GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 6250 Fall 2018
Problem Set #7

Assigned: 17-Oct-18
Due Date: 24-Oct-18

As stated in the syllabus, unauthorized use of previous semester course materials is strictly prohibited in this course.

All assignments will be due at the begining of class on the due date.

PROBLEM 7.1:
Using your class notes, prepare a 1-2 paragraph summary of what we talked about in class in the last week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better.

PROBLEM 7.2:
Consider the space $\mathcal{V} = \overline{\mathrm{Span}}(\{\phi(t-n)\}_{n\in\mathbb{Z}})$, where

$$\phi(t) = \left(\frac{2}{\pi}\right)^{1/4} e^{-t^2}.$$

Is $\{\phi(t-n)\}_{n\in\mathbb{Z}}$ a Riesz basis for $\mathcal{V}$ and if so, what are the constants A, B? (You should be able to compute these to a high degree of accuracy.) You might find the following fact useful in your analysis:

$$\Phi(j\Omega) = \int_{-\infty}^{\infty} \left(\frac{2}{\pi}\right)^{1/4} e^{-t^2} e^{-j\Omega t}\ \mathrm{d}t = (2\pi)^{1/4} e^{-\Omega^2/4}.$$

PROBLEM 7.3:
The dual B-spline functions can be written as

$$\widetilde{b}_L(t) = \sum_{\ell=\infty}^{\infty} h_L[\ell] b_L(t-\ell).$$

1. Compute $h_L[n]$ for $-20 \leq n \leq 20$ for $L = 1, 2, 3, 4$. Turn in a stem plot of your answers and your code. To do this, you will need to approximate the integral

$$h_L[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_L(e^{j\omega}) e^{j\omega n} \ \mathrm{d}\omega,$$

for those values of n. You can do this using any numerical integration technique with which you are familiar, or by simply taking a sum on a very fine grid and normalizing appropriately to account for the width of the intervals. $h[-20]$ and $h[20]$ should be very close to zero. Also, since H_L is real and even, you can replace $e^{j\omega n}$ in the expression above with $\cos(\omega n)$.

2. Using your answer above and the provided `bspline` functions, plot (a very good approximation of) the dual B-spline functions $\widetilde{b}_L(t)$ for $L = 1, 2, 3, 4$.

PROBLEM 7.4:
Let $x(t)$ be the signal

$$x(t) = \begin{cases} 1/2, & 0 \leq t \leq 10, \\ -\sin(\pi t/10), & 10 \leq t \leq 20 \end{cases}$$

Compute and use MATLAB to plot the solutions to

$$\min_{\boldsymbol{y} \in \mathcal{V}_L} \int_{-\infty}^{\infty} |x(t) - y(t)|^2 \ \mathrm{d}t,$$

for $L = 1, 2, 3, 4$, where

$$\mathcal{V}_L = \overline{\mathrm{Span}}(\{b_L(t-n)\}_{n\in\mathbb{Z}}).$$

To do this, you will need to integrate $x(t)$ against the dual B-splines. You can compute this integral using

$$\int_{-\infty}^{\infty} x(t)\widetilde{b}_L(t-n) \ \mathrm{d}t = \sum_{\ell=-\infty}^{\infty} h_L[\ell] \int_{-\infty}^{\infty} x(t) b_L(t-n-\ell) \ \mathrm{d}t.$$

So the first step is to write a function that numerically integrates $x(t)$ with shifts of $b_L(t)$ over an appropriate range.