GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 6250 Fall 2018
Problem Set #2

Assigned: 29-Aug-18
Due Date: 5-Sep-18

As stated in the syllabus, unauthorized use of previous semester course materials is strictly prohibited in this course.

All assignments will be due at the begining of class on the due date.

PROBLEM 2.1:
Using your class notes, prepare a 1-2 paragraph summary of what we talked about in class in the last week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better.

PROBLEM 2.2:
Consider the vector space $\mathbb{R}^{2\times 2}$ and the four vectors in this space:

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{v}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

1. Show that $\text{span}(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4) = \mathbb{R}^{2\times 2}$.
2. Show that $(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4)$ are linearly independent in $\mathbb{R}^{2\times 2}$.
3. Show that $(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4)$ is a basis for $\mathbb{R}^{2\times 2}$.
4. Find the dimension of $\mathbb{R}^{2\times 2}$.

PROBLEM 2.3:
Let $\mathcal{C} = \mathcal{C}[-1,1]$ be the vector space of all continuous real valued functions on the interval $[-1,1]$. A function f in $\mathcal{C}$ is even if $f(-x) = f(x)$ for all $x \in [-1,1]$; it is odd if $f(-x) = -f(x)$ for all $x \in [-1,1]$. Let $\mathcal{C}_o = \{f \in C : f \text{ is odd}\}$ and $\mathcal{C}_e = \{f \in C : f \text{ is even}\}$.

1. Prove that $\mathcal{C}_o$ and $\mathcal{C}_e$ are both subspaces of $\mathcal{C}$.
2. Prove that $\mathcal{C}_o$ and $\mathcal{C}_e$ are *orthogonal* subspaces, meaning that any $f \in \mathcal{C}_o$ and $g \in \mathcal{C}_e$ must satisfy
$$\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx = 0.$$

Last updated 13:13, August 30, 2018

PROBLEM 2.4:
Let V be the set of all real numbers x such that $x > 0$.

1. Define an operation of "addition" by

$$x \boxplus y = xy + 1 \qquad \text{for all } x, y \in V.$$

   Define an operation of "scalar multiplication" by

$$a \boxdot x = a^2 x \qquad \text{for all } x \in V \text{ and } a \in \mathbb{R}.$$

   Under the operations $\boxdot$ and $\boxtimes$ is the set V is a vector space? Support your answer; if you determine that this is not a vector space, list all axioms of vector spaces which fail to hold.

2. Define an operation of "addition" by

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   Define an operation of "scalar multiplication" by

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   Under the operations $\boxdot$ and $\boxtimes$ is the set V is a vector space? Support your answer; if you determine that this is not a vector space, list all axioms of vector spaces which fail to hold.

PROBLEM 2.5:
In a normed vector space, the triangle inequality holds

$$\|\boldsymbol{x} + \boldsymbol{y}\| \ \leq \ \|\boldsymbol{x}\| - \|\boldsymbol{y}\|.$$

Prove the "reverse triangle inequality" also holds in a normed vector space

$$\|\boldsymbol{x}\| - \|\boldsymbol{y}\| \ \leq \ \|\boldsymbol{x} - \boldsymbol{y}\|.$$

What can we say about $|\ \|\boldsymbol{x}\| - \|\boldsymbol{y}\|\ |$ in relation to $\|\boldsymbol{x} - \boldsymbol{y}\|$?

PROBLEM 2.6:
Let $\boldsymbol{A}$ be the 2×2 matrix

$$\boldsymbol{A} = \begin{bmatrix} 2 & -1/2 \\ -1/2 & 2 \end{bmatrix}.$$

For $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^2$, define $\langle \boldsymbol{x}, \boldsymbol{y} \rangle_A = \boldsymbol{y}^T \boldsymbol{A} \boldsymbol{x}$.

1. Show that $\langle \cdot, \cdot \rangle_A$ is indeed a valid inner product.

2. Sketch the *unit ball* $B_A = \{\boldsymbol{x} : \|\boldsymbol{x}\|_A \leq 1\}$ corresponding to the induced norm defined by $\|\boldsymbol{x}\|_A = \sqrt{\langle \boldsymbol{x}, \boldsymbol{x} \rangle_A}$. Feel free to use MATLAB.