Chapter 2: Linear Inequalities

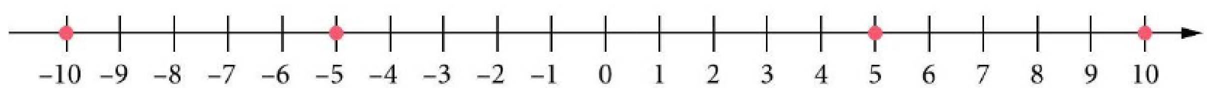
At the end of this chapter, you will be able to:

- ☐ Identify linear inequalities and differentiate between an equation and an inequality
- ☐ Solve simple inequalities in the form $ax+b\leq c$ and $ax+b<c$ and represent solutions on the number line
- ☐ Apply linear inequalities to solve real-world problems

*****_please tick the checkboxes to monitor your progress!_

2.1 Simple inequalities

RECAP! We have learnt how to represent numbers on a number line. A number on the number line is more than any number on its left and less than any number on its right.



Since the number 10 is to the right of the number 5, 10 is more than 5 (we write $10 > 5$). Similarly, −5 is to the right of −10, so −5 is more than −10 (we write $-5 > -10$). Alternatively, we can say that −10 is less than −5 (we write $-10 < -5$). $10 > 5$ and $-10 < -5$ are known as **inequalities**.

IMPORTANT!

Notation	Meaning
$a > b$	a is more than b.
$a < b$	a is less than b.
$a \geqslant b$	a is more than or equal to b.
$a \leqslant b$	a is less than or equal to b.

Understanding and exploring the properties of Inequalities

In this Investigation, we shall explore some properties of inequalities.
Fill in each blank with a '$>$' or '$<$'.

Part 1: Adding/subtracting a real number on/from both sides of an inequality

1. Consider the inequality $6 < 12$.

(a) **(i)** $6 + 2$ ☐ $12 + 2$ **(ii)** $6 - 4$ ☐ $12 - 4$

(b) If $6 < 12$ and a is a real number, then $6 + a$ ☐ $12 + a$ and $6 - a$ ☐ $12 - a$.

(c) If $12 > 6$ and a is a real number, then $12 + a$ ☐ $6 + a$ and $12 - a$ ☐ $6 - a$.

2. Consider the inequality $-6 < 12$.

(a) **(i)** $-6 + 2$ ☐ $12 + 2$ **(ii)** $-6 - 4$ ☐ $12 - 4$

(b) If $-6 < 12$ and a is a real number, then $-6 + a$ ☐ $12 + a$ and $-6 - a$ ☐ $12 - a$.

(c) If $12 > -6$ and a is a real number, then $12 + a$ ☐ $-6 + a$ and $12 - a$ ☐ $-6 - a$.

3. Consider the inequality $6 > -12$.

(a) **(i)** $6 + 2$ ☐ $-12 + 2$ **(ii)** $6 - 4$ ☐ $-12 - 4$

(b) What conclusions can you draw from your answers in part **(a)**?

4. From Questions 1 to 3, can you generalise the conclusions for $x < y$ and $x > y$? What about $x \leqslant y$ and $x \geqslant y$?

Part 2: Multiplying/dividing by a real number on both sides of an inequality

5. Consider the inequality $6 < 12$.

(a) **(i)** 6×2 ☐ 12×2 **(ii)** $6 \times (-4)$ ☐ $12 \times (-4)$

(b) If $6 < 12$ and a is a *positive* real number, then $6 \times a$ ☐ $12 \times a$.
If $6 < 12$ and a is a *negative* real number, then $6 \times a$ ☐ $12 \times a$.

(c) If $12 > 6$ and a is a *positive* real number, then $12 \times a$ ☐ $6 \times a$.
If $12 > 6$ and a is a *negative* real number, then $12 \times a$ ☐ $6 \times a$.

6. Consider the inequality $-6 < 12$.

(a) **(i)** -6×2 ☐ 12×2 **(ii)** $-6 \times (-4)$ ☐ $12 \times (-4)$

(b) If $-6 < 12$ and a is a *positive* real number, then $-6 \times a$ ☐ $12 \times a$.
If $-6 < 12$ and a is a *negative* real number, then $-6 \times a$ ☐ $12 \times a$.

(c) If $12 > -6$ and a is a *positive* real number, then $12 \times a$ ☐ $-6 \times a$.
If $12 > -6$ and a is a *negative* real number, then $12 \times a$ ☐ $-6 \times a$.

7. Consider the inequality $6 > -12$.

(a) **(i)** 6×2 ☐ -12×2 **(ii)** $6 \times (-4)$ ☐ $-12 \times (-4)$

(b) What conclusions can you draw from your answers in part **(a)**?

8. Consider each of the inequalities in Questions 5 to 7.

(a) Divide by a *positive* real number on both sides of the inequality.
Is the inequality sign reversed in each case?

(b) Divide by a *negative* real number on both sides of the inequality.
Is the inequality sign reversed in each case?

9. From Questions 5 to 8, can you generalise the conclusions for $x < y$ and $x > y$? What about $x \leqslant y$ and $x \geqslant y$?

We can conclude:

- When we **add** or **subtract** a positive or a negative number on both sides of an inequality, the inequality sign does *not* change.
- When we multiply or divide by a **positive** number on both sides of an inequality, the inequality sign does *not* change.
- When we multiply or divide by a **negative** number on both sides of an inequality, the inequality sign is *reversed*.

For example, if $x < 2$ and a is a real number, then $x + a < 2 + a$ and $x - a < 2 - a$;

if $x < 2$ and $a > 0$, then $ax < 2a$ and $\frac{x}{a} < \frac{2}{a}$;

if $x < 2$ and $a < 0$, then $ax > 2a$ and $\frac{x}{a} > \frac{2}{a}$.

2.2 Solving simple linear inequalities

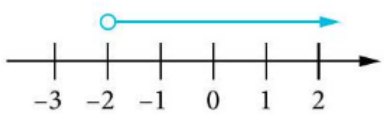
For an inequality with an unknown x, all that satisfy the inequality are called the solutions of the inequality.

Consider $x > -2$. We read this as x more than –2. Any values more than –2 will satisfy the inequality.

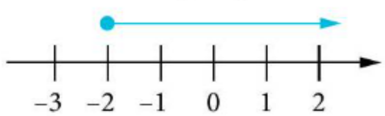
We can represent the solutions to an inequality using the number line.

For example,

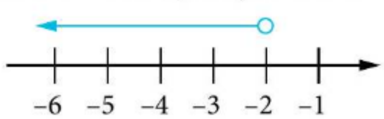
- the solutions of the inequality $x > -2$ are represented as



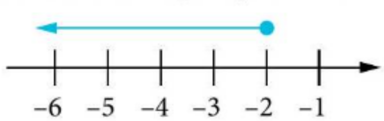
- the solutions of the inequality $x \geqslant -2$ are represented as



- the solutions of the inequality $x < -2$ are represented as



- the solutions of the inequality $x \leqslant -2$ are represented as



Attention

On a number line, a circle ○ is used to indicate that x cannot take on a particular value whereas a dot ● is used to indicate that x can take on the particular value.

Big Idea

Diagrams

Using a number line helps us to visualise the solution, or the range of possible values, to a given inequality. We follow conventions to construct these diagrams so that the information can be correctly represented and interpreted.

When solving linear inequalities, it is similar to when solving an equation.

Worked Example 1 (textbook 2A, pg 38)

Solving linear inequalities

Solve each of the following inequalities and represent the solutions on a number line.

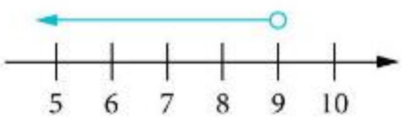
(a) $3x < 27$ **(b)** $-2x \geqslant 4$

Solution

(a) $3x < 27$

$\frac{3x}{3} < \frac{27}{3}$ divide both sides by 3; no change in the inequality sign since $3 > 0$

$\therefore x < 9$



Big Idea

Equivalence

Similar to solving equations, we convert inequalities into equivalent forms to solve them. The inequality $3x < 27$ is equivalent to the inequality $x < \frac{27}{3}$. Likewise for **(b)**, all subsequent inequalities are equivalent to the original inequality $-2x \geqslant 4$.

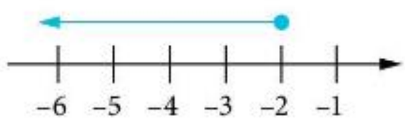
(b) $-2x \geqslant 4$

$-1 \times (-2x) \leqslant -1 \times 4$ multiply both sides by -1; reverse the inequality sign since $-1 < 0$

$2x \leqslant -4$

$\frac{2x}{2} \leqslant \frac{-4}{2}$ divide both sides by 2; no change in the inequality sign since $2 > 0$

$\therefore x \leqslant -2$



Big Idea

Notations

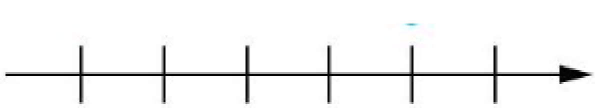
Some problems have multiple solutions within a range of values, or even infinitely many solutions. Using an inequality symbol, we can write an inequality such as $x \leqslant -2$ to express these ideas in a clear and concise manner.

TRAINING ARENA!

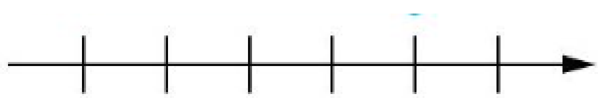
Practice Now 1 (textbook 2A, pg 38)

Solve each of the following inequalities and represent the solutions on a number line.

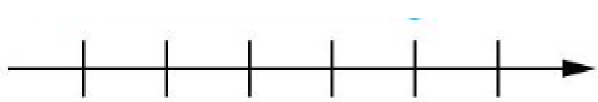
(a) $5x > 30$



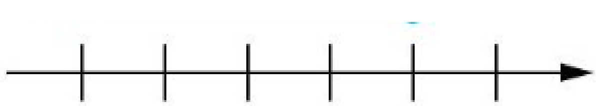
(b) $-4x \geq 20$



(c) $15x \leq 45$



(d) $-6x < 15$



Worked Example 2 (textbook 2A, pg 39)

Solving linear inequalities

Solve each of the following inequalities, representing each solution on a number line.

(a) $x + 4 < 3$ **(b)** $-4y - 5 \geqslant 11$

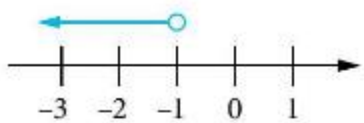
Solution

(a)

$$x + 4 < 3$$

$$x + 4 - 4 < 3 - 4 \quad \text{subtract 4 from both sides}$$

$$x < -1$$



(b)

$$-4y - 5 \geqslant 11$$

$$-4y - 5 + 5 \geqslant 11 + 5 \quad \text{add 5 to both sides}$$

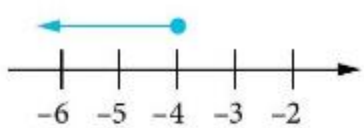
$$-4y \geqslant 16$$

$$-1 \times (-4y) \leqslant -1 \times 16 \quad \text{multiply both sides by } -1\text{; reverse the inequality sign since } -1 < 0$$

$$4y \leqslant -16$$

$$\frac{4y}{4} \leqslant \frac{-16}{4} \quad \text{divide both sides by 4; no change in the inequality sign since } 4 > 0$$

$$y \leqslant -4$$

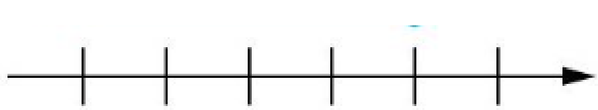


TRAINING ARENA!

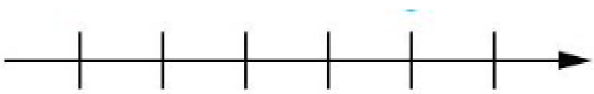
Practice Now 2 (textbook 2A, pg 39)

Solve each of the following inequalities and represent the solutions on a number line.

(a) $x - 3 \geq 7$



(b) $-2y+4>3$



Practice Now 3 (textbook 2A, pg 40)

Solve the inequality $5 - x < -9$ and represent the solution on a number line.

(i) If x is a prime number, write down the smallest possible value of x that satisfies the inequality.

(ii) Given that x is a perfect cube, find the smallest possible value of x.

Answer (i) ....

(ii) ....

Solving more complicated linear inequalities

Worked Example 4 (textbook 2A, pg 40)

Solve each of the following inequalities.

(a) $-3 > 2(1 - x)$ **(b)** $\frac{1}{4} \leqslant \frac{y+1}{7}$

Solution

(a)

$-3 > 2(1-x)$

$-3 > 2 - 2x$ expand the RHS

$-3 - 2 > 2 - 2x - 2$ subtract 2 from both sides

$-5 > -2x$

$-2x < -5$ change sides

$\frac{-2x}{-2} > \frac{-5}{-2}$ divide both sides by -2; reverse the inequality sign since $-2 < 0$

$x > \frac{5}{2}$

Attention

You can leave your answer as $x > \frac{5}{2}$ or $x > 2\frac{1}{2}$.

(b)

$\frac{1}{4} \leqslant \frac{y+1}{7}$

$4 \times 7 \times \frac{1}{4} \leqslant 4 \times 7 \times \frac{y+1}{7}$ multiply both sides by 4×7; no change in the inequality sign since $4 \times 7 > 0$

$7 \leqslant 4(y+1)$

$7 \leqslant 4y + 4$ expand the RHS

$7 - 4 \leqslant 4y + 4 - 4$ subtract 4 from both sides

$3 \leqslant 4y$

$4y \geqslant 3$ change sides

$\frac{4y}{4} \geqslant \frac{3}{4}$ divide both sides by 4; no change in the inequality sign since $4 > 0$

$y \geqslant \frac{3}{4}$

Problem-solving Tip

The LCM of 4 and 7 is 4×7.

TRAINING ARENA!

Practice Now 4 (textbook 2A, pg 40)

1. Solve each of the following inequalities.

(a) $5(3+x) \geq 9$

(b) $\frac{1}{3} > \frac{y+1}{2}$

(c) $\frac{1}{3}(z+1)+2 \leq \frac{1}{2}$

Reflection:

1. What have I learnt about the properties of inequalities earlier that helped me solve the problems?
2. How is solving a linear inequality similar to or different from solving a linear equation?

Marked Assignment:

Workbook 2A
Worksheet 2A (pp.25-28): Q4c, Q5, Q11

2.3 Solving problems involving linear inequalities

Worked Example 5 (textbook 2A, pg 42)

Solving problem involving inequalities

A curry puff costs \$1.60. By forming an inequality, find the maximum number of curry puffs that can be bought with \$20.

Solution

Let the number of curry puffs that can be bought with \$20 be x.

Then $160x \leqslant 2000$ \$1.60 = 160 cents, \$20 = 2000 cents

$$\frac{160x}{160} \leqslant \frac{2000}{160}$$ divide both sides by 160; no change in the inequality sign since $160 > 0$

$$x \leqslant 12\frac{1}{2}$$ Leave answer in exact form.

$\therefore$ the maximum number of curry puffs that can be bought with \$20 is 12.

Attention

In such cases, it is better to leave the answer in mixed numbers so that the largest integer value can be easily determined.

TRAINING ARENA!

Practice Now 5 (textbook 2A, pg 42)

A bus can ferry a maximum of 45 students. By forming an inequality, find the minimum number of buses that are needed to ferry 520 students.

Practice Now 6 (textbook 2A, pg 42)

The minimum mark to obtain a Grade A is 75. Joyce managed to achieve an average of Grade A for three of her Science quizzes. What is the minimum mark she scored in her first quiz if she scored 76 and 89 marks in her second and third quiz respectively?

Marked Assignment:

Workbook 2A
Worksheet 2B (pp.29-30): Q1, Q4

We have come to the end of this chapter!

Let's see what you have learnt…

Complete this Summary page for Linear Inequalities

1. **Properties of inequalities:**

Case	Adding a number	Subtracting a number	Multiplying or dividing by a number	
			$c > 0$	$d < 0$
$x > y$	$x + a > y + a$	$x - b > y - b$	$cx > cy$ $\frac{x}{c} > \frac{y}{c}$	$dx < dy$ $\frac{x}{d} \square \frac{y}{d}$
$x \geqslant y$	$x + a \geqslant y + a$	$x - b \square y - b$	$cx \geqslant cy$ $\frac{x}{c} \square \frac{y}{c}$	$dx \square dy$ $\frac{x}{d} \leqslant \frac{y}{d}$
$x < y$	$x + a < y + a$	$x - b \square y - b$	$cx \square cy$ $\frac{x}{c} \square \frac{y}{c}$	$dx > dy$ $\frac{x}{d} > \frac{y}{d}$
$x \leqslant y$	$x + a \leqslant y + a$	$x - b \square y - b$	$cx \leqslant cy$ $\frac{x}{c} \leqslant \frac{y}{c}$	$dx \square dy$ $\frac{x}{d} \square \frac{y}{d}$

2. **Solving of linear inequality:**

Solving a linear inequality

To solve a linear inequality in the form $ax + b \leqslant c$ or $ax + b < c$, we isolate the variable x on the left-hand side. This gives us the *solution* of the inequality.

For example,

$$\begin{aligned} 3x - 8 &\leqslant 4 \\ 3x &\leqslant 12 \\ \frac{3x}{3} &\leqslant \frac{12}{3} && \text{divide both sides by 3; no change in the inequality sign since } 3 > 0 \\ x &\leqslant 4 \end{aligned}$$

$$\begin{aligned} -2x + 10 &> 5 \\ -2x &> -5 \\ \frac{-2x}{-2} &< \frac{-5}{-2} && \text{divide both sides by } -2\text{; reverse the inequality sign since } -2 < 0 \\ x &< \frac{5}{2} \end{aligned}$$