$$y_j = \Phi_B x_j \tag{4}$$

where y_j is the compressed block of the x_j^{th} block in image X and Φ_B is a random matrix of size $M \times B^2$. We define measurement rate as shown in Eq. (5).

$$Measurment\ Rate = \frac{M}{N} \qquad \text{Where } N = B^2 \tag{5}$$

C. Measurement Matrices

CS mainly depends on the selection of good measurement matrix Φ for exact recovery of X from Y under the condition $M << N$. In [2], authors introduced the restricted isometry property (RIP) as a sufficient condition on Φ matrices which can recover the best performance signal. The measurement matrices which satisfy the RIP are Gaussian random matrices, Partial Fourier matrices and Bernoulli random matrices [16]. Bernoulli random matrices require very less resources to store values and we can implement CS encoding in Eq. (4) using only adders. We can write Bernoulli random matrices as

$$\Phi_B(i,j) = \begin{cases} +1 & \text{with probability 1/2} \\ -1 & \text{with probability 1/2} \end{cases} \tag{6}$$

D. CS Reconstruction

The CS reconstruction presented in Eq. (3) can be implemented using the BP. Because of high computational complexities involved in BP, other less complex methods are preferred like orthogonal matching pursuit (OMP), gradient projection for sparse reconstruction (GPSR) [12] and sparsity adaptive matching pursuits (SAMP) [5]. Recently, projected Landweber (PL) based method for CS image reconstruction was presented in [6], [10]. In [6], [10], the authors incorporated Wiener filter into PL method at each iteration to remove the blocking artifacts. The overall method is known as block based compressive sensing with smoothed projected Landweber (BCS-SPL). Most reconstruction methods consider measurement matrix Φ as orthogonal matrix to reduce the complexity. The non-orthogonal based BCS-SPL method require additional $O(M^2)$ multiplications and additions for each block at each iteration. The BCS-SPL method starts at initial step $x_j^0 = \Phi_B y_j$ and iteratively calculates image X^i until it converges. In this paper, we are not presenting the convergence details (see [10]). The BCS-SPL method of non-orthogonal matrices at i^{th} iteration is given in Table-I. Here, X^i and x_j^i are the i^{th} iteration image and j^{th}-block. More details on Wiener( ) and Threshold( ) operators are presented in [10]. The $(\Phi_B * \Phi'_B)^{-1}$ becomes identity matrix, if Φ_B is orthogonal matrix otherwise BCS-SPL algorithm requires additional $2*M^2$ multiplications and $M*(N-1)$ additions for each block. We can pre-compute the $\Phi'_B(\Phi_B * \Phi'_B)^{-1}$ matrix and store in memory for reduction of additional multiplications but CS method uses adders and subtractors based Φ-matrix to reduce resources at CS compressions stage [3] . In such a case, additions based $\tilde{x}_j^i$ and x_j^i-terms require multiplications because of $(\Phi_B * \Phi'_B)^{-1}$ matrix.

TABLE I: Non-Orthogonal matrix based BSC-SPL Algorithm

$$\begin{aligned}
x_i^0 &= \Phi'_B y_j && (7)\\
X^{i-1} &= Wiener(X^{i-1}) && (8)\\
\tilde{x}_j^i &= x_j^{i-1} + \Phi'_B(\Phi_B\Phi'_B)^{-1}(y_j - \Phi_B x_j^{i-1}) && (9)\\
\breve{x}_j^i &= \Psi x_j^{i}\ \tilde{} && (10)\\
\breve{x}_j^i &= Threshold(\breve{x}_j^i, \lambda) && (11)\\
\hat{x}_j^i &= \Psi^{-1}\breve{x}_j^i && (12)\\
x_j^i &= \hat{x}_j^i + \Phi'_B(\Phi_B\Phi'_B)^{-1}(y_j - \Phi_B \hat{x}_j^i) && (13)
\end{aligned}$$

In [7], authors proposed a form of PL method for CS signal reconstruction without Wiener filter. This algorithm successively iterates and thresholds the signals. This algorithm is similar to the BCS-SPL, but requires less computations for non-orthogonal matrices. The CS reconstruction method in [7] is given as

$$\hat{x}^i = x^{i-1} + \frac{1}{\gamma}\Psi^{-1}\Phi'_B(y - \Phi_B \Psi x^{i-1}) \tag{14}$$

$$x^i = Threshold(\hat{x}_j^i, \lambda) \tag{15}$$

where γ is scaling factor and largest eigen value of $\Phi'_B\Phi_B$. This method requires constant multiplication ($1/\gamma$) for non-orthogonal matrix based CS reconstruction but it lacks the smoothing operator $Wiener$. In the next section, we propose a new and improved CS reconstruction algorithm for non-orthogonal matrices by combining the two methods presented in [10] and [7].

III. Block-Based CS Image Reconstruction with Modified PL

CS reconstruction method in Table-I increases complexity for non-orthogonal matrices by introducing $(\Phi'_B\Phi_B)^{-1}$ matrix. We can replace the $(\Phi'_B\Phi_B)^{-1}$-matrix in Eq. (9) and (13) with $\frac{1}{\gamma}$ term presented in Eq. (14). The modified equations of $\tilde{x}_j^i$ and x_j^i terms in Eq. (9) and (13) are written as

$$\tilde{x}_j^i = x_j^{i-1} + \frac{1}{\gamma}\Phi'_B(y_j - \Phi_B x_j^{i-1}) \tag{16}$$

$$x_j^i = \hat{x}_j^i + \frac{1}{\gamma}\Phi'_B(y_j - \Phi_B \hat{x}_j^i) \tag{17}$$

We can use Eqs. (16) and (17) for computing the $\tilde{x}_j^i$ and x_j^i terms in CS reconstruction presented in Table-I. The proposed CS reconstruction requires additional $2N$ constant multipliers ($\frac{1}{\gamma}$) in place of $2 * M^2$ multiplications and $M * (N-1)$ additions. We can convert constant multiplications into shifting operations by replacing γ value as closest powers of two and referred as γ'. The calculation of γ' value defining in Eq. (18).

$$\gamma' = 2^{\lfloor \log_2 \gamma \rfloor} \tag{18}$$

We can replace γ in Eqs. (16) and (17) with γ'. The computations of proposed $\tilde{x}_j^i$ and x_j^i terms requires only additions with constant shifts, provided Φ_B-matrix with **+1** and **-1** values. This proposed method is less complex than the methods presented in [7],[10].

The CS method was introduced for fast compression and reduction of communication cost. However, CS compression in Eq. (1) increases the bit size of Y compared to the input X, this increases the communication cost. We are proposing the uniform quantization method by transferring the part of scaling factor γ' to the CS compression stage. We are defining the γ' as a product of g and q shown in Eq. (19). We will use q as uniform quantization factor, the calculation of g and q are shown in Eq. (20) and (21) respectively.

$$\begin{aligned} \gamma' &= g * q &(19)\\ g &= 2^{\lfloor \log_2 \sqrt{\gamma} \rfloor} &(20)\\ q &= 2^{\lceil \log_2 \sqrt{\gamma} \rceil} &(21) \end{aligned}$$

The CS compression equation using uniform quantization term q and Φ_B shown in Eq. (22). The q in Eq. (22) does not require any computations, it will remove $\lceil \log_2 \sqrt{\gamma} \rceil$ number of LSB bits in each value of $\Phi_B x_j$ vector. We have empirically studied the g and q values for many Φ_B matrices and surprisingly the g and q values are constant for given block size. The table-II shows the g and q values for different block sizes of images. Through empirical studies, we found that 10-bit long values of y_js are enough for reconstruction of images with better quality. We can further reduce the transmission bit sizes by using method (DPCM) in [11].

$$y_j = \frac{1}{q} \Phi_B x_j \tag{22}$$

TABLE II: g,q-values and size of $y_j(k)$ for different block sizes of images;where $y_j(k)$ represents the k^{th} element of y_j

Block-Size	g	q	Minimum size of $y_j(k)$
8×8	8	16	10-bits
16×16	16	32	10-bits
32×32	32	64	10-bits

The $\tilde{x}_j^i$ term in Eq. (16) is modified as shown in Eq. (23). The $\Phi'_B y_j$ term divides by only g value, because the y_j term is already divided by q value as shown in Eq. (22).

$$\tilde{x}_j^i = x_j^{i-1} + \frac{1}{g} \Phi'_B (y_j - \frac{1}{q} \Phi_B x_j^{i-1}) \tag{23}$$

The proposed CS compression and reconstruction method is presented in Algorithm-1. The X^i in CS reconstruction represents the i^{th} iteration image formed by merging the all x_j^i blocks of image. The CS reconstruction process is iterative and continues until it converges. The detail of the converges and the Threshold() operator are presented in [10]. The main advantages of our proposed method are reduction of the communication cost with-out any additional circuit and the uses of only additions with fixed shifts for computing $\tilde{x}_j^i$ and x_j^i in Algorithm-1. The proposed method is useful for resource constrained devices and less complex than the existing methods in [7], [10].

Algorithm-1: Proposed Block-Based Compressive Sensing method for non-orthogonal matrices

CS Compression

$$y_j = \frac{1}{q} \Phi_B x_j$$

CS Reconstruction

$$\begin{aligned} &x_j^0 = \Phi'_B y_j\\ &X^{i-1} = Wiener(X^{i-1})\\ &\tilde{x}_j^i = x_j^{i-1} + \frac{1}{g} \Phi'_B (y_j - \frac{1}{q} \Phi_B x_j^{i-1})\\ &\check{x}_j^i = \Psi \tilde{x}_j^i\\ &\breve{x}_j^i = Threshold(\check{x}_j^i, \lambda)\\ &\hat{x}_j^i = \Psi^{-1} \breve{x}_j^i\\ &x_j^i = \hat{x}_j^i + \frac{1}{g} \Phi'_B (y_j - \frac{1}{q} \Phi_B \hat{x}_j^i) \end{aligned}$$

Remark 1. *The proposed CS compression method in Eq. (22) is directly applicable to the CS reconstruction for non-orthogonal matrices presented in Table-I with little modification to y_j term. The y_j-term has to be multiplied with uniform quantization value q, i.e $y_j = q * y_j$, before starting the reconstruction of the image using Eq. (22). This way we can reduce the communication cost and reconstruct the images with same quality as in [7] and [10].*

IV. Experimental Results

We have implemented our proposed BCS-SPL method by selecting two different compression matrices Ψ namely DCT and DWT. We refer to the proposed methods with DCT and DWT as Prop-BCS-SPL-DCT and Prop-BCS-SPL-DWT respectively. Fig. 1 shows the reconstructed images with different measurement rates using 10-bit long $y_j(k)$s and block size of 32×32. The performance of our method is better for higher block sizes of the image.

Table-III compares the peak signal-to-noise ratio (PSNR) in dB for different 512×512 images at different measurement rates. We compared our results with existing works presented in [1], [10], [15]. The quality of reconstruction images varies with measurement-matrix Φ_B and hence, we have considered averaged PSNR values over 5 independent trials. The results indicate that the proposed method performs better when the

Fig. 1: Reconstructed images with $\frac{M}{N} = 0.1, 0.2, 0.3, 0.4\ and\ 0.5$ from left-to-right using proposed BCS-SPL-DCT

TABLE III: Comparison of PSNR (dB) values of different images with different measurement rates

Algorithm	Measurement Rate (M/N)				
	0.1	0.2	0.3	0.4	0.5
Lena					
Prop-BCS-SPL-DCT	26.50	29.51	31.17	32.50	33.55
Prop-BCS-SPL-DWT	26.81	29.65	31.40	32.80	33.84
BCS-SPL-DCT [10]	27.70	30.45	32.46	34.19	35.77
BCS-SPL-DWT [10]	27.81	30.89	32.94	34.61	36.15
SAMP [10]	25.94	28.54	32.04	33.93	35.37
GPSR [10]	24.69	28.54	31.53	33.69	35.82
LCS [15]	—	27.63	30.73	31.78	32.54
BCS [1]	28.21	31.03	33.11	—	—
Barbara					
Prop-BCS-SPL-DCT	21.67	23.82	24.95	25.90	26.93
Prop-BCS-SPL-DWT	22.19	23.42	24.33	25.23	26.11
BCS-SPL-DCT [10]	22.76	24.38	25.91	27.42	29.05
BCS-SPL-DWT [10]	22.62	23.94	25.20	26.56	28.05
SAMP [10]	20.97	22.83	25.04	27.68	30.08
GPSR [10]	20.23	22.66	24.99	27.42	30.15
Goldhill					
Prop-BCS-SPL-DCT	25.83	27.66	28.85	29.83	30.37
Prop-BCS-SPL-DWT	26.15	27.94	29.18	30.11	30.96
BCS-SPL-DCT [10]	26.10	28.32	29.63	30.98	32.57
BCS-SPL-DWT [10]	26.71	28.68	30.13	31.53	32.85
SAMP [10]	24.31	26.30	28.07	29.45	30.86
GPSR [10]	23.63	26.14	28.09	30.02	31.72

measurement rate is less than 0.5. The performance of our methods is better compared to the SAMP and GPSR methods. The PSNR values of proposed BCS-SPL-DCT and BCS-SPL-DWT methods are less than the methods in [10]. Our PSNR values differ from -0.5 dB to -2 dB compared to the the PSNR values in [10]. The higher the measurement rate, the larger is the difference in PSNR values. The performance of our method is almost the same as the methods in [10]. For better performance, we suggest to use the measurement rate between 0.2 to 0.4.

V. Conclusions

In this paper, we have presented a modified projected Landweber method to reconstruct the images using non-orthogonal matrices with less complexity. In addition to this, we have adopted a uniform quantization method for different block sizes of images. This method works very well with non-orthogonal random matrices (which are composed of -1s and +1s). The Prop-BCS-SPL provides almost the same gain as in [10] for lower measurement rates. Overall, the proposed algorithm reduces the communication cost by using uniform quantization after the compression and is less complex to reconstruct the images.

References

[1] A. Adler, D. Boublil, M. Elad, and M. Zibulevsky, "A deep learning approach to block-based compressed sensing of images," *arXiv preprint arXiv:1606.01519*, 2016.

[2] E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," *IEEE signal processing magazine*, vol. 25, no. 2, pp. 21–30, 2008.

[3] F. Chen, A. P. Chandrakasan, and V. M. Stojanovic, "Design and analysis of a hardware-efficient compressed sensing architecture for data compression in wireless sensors," *IEEE Journal of Solid-State Circuits*, vol. 47, no. 3, pp. 744–756, 2012.

[4] M. Dadkhah, M. J. Deen, and S. Shirani, "Block-based CS in a CMOS image sensor," *IEEE Sensors Journal*, vol. 14, no. 8, pp. 2897–2909, 2014.

[5] T. Do, L. Gan, N. Nguyen, and T. Tran, "Sparsity adaptive matching pursuit algorithm for practical compressed sensing," in *2008 42nd Asilomar Conference on Signals, Systems and Computers*.

[6] L. Gan, "Block compressed sensing of natural images," in *2007 15th International conference on digital signal processing*. IEEE, 2007, pp. 403–406.

[7] J. Haupt and R. Nowak, "Signal reconstruction from noisy random projections," *IEEE Transactions on Information Theory*, vol. 52, no. 9, pp. 4036–4048, 2006.

[8] L. Jacques, D. K. Hammond, and J. M. Fadili, "Dequantizing compressed sensing: When oversampling and non-gaussian constraints combine," *IEEE Transactions on Information Theory*, vol. 57, no. 1, pp. 559–571, 2011.

[9] L. Jacques, P. Vandergheynst, A. Bibet, V. Majidzadeh, A. Schmid, and Y. Leblebici, "CMOS compressed imaging by random convolution," in *2009 IEEE International Conference on Acoustics, Speech and Signal Processing*. IEEE, 2009, pp. 1113–1116.

[10] S. Mun and J. E. Fowler, "Block compressed sensing of images using directional transforms," in *2009 16th IEEE international conference on image processing (ICIP)*. IEEE, 2009, pp. 3021–3024.

[11] ——, "DPCM for quantized block-based compressed sensing of images." in *Signal Processing Conference (EUSIPCO), 2012 Proceedings of the 20th European*. IEEE, 2012, pp. 1424–1428.

[12] R. D. Nowak, S. J. Wright *et al.*, "Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems," *IEEE Journal of selected topics in signal processing*, vol. 1, no. 4, pp. 586–597, 2007.

[13] Y. Oike and A. El Gamal, "CMOS image sensor with per-column $\sigma\delta$ adc and programmable compressed sensing," *IEEE Journal of Solid-State Circuits*, vol. 48, no. 1, pp. 318–328, 2013.

[14] L. Wang, X. Wu, and G. Shi, "Binned progressive quantization for compressive sensing," *IEEE Transactions on Image Processing*, vol. 21, no. 6, pp. 2980–2990, 2012.

[15] S. Yang, M. Wang, L. Jin, S. Wang, F. Liu, and L. Jiao, "Learning compressive sampling via multiscale and steerable support value transform," *Knowledge-Based Systems*, vol. 82, pp. 128–138, 2015.

[16] G. Zhang, S. Jiao, X. Xu, and L. Wang, "Compressed sensing and reconstruction with bernoulli matrices," in *Information and Automation (ICIA), 2010 IEEE International Conference on*. IEEE, 2010, pp. 455–460.