GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Quiz #2

Date: November 3, 2017 Course: ECE 6250

Name: ______________________________
Last, First

- Closed book, closed notes, two $8\frac{1}{2}'' \times 11''$ handwritten sheets are allowed. Fifty (50) minute time limit.
- No calculators. None of the problems require involved calculations.
- There are 4 questions. Each question will indicate how many points it is worth. In multipart questions, each subproblem is equally weighted.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.
- This quiz will be conducted under the rules and guidelines of the Georgia Tech Honor Code and no cheating will be tolerated.
- **Write your final answers in the boxes provided.**

1. Short answer (30 points – 5 points each): Concisely answer the following conceptual questions with **at most** 1–2 short sentences, equations, or a picture.

   (a) Different Cosine bases are derived from implicit periodic *extensions* of the signal $x(t)$. Describe (or sketch) the extension used for the Cosine-I basis. Why might the Cosine-I basis be superior to the Fourier series?

   Answer:

   (b) What advantage does the lapped orthogonal transform (LOT) have compared to taking the standard DCT of non-overlapping blocks?

   Answer:

   (c) The discrete wavelet transform can be computed using a filterbank. Why should we care? (I.e., why is this useful?)

   Answer:

(d) Some wavelets are designed to have *vanishing moments*. What does this mean, and why is it advantageous?

Answer:

(e) Given a matrix $\boldsymbol{A} \in \mathbb{R}^{N \times N}$ and its eigen-decomposition $\boldsymbol{A} = \sum_{i=1}^{N} \lambda_i \boldsymbol{v}_i \boldsymbol{v}_i^T$, write down an expression for its inverse in terms of $\{\lambda_i\}$ and $\{\boldsymbol{v}_i\}$.

Answer:

(f) Find the eigenvectors and corresponding eigenvalues of the symmetric matrix

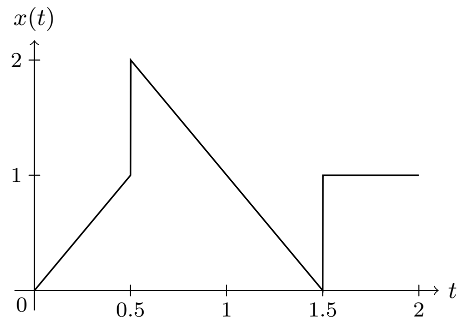
$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

That is, find orthonormal $\boldsymbol{V}$ and diagonal $\boldsymbol{\Lambda}$ such that $\boldsymbol{A} = \boldsymbol{V} \boldsymbol{\Lambda} \boldsymbol{V}^{\mathrm{T}}$.

$\boldsymbol{V} =$	$\boldsymbol{\Lambda} =$

2. (20 points – 10 points each) Consider the function $x(t)$ on $[0,2]$ by

$$x(t) = \begin{cases} 2t & 0 \le t \le \frac{1}{2} \\ 2 - 2\left(t - \frac{1}{2}\right) & \frac{1}{2} < t \le \frac{3}{2} \\ 1 & \frac{3}{2} < t \le 2 \end{cases}$$



(a) Find the Haar scaling coefficients at level 2, i.e., compute $s_{2,n} = \langle x, \phi_{2,n} \rangle$ for $n = 0, \ldots, 7$. Recall that $\phi_{j,n}(t) = 2^{j/2}\phi_0(2^j t - n)$, where the Haar scaling function is

$$\phi_0(t) = \begin{cases} 1 & 0 \le t \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{bmatrix} s_{2,0} \\ s_{2,1} \\ s_{2,2} \\ s_{2,3} \\ s_{2,4} \\ s_{2,5} \\ s_{2,6} \\ s_{2,7} \end{bmatrix} =$$

(b) Find the Haar scaling coefficients at level 0, along with the wavelet coefficients at levels 0 and 1. Recall that $w_{j,n} = \langle x, \psi_{j,n} \rangle$ where $\psi_{j,n}(t) = 2^{j/2}\psi_0(2^j t - n)$, where the Haar wavelet function is

$$\psi_0(t) = \begin{cases} 1 & 0 \leq t \leq \frac{1}{2}, \\ -1 & \frac{1}{2} \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

[Hint: You can solve this problem using your answer to part (a).]

$$\begin{bmatrix} s_{0,0} \\ s_{0,1} \end{bmatrix} = \qquad \begin{bmatrix} w_{0,0} \\ w_{0,1} \end{bmatrix} = \qquad \begin{bmatrix} w_{1,0} \\ w_{1,1} \\ w_{1,2} \\ w_{1,3} \end{bmatrix} =$$

3. (10 points) Suppose that we observe a pair of points in the xy-plane, $(x_i, y_i), i = 1, 2$. We want to fit a line that passes through these two data points, i.e., a slope α and intercept β such that $y_i = \alpha x_i + \beta$. Show how to set up this problem as a linear inverse problem. That is, determine a vector $\boldsymbol{y}$ and a matrix $\boldsymbol{A}$ so that the solution to

$$\boldsymbol{A}\boldsymbol{z} = \boldsymbol{y}$$

gives a solution $\boldsymbol{z} = [\alpha, \beta]^T$ corresponding to the line that interpolates these points.

$\boldsymbol{y} =$	$\boldsymbol{A} =$

4. (10 points) Consider the function

$$\varphi(t) = \begin{cases} 1, & -1 \leq t \leq 1 \\ 0, & |t| > 1, \end{cases}$$

and set $\mathcal{V} = \overline{\text{Span}}\left(\{\varphi(t-n)\}_{n\in\mathbb{Z}}\right)$. Is $\{\varphi(t-n)\}_{n\in\mathbb{Z}}$ a Riesz basis for $\mathcal{V}$?

Circle one: **YES** or **NO**

Justification:

[HINT: There are multiple approaches to this problem. One approach is to consider the signal given by

$$x_N(t) = \sum_{n=0}^{N} (-1)^n \varphi(t-n),$$

where $N > 0$ is fixed. Recall that by definition, for $\{\varphi(t-n)\}_{n\in\mathbb{Z}}$ a Riesz basis for $\mathcal{V}$, there must be constants $A, B > 0$ such that

$$A \sum_{n=1}^{\infty} |\alpha_n|^2 \;\leq\; \left\| \sum_{n=1}^{\infty} \alpha_n \varphi_n(t) \right\|_2^2 \;\leq\; B \sum_{n=1}^{\infty} |\alpha_n|^2.$$

What does $x_N(t)$ tell us about A in this case?]

Additional workspace:

Additional workspace:

Additional workspace:

1(a)		1(e)		3	
1(b)		1(f)		4	
1(c)		2(a)			
1(d)		2(b)		Total	