Chapter 1: Linear Graphs and Simultaneous Linear Equations

At the end of this chapter, you will be able to:

- ☐ Determine the equation of a horizontal and a vertical line
- ☐ Draw the graphs of linear equations in the form $ax + by = k$
- ☐ Solve simultaneous linear equations using
    - ☐ 1. Graphical method
    - ☐ 2. Elimination method
    - ☐ 3. Substitution method

******please tick the checkboxes to monitor your progress!*

1.1 Equations of straight lines (Recap!)

You have learnt how that the equation of a straight line is in the form:

$$y = mx + c$$

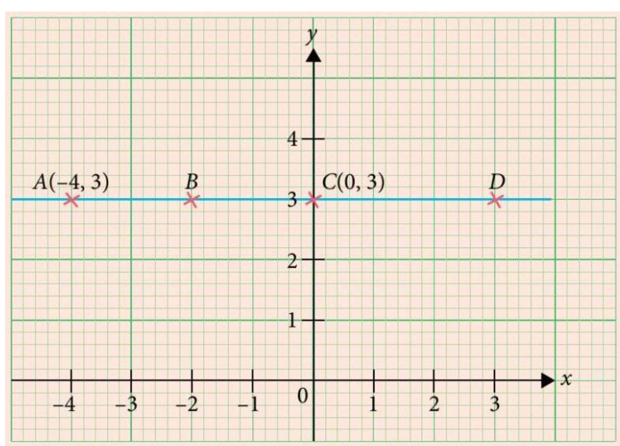
gradient of line

y – intercept

where ***m*** and ***c*** are constants.

Recall that if a pair of coordinates (x, y) lies on the straight line, then the x– coordinate and y– coordinates satisfy the equation of the function.

A. Equation of horizontal lines



What is the gradient of the horizontal line?

What is the coordinates of B and D ? B (,) D (,)

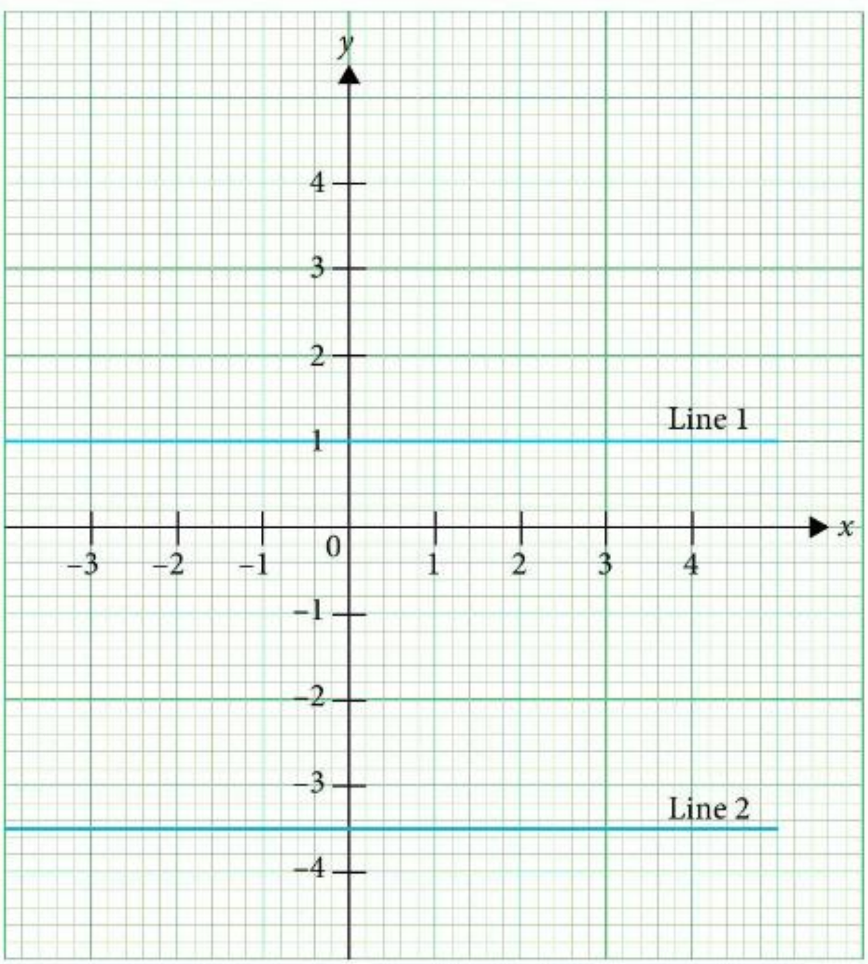
What is the equation of the horizontal line?

The gradient m of a horizontal line is 0 and,

> the equation of a horizontal line is
>
> $y = c$.

where **c** is the value the line cuts the y – axis.

Practice Now 1A (textbook 2A, pg 4)



(a) Write down the equation of each of the given horizontal lines.

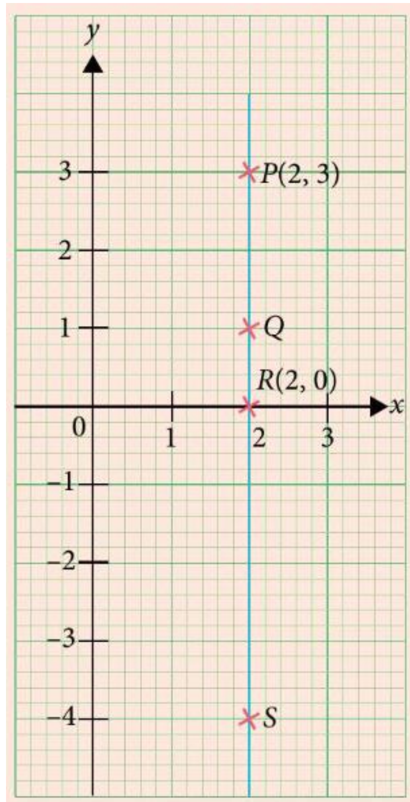
(b) On the graph, draw each of the lines with the following equations.

(i) $y = 2$

(ii) $y = 0$

Describe the lines.

B. Equation of vertical lines



What is the gradient of the vertical line?

What is the coordinates of Q and S ? Q (,) S (,)

What is the equation of the vertical line?

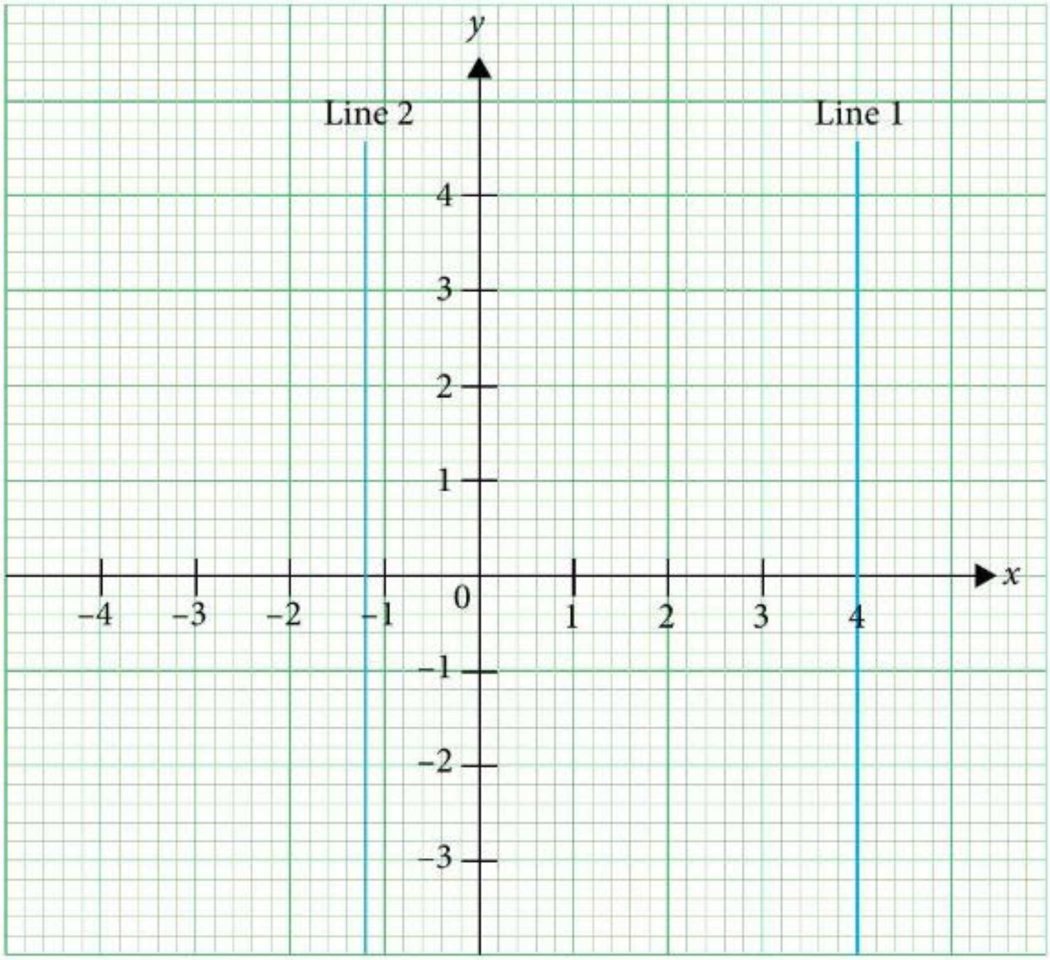
The gradient m of a vertical line is **undefined** and,

the equation of a vertical line is

$x = a.$

where a is the value the line cuts the x – axis.

Practice Now 1B (textbook 2A, pg 6)



(a) Write down the equation of each of the given vertical lines.

(b) On the graph, draw each of the lines with the following equations.

(i) $x = -3.5$

(ii) $x = 0$

Describe the lines.

1.2 Graphs of linear equations in the form $ax + by = k$

We have learnt how to draw graphs of linear equations in the form $y = mx + c$, where *m* and *c* are constants.

How is $y = mx + c$ related to $ax + by = k$? **They are equivalent!**

Proof:

$$ax + by = k$$

$$by = -ax + k$$

$$y = -\frac{a}{b}x + \frac{k}{b}$$

$$y = mx + c$$

Big Idea

Equivalence

Equations of the form $ax + by = k$ can be rewritten in the form $y = mx + c$, where $m = -\frac{a}{b}$ and $c = \frac{k}{b}$. We say that these two equations are equivalent. Equivalence of equations allows us to rewrite one equation into another form that is easier for us to work with.

Worked Example 1 (textbook 2A, pg 7)

Drawing the graph of $ax + by = k$

The variables x and y are connected by the equation $2x - 3y = 2$. Some values of x and the corresponding values of y are given in the table.

x	-2	-0.5	4
y	-2	p	2

(a) Calculate the value of p.

(b) On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on the x-axis and 2 cm to represent 1 unit on the y-axis, draw the graph of $2x - 3y = 2$ for $-2 \leqslant x \leqslant 4$.

Attention

$-2 \leqslant x \leqslant 4$ represents values of x that are more than or equal to -2 but less than or equal to 4.

(c) The point $(1, q)$ lies on the graph in part **(b)**. Find the value of q.

(d) **(i)** On the same axes in part **(b)**, draw the graph of $y = 1$.

(ii) State the x-coordinate of the point on the graph of $2x - 3y = 2$ that has a y-coordinate of 1.

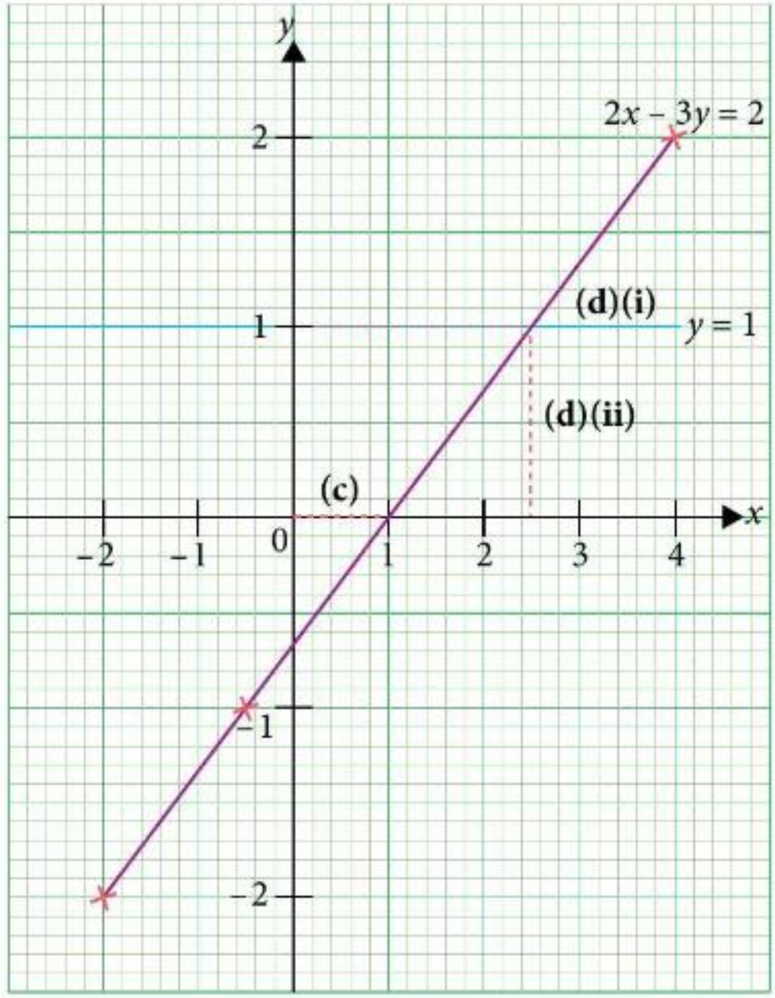
Solution:

(a)

Equation of line:
$2x - 3y = 2$
When $x = -0.5,\ y = p,$

$2(-0.5) - 3p = 2$
$3p = -3$
$\therefore\ p = -1$

(b)



(c) Working line (remember to draw dotted lines to show how you obtained your value)

when $x = 1$,
from graph,
$q = y = 0$

(d) (i)

$y = 1$
is a horizontal line
cutting the $y-$axis at 1.

(d) (ii) Working line (remember to draw dotted lines to show how you obtained your value) $x-$coordinate $= 2.5$

TRAINING ARENA!

Practice Now 1C (textbook 2A, pg 8)

The variables x and y are connected by the equation $3x + y = 1$. Some values of x and the corresponding values of y are given in the table.

x	−2	0	2
y	p	1	−5

(a) Find the value of p.

(b) On a sheet of graph paper, using a scale of 4 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $3x + y = 1$ for $-2 \leqslant x \leqslant 2$.

(c) The point $(-1, q)$ lies on the graph in part **(b)**. Find the value of q.

(d) **(i)** On the same axes in part **(b)**, draw the graph of $x = -0.5$.

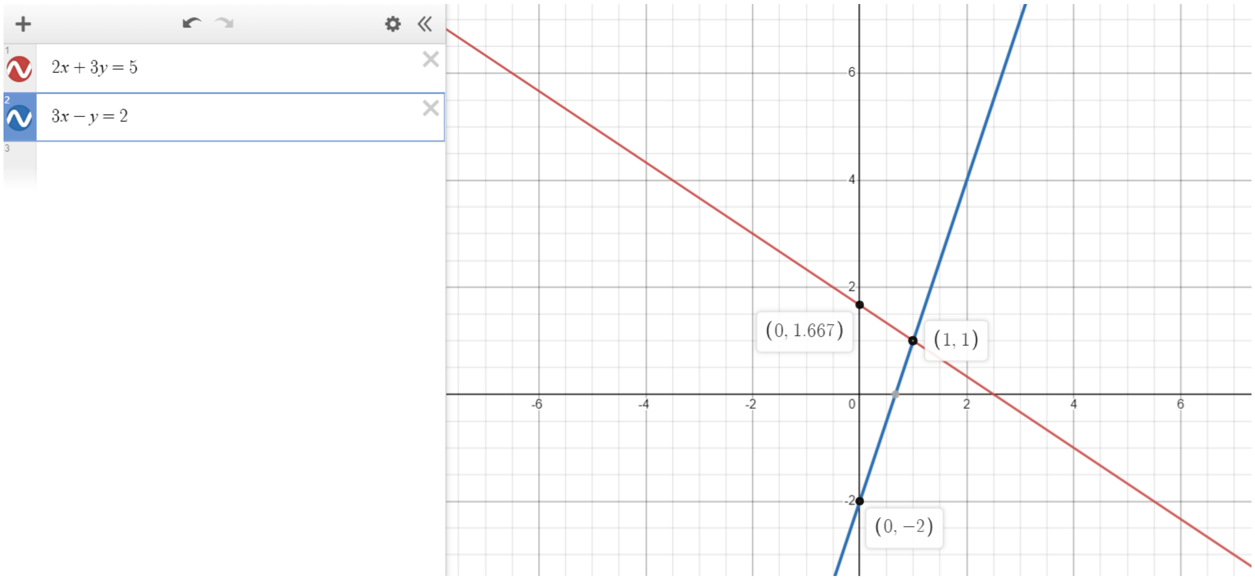
(ii) State the y-coordinate of the point on the graph of $3x + y = 1$ that has an x-coordinate of -0.5.

Marked Assignment:

Workbook 2A
Worksheet 1A (pp.1-2): Q1, Q2
Worksheet 1B (pp.1-2): Q2

1.3 Solving simultaneous linear equations using **graphical method**

Let us consider the linear equations $2x+3y=5$ and $3x-y=2$ on desmos.



(i) What is the value of *y* when *x* = 0 on both linear equations?

(ii) What are the coordinates of the point of intersection?

(,)

(iii) What do you notice about the relationship between the x and y coordinates of the point of intersection and the line of the two graphs?

In the above Investigation, the graphs of $2x + 3y = 5$ and $3x - y = 2$ intersect at the point (1, 1). The set of values of $x = 1$ and $y = 1$ satisfies the two linear equations simultaneously. We say that $x = 1$ and $y = 1$ is the **solution** of the **simultaneous linear equations** $2x + 3y = 5$ and $3x - y = 2$.

Worked Example 2 (textbook 2A, pg 11)

Solving simultaneous linear equations using graphical method

Using the graphical method, solve the simultaneous equations

$$2x - 5y = 32,$$
$$2x + 3y = 0.$$

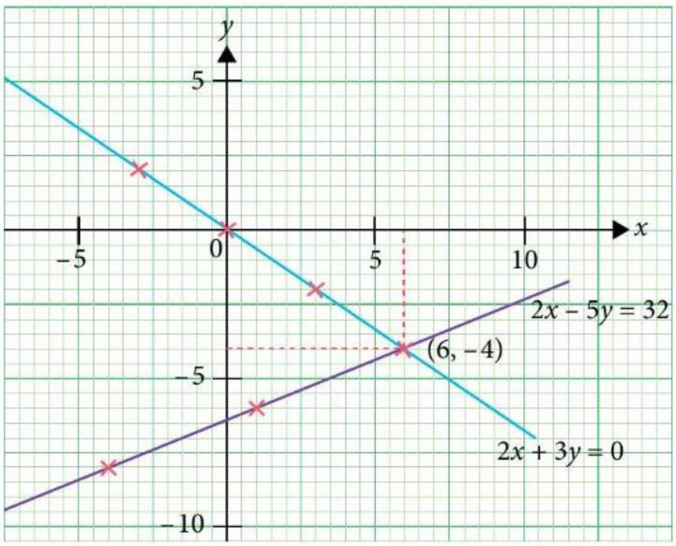
Solution

$2x - 5y = 32$

x	−4	1	6
y	−8	−6	−4

$2x + 3y = 0$

x	−3	0	3
y	2	0	−2



The graphs intersect at the point (6, −4).
∴ the solution is $x = 6$ and $y = -4$.

Recall
We only need to plot 3 points to obtain the graph of a linear equation. In fact, a straight line can be determined by plotting 2 points. We use the 3rd point to check for mistakes in the graph.

Problem-solving Tip
An appropriate scale to use here would be 2 cm to 5 units for both axes.

Big Idea
Diagrams
The Cartesian coordinate system allows us to represent a pair of simultaneous equations graphically so that we can find the point of intersection which also gives the solution that satisfies both equations.

TRAINING ARENA!

Practice Now 2 (textbook 2A, pg 12)

1. Using the graphical method, solve the simultaneous equations

$$x + y = 3,$$
$$3x + y = 5.$$

Step 1: Table of values for both equations

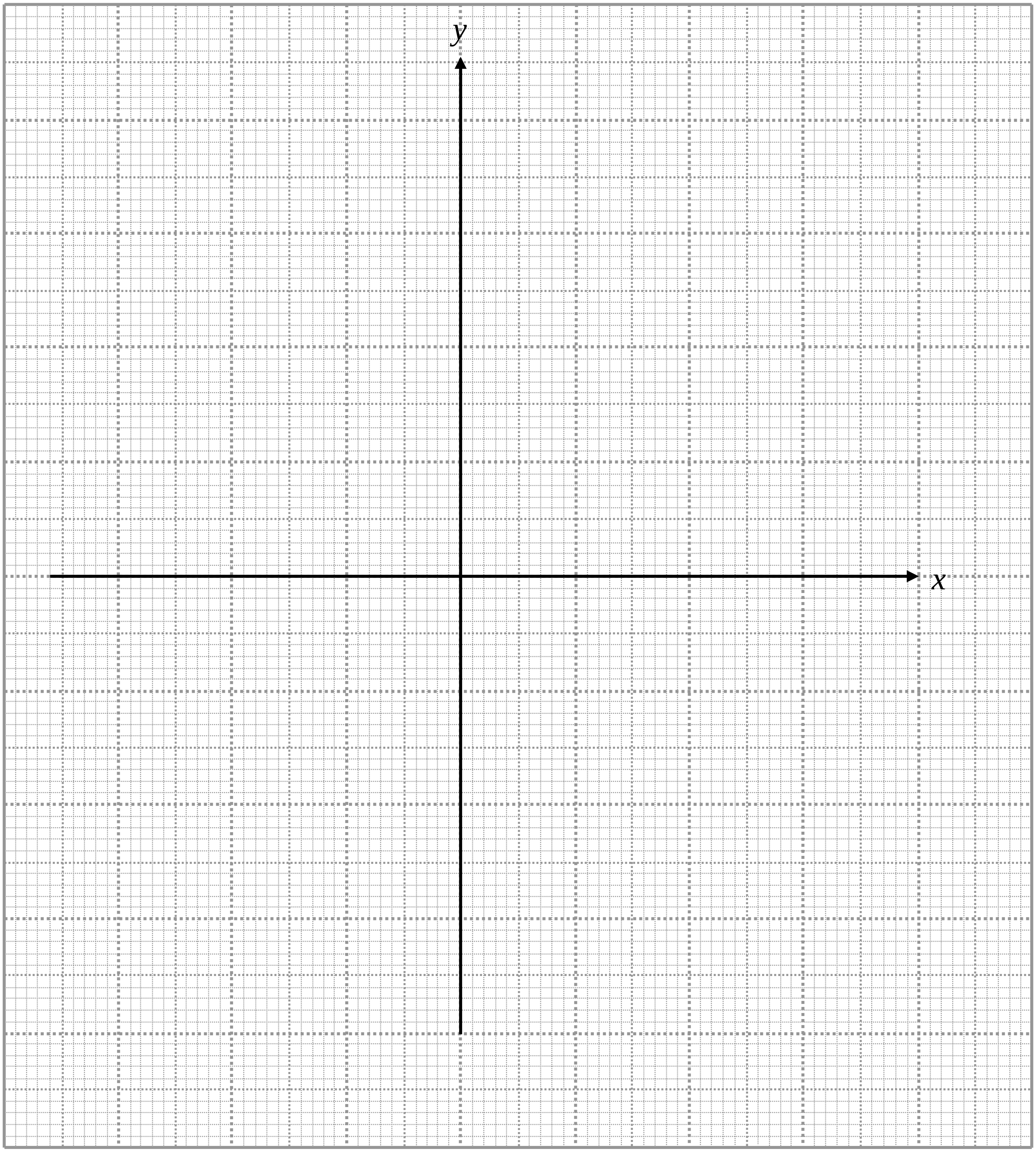
$x + y = 3$

x	−2	0	2
y			

$3x + y = 5$

x	−2	0	2
y			

Step 2: Choose an appropriate scale to use.
Step 3: Plot points from the table above and connect with a straight line.



Marked Assignment:
Workbook 2A
Worksheet 1C (pp.7-12): Q1, 4

1.4A Solving simultaneous linear equations using elimination method

Let us use the **elimination method** to solve the following pair of equations.

We shall label the equations as equation (1) and equation (2).

$$3x - y = 12 \quad \text{—— (1)}$$
$$2x + y = 13 \quad \text{—— (2)}$$

The elimination method is usually used when the absolute values of the coefficients of one variable in both the equations are the same. For example, the absolute values of the coefficients of y in equations (1) and (2) are the same.

What happens when we add the two equations?

(1) + (2): $\quad (3x - y) + (2x + y) = 12 + 13$

$$3x + 2x - y + y = 25$$

Notice that the terms in y are eliminated. We now have a linear equation in one variable x.

$$5x = 25$$
$$x = 5$$

Substitute $x = 5$ into (1): $\quad 3(5) - y = 12$

$$y = 3$$

$\therefore$ the solution of the simultaneous equations is $x = 5$ and $y = 3$.

Check: Substitute $x = 5$ and $y = 3$ into (1) and (2):

In (1), LHS $= 3(5) - 3$
$= 12$
$=$ RHS

In (2), LHS $= 2(5) + 3$
$= 13$
$=$ RHS

Since $x = 5$ and $y = 3$ satisfies both the equations, it is the solution of the simultaneous equations.

Recall

The absolute value of -1 is 1, and the absolute value of 1 is 1.

Big Idea

Equivalence

We can add equations (1) and (2) based on the idea of equivalence. Do you know why?

Problem-solving Tip

It is a good practice to check your solution by substituting the values of the unknowns found into the original equations.

NOTE! For elimination method the absolute value of one of the variable should be equal. If one of the value is positive and the value in the other equation is negative, we ADD both equations. If both the values in both equations are the same, we take one equation and SUBTRACT the other.

TRAINING ARENA!

Practice Now 3 (textbook 2A, pg 15)

Using the elimination method, solve each of the following pairs of simultaneous equations

Q1a.

$$x - y = 3$$
$$4x + y = 17$$

Step 1: Label both equations

Step 2: Is the absolute value of one of the variables the same in both equations?

Step 3: Are they of the same value? (positive/negative)

Step 4: Add/Subtract the equation.

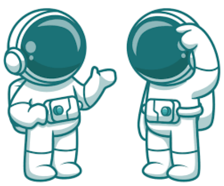
Step 5: Solve for one variable and substitute the value into any one of the original equations to find the value of the other variable

Q1d.

$$4x - 5y = 17$$
$$2x - 5y = 8$$

Q2.

$3x - y + 14 = 0$
$2x + y = 1 = 0$



What do we do if none of the absolute value of the variables in both equations is the same?

e.g.

$3x + 2y = 8$
$4x - y = 7$

Can we still use the elimination method?

YES WE CAN! But it is necessary to manipulate one of the equation before we can eliminate a variable.

Worked Example 4 (textbook 2A, pg 16)

Solving simultaneous linear equations using elimination method

Using the elimination method, solve the simultaneous equations

$3x + 2y = 8,$
$4x - y = 7.$

Solution

$3x + 2y = 8$ —— (1)
$4x - y = 7$ —— (2)
$2 \times (2)$: $8x - 2y = 14$ —— (3)
$(1) + (3)$: $(3x + 2y) + (8x - 2y) = 8 + 14$
$11x = 22$
$x = 2$

Substitute $x = 2$ into (2): $4(2) - y = 7$
$y = 1$

$\therefore$ the solution is $x = 2$ and $y = 1$.

TRAINING ARENA!

Practice Now 4 (textbook 2A, pg 16)

Using the elimination method, solve each of the following pairs of simultaneous equations

(a)

$$2x + 3y = 18$$
$$3x - y = 5$$

Step 1: Label both equations

Step 2: Is the absolute value of one of the variables the same in both equations?

Step 3: Which variable is easier to make the absolute value the same? What value must be multiplied to the equation? (Additional Step)

Step 4: Are they of the same value? (positive/negative)

Step 5: Add/Subtract the equation.

Step 6: Solve for one variable and substitute the value into any one of the original equations to find the value of the other variable

(b)

$$x + 4y = 11$$
$$2x + 3y = 7$$

Practice Now 5 (textbook 2A, pg 17)

Using the elimination method, solve each of the following pairs of simultaneous equations

(a)

$$9x + 2y = 5$$
$$7x - 3y = 13$$

(b)

$$5x - 4y = 17$$
$$2x - 3y = 11$$

Step 1: Label both equations

Step 2: Is the absolute value of one of the variables the same in both equations?

Step 3: Which variable is easier to make the absolute value the same? What value must be multiplied to the equation? (Additional Step)

Step 4: Are they of the same value? (positive/negative)

Step 5: Add/Subtract the equation.

Step 6: Solve for one variable and substitute the value into any one of the original equations to find the value of the other variable

Solving simultaneous fractional equations using elimination method

Practice Now 6 (textbook 2A, pg 17)

Using the elimination method, solve each of the following pairs of simultaneous equations

$$\frac{x}{2}-\frac{y}{3}=4$$

$$\frac{2}{5}x-\frac{y}{6}=3\frac{1}{2}$$

Step 1: Label both equations

Step 2: Is the absolute value of one of the variables the same in both equations?

Step 3: What value can be multiplied throughout to get integer coefficients for both equations?

Step 4: Which variable is easier to make the absolute value the same? What value must be multiplied to the equation? (Additional Step)

Step 5: Are they of the same value? (positive/negative)

Step 6: Add/Subtract the equation.

Step 7: Solve for one variable and substitute the value into any one of the original equations to find the value of the other variable

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1.4B Solving simultaneous linear equations using <u>substitution method</u>

We can also solve a pair of simultaneous equations using the **substitution method**.

In this method, we first rearrange one equation to express one variable in terms of the other variable. Next, we substitute this expression into the other equation to obtain an equation in only one variable.

Worked Example 7 (textbook 2A, pg 18)

Solving simultaneous linear equations using substitution method

Using the substitution method, solve the simultaneous equations

$7x - 2y = 21,$

$4x + y = 57.$

HTML5

Solution

$7x - 2y = 21$ — (1)

$4x + y = 57$ — (2)

From (2), $y = 57 - 4x$ — (3) *rearrange (2) to express y in terms of x*

Substitute (3) into (1): $7x - 2(57 - 4x) = 21$ *replace y in (1) with expression in x to obtain an equation in x*

$7x - 114 + 8x = 21$ *solve the equation in x*

$15x = 135$

$x = 9$

Substitute $x = 9$ into (3): $y = 57 - 4(9)$ *substitute the solution of x into the expression for y*

$= 21$

$\therefore$ the solution is $x = 9$ and $y = 21$.

Problem-solving Tip

It is easier to obtain the value of y by substituting the value of x into equation (3) instead of equation (1) or (2).

TRAINING ARENA!

Practice Now 7 (textbook 2A, pg 18)

Using the substituition method, solve each of the following pairs of simultaneous equations

$3y - x = 7$

$2x + 3y = 4$

Step 1: Label both equations

Step 2: Which variable is easier to make the rearrange and express it in terms of the other variable?

Step 3: Substitute into the other equation

Step 4: Solve for one variable and substitute the value into any one of the original equations to find the value of the other variable

Practice Now 8 (textbook 2A, pg 19)

Using the substituition method, solve each of the following pairs of simultaneous equations

$3x - 2y = 8$

$4x + 3y = 5$

Solving simultaneous fractional equations using substitution method.

Practice Now 9 (textbook 2A, pg 20)

Using the substituition method, solve each of the following pairs of simultaneous equations

$$\frac{x-1}{y-3}=\frac{2}{3}$$

$$\frac{x-2}{y-1}=\frac{1}{2}$$

Step 1: Label both equations

Step 2: What should be done to simplify the fractional equation?

Step 2: Which variable is easier to make the rearrange and express it in terms of the other variable?

Step 3: Substitute into the other equation

Step 4: Solve for one variable and substitute the value into any one of the original equations to find the value of the other variable

Both the elimination method and the substitution method use the same principle: reduce the two given simultaneous equations in two variables to an equation in one variable.

Reflection:

1. Which method do I prefer?
2. How can I decide when to use the elimination or the substitution method to solve a pair of simultaneous linear equations?
3. When will I use the graphical method to solve a pair of simultaneous linear equations?

Marked Assignment:

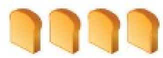
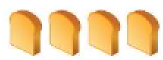
Workbook 2A
Worksheet 1D (pp.13-18): Q1b, 1(n), Q2(b), 2(d)

1.5 Applications of simultaneous equations in real-world contexts

Consider the following problem:

> 7 cups of coffee and 4 pieces of toast cost \$10.60. 5 cups of coffee and 4 pieces of toast cost \$8.60. Find the cost of each item.

In primary school, we have learnt how to solve the problem by drawing a diagram (see **Method 1**). This is essentially the same as using algebra to solve the problem (see **Method 2**).

Method 1:	**Method 2:** Let the cost of 1 cup of coffee be \$$x$ and the cost of 1 piece of toast be \$$y$.
 +  = \$10.60 Cost of 7 cups of coffee Cost of 4 pieces of toast	$7x + 4y = 10.6$ —— (1)
 +  = \$8.60 Cost of 5 cups of coffee Cost of 4 pieces of toast	$5x + 4y = 8.6$ —— (2)
Cost of 2 cups of coffee = \$10.60 – \$8.60 = \$2 ∴ cost of 1 cup of coffee = \$1	(1) – (2): $2x = 10.6 - 8.6$ $2x = 2$ $x = 1$ ∴ cost of 1 cup of coffee = \$1
Cost of 4 pieces of toast = \$8.60 – 5 × \$1 = \$3.60 ∴ cost of 1 piece of toast = \$0.90	Substitute $x = 1$ into (2): $5(1) + 4y = 8.6$ $4y = 8.6 - 5$ $4y = 3.6$ $y = 0.9$ ∴ cost of 1 piece of toast = \$0.90

Worked Example 10 (textbook 2A, pg 24)

Finding two numbers given sum and difference

The sum of two numbers is 67 and their difference is 3. Find the two numbers.

Solution

Let the smaller number be x and the greater number be y.

$x + y = 67$ —— (1)

$y - x = 3$ —— (2)

(1) + (2): $2y = 70$

$y = 35$

Substitute $y = 35$ into (1): $x + 35 = 67$

$x = 32$

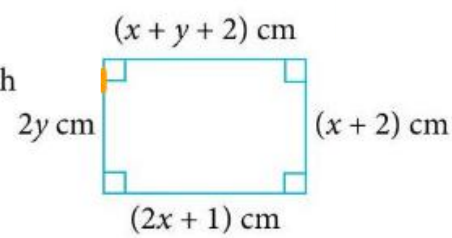
∴ the two numbers are 32 and 35.

TRAINING ARENA!

Practice Now 10 (textbook 2A, pg 24)

1. The sum of two numbers is 36 and their difference is 9. Find the two numbers.

3. The figure shows a rectangle with its length and breadth as indicated. Find the perimeter of the rectangle.



Practice Now 11 (textbook 2A, pg 25)

1. In five years' time, Li Ting's father will be three times as old as Li Ting. Four years ago, her father was six times as old as her. Find their present ages.

Marked Assignment:

Workbook 2A
Worksheet 1E (pp.19-22): Q2, Q3

We have come to the end of this chapter!

Let's see what you have learnt...

In the space provided, summarize your learning

(eg. Mind map, point form, drawings etc.)