Chapter 3: Expansion and Factorisation of Algebraic Expressions

At the end of this chapter, you will be able to:

- ☐ Identify quadratic expressions
- ☐ Add and subtract quadratic expressions
- ☐ Expand the product of algebraic expressions
- ☐ Expand and simplify quadratic expressions
- ☐ Expand and algebraic expression in two or more variables involving squares and cubes
- ☐ Factorise quadratic expressions
- ☐ Factorise algebraic expressions in the form $ab+ad+bc+bd$

**please tick the checkboxes to monitor your progress!*

3.1 Addition and subtraction of quadratic expressions

RECAP! Algebraic expression is an expression containing letters, numbers and/or operations.

Examples of algebraic expressions are: $x+8$, $\frac{3}{2}y-7$, $\frac{6-x}{2}$ and $x-3xy+\frac{4y}{3}-7$

A linear expression in one variable is an algebraic expression that contains only one variable term, with or without a constant term. $x+8$, $5x$, $\frac{3}{2}y-7$ and $\frac{6-x}{2}$.

Linear Expressions

Class Discussion

Recap of linear expressions

Which of the following are linear expressions? Give your reasons.

(a) $7-4x$ (b) $\frac{5}{3}y+8$ (c) $\frac{2-x}{9}$ (d) $4x+y-8$

(e) 4 (f) $2x-3xy+7$ (g) x^2-5x+6

Quadratic Expressions

Quadratic expression

A quadratic expression in one variable x is of the form ax^2+bx+c, where a, b and c are constants and $a \neq 0$.

If $a=0$, then it becomes a linear expression.

*** An expression becomes a Quadratic expression when the highest power of the variable is 2.**

Addition and Subtraction of quadratic expressions

Worked Example 1 (textbook 2A, pg 50)

Adding and subtracting algebraic terms

Without using a calculator, simplify the following.

(a) $5x^2 + (-11x^2)$ **(b)** $-8x^2 + 4x^2$

(c) $-2y^2 - 8y^2$ **(d)** $-3y^2 - (-9y^2)$

Solution

(a) $5x^2 + (-11x^2) = 5x^2 - 11x^2$
$= -6x^2$

(b) $-8x^2 + 4x^2 = 4x^2 - 8x^2$
$= -4x^2$

(c) $-2y^2 - 8y^2 = -10y^2$

(d) $-3y^2 - (-9y^2) = -3y^2 + 9y^2$
$= 9y^2 - 3y^2$
$= 6y^2$

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Practice Now 1 (textbook 2A, pg 50)

Without using a calculator, simplify the following.

(a) $x^2 + (-6x^2)$ **(c)** $-13y^2 + 3y^2$

(e) $-8w^2 - 4w^2$ **(g)** $-x^2 - (-30x^2)$

Worked Example 2 (textbook 2A, pg 50)

Adding and subtracting algebraic terms

Without using a calculator, simplify each of the following expressions.

(a) $-3x^2 + (-4x^2) + 2 - 8$ **(b)** $-y^2 - 3xy - 5y^2 - (-4yx)$

Solution

(a) $-3x^2 + (-4x^2) + 2 - 8 = -7x^2 + (-6)$

$= -7x^2 - 6$

(b) $-y^2 - 3xy - 5y^2 - (-4yx) = -y^2 - 5y^2 - 3xy - (-4xy)$ group like terms and change yx to xy

$= -6y^2 - 3xy + 4xy$

$= -6y^2 + xy$

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Practice Now 2 (textbook 2A, pg 50)

Without using a calculator, simplify the following.

(a) $-5x^2 + (-2x^2) + 3 - 7$

(c) $-4y^2 - yx + 3y^2 - (-5xy)$

(e) $10a^2 + (-12b^2) - 9 - (-3b^2) + 5 + (-6a^2)$

3.2 Expansion of algebraic expressions of the form $(a+b)(c+d)$

A. Expansion of $a(b+c)$

The Distributive Law

$a(b+c) = ab + ac$

This is called the Distributive Law because the first factor a is *distributed*, or multiplied separately, to each of the two terms, b and c, in the second factor $(b + c)$. In particular,

Negative of $(x + y)$

$-(x+y) = -x - y$

Negative of $(x - y)$

$-(x-y) = -x + y$

Worked Example 3 (textbook 2A, pg 51)

Expanding expressions using Distributive Law

Expand each of the following expressions.

(a) $3(x + 2)$ **(b)** $-4(5x - y)$ **(c)** $8 - a(-3b + 2c)$

Solution

(a) $3(x + 2) = 3x + 6$

(b) $-4(5x - y) = -20x + 4y$ Distributive Law: $-4 \times (-y) = +4y$

(c) $8 - a(-3b + 2c) = 8 + 3ab - 2ac$

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Practice Now 3 (textbook 2A, pg 51)

Expand each of the following expressions simplify the following.

(a) $2(x+5)$ **(b)** $-3(6x-y)$

(c) $5-a(-2b+3c)$

Worked Example 4 (textbook 2A, pg 52)

Simplifying algebraic expressions using Distributive Law

Simplify $-4x(y + 2z) - 3x(5y - z)$.

Solution

$-4x(y + 2z) - 3x(5y - z)$

$= -4xy - 8xz - 15xy + 3xz$ Distributive Law

$= -4xy - 15xy - 8xz + 3xz$ group like terms

$= -19xy - 5xz$

Big Idea

Equivalence

Using the Distributive Law to write $-4x(y + 2z) - 3x(5y - z)$ in its equivalent form helps us to simplify the expression here.

TRAINING ARENA!

Practice Now 4 (textbook 2A, pg 52)

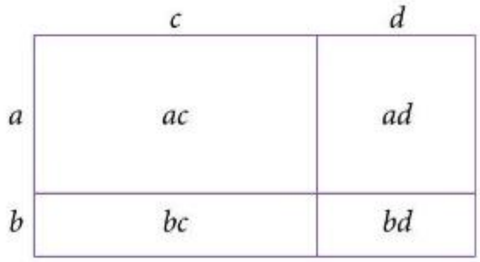
Simplify each of the following expressions.

(a) $-3x(y+4z)-5x(2y-z)$

(b) $2p(-4q-3r)-6q(3p+2r)$

B. Expansion of $(a+b)(c+d)$

We can use the rectangle to visualize the expansion of $(a+b)(c+d)$.



Attention

If a, b, c and d are positive, we can think of this expansion as finding the area of a rectangle of sides $(a + b)$ and $(c + d)$. However, the result is the same for all values of a, b, c and d.

$$(a + b)(c + d) = ac + ad + bc + bd$$

The Distributive Law for $(a + b)(c + d)$

$$(a + b)(c + d) = ac + ad + bc + bd$$

Worked Example 5 (textbook 2A, pg 53)

Expanding expressions using Distributive Law

Expand each of the following expressions.

(a) $(a + b)(7x + 5y)$ **(b)** $(3a + 1)(x - 4y)$

(c) $(x - 6y)(3c + 2d)$ **(d)** $(8p - 3q)(2r - 5s)$

Solution

(a) $(a + b)(7x + 5y) = 7ax + 5ay + 7bx + 5by$

(b) $(3a + 1)(x - 4y) = 3ax - 12ay + x - 4y$

(c) $(x - 6y)(3c + 2d) = 3cx + 2dx - 18cy - 12dy$

(d) $(8p - 3q)(2r - 5s) = 16pr - 40ps - 6qr + 15qs$

TRAINING ARENA!

Practice Now 5 (textbook 2A, pg 52)

Expand each of the following expressions.

(a) $(a+b)(8x+7y)$

(c) $(5a+2)(x-2y)$

(e) $(x-4y)(2c+3d)$

(g) $(6p-5q)(3r-4s)$

***(j)** $(-4r-3s)(3-2t-5u)$

Worked Example 6 (textbook 2A, pg 53)

Simplifying algebraic expressions using Distributive Law

Simplify the expression $2ac - (3a - b)(c + 4b)$.

Solution

$2ac - (3a - b)(c + 4b)$

$= 2ac - (3ac + 12ab - bc - 4b^2)$ Distributive Law

$= 2ac - 3ac - 12ab + bc + 4b^2$ $-(x + y) = -x - y$ and $-(x - y) = -x + y$

$= 4b^2 - ac - 12ab + bc$

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Practice Now 6 (textbook 2A, pg 53)

Simplify each of the following expressions.

(a) $2ac - (3a + b)(c - 4d)$

(b) $2x(3y - 4z) - (3x + y)(y - 3z)$

(c) $(3p - q)(2r + s) - (p - 2q)(5r - 4s)$

(d) $(h+6k)(2m-h)+(3h-2m)(2k+h)$

Marked Assignment:

Workbook 2A
Worksheet 3A (p.33-34): Q1(a), 1(g), 2(a), 2(e)
Worksheet 3B (p. 35-36): Q1(h), 2(c), 2(d), 3(a), 3(h)

3.3 Expansion of quadratic expressions

A. Expansion of quadratic expressions of the form $px(qx+r)$

e.g. $2(x+3)=2x+6$

Method 1: Distributive Law

$$2(x+3)=2(x)+2(3)$$
$$=2x+6$$

Method 2: Multiplication frame

$\times$	x	$+3$
2	$2x$	$+6$

Worked Example 7 (textbook 2A, pg 56)

Expanding expressions of the form $px(qx+r)$ using Distributive Law

Without using algebra discs or multiplication frames, expand each of the following expressions.

(a) $-7x(4x+3)$ **(b)** $-3y(10-9y)$

Solution

(a) $-7x(4x+3)=-28x^2-21x$

(b) $-3y(10-9y)=-30y+27y^2$
$=27y^2-30y$

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Practice Now 7 (textbook 2A, pg 56)

Expand each of the following expressions.

(a) $4x(2x+3)$

(d) $-n(12n-29)$

Worked Example 8 (textbook 2A, pg 56)

Simplifying expressions of the form $px(qx + r)$

Simplify $x(2x - 1) - 2(x + 5)$.

Solution

$$x(2x - 1) - 2(x + 5) = 2x^2 - x - 2x - 10 \quad \text{Distributive Law}$$
$$= 2x^2 - 3x - 10$$

TRAINING ARENA!

Practice Now 8 (textbook 2A, pg 57)

Simplify each of the following expressions.

(a) $x(7x - 4) - 3(x + 2)$

Worked Example 9 (textbook 2A, pg 57)

Expanding and simplifying algebraic expressions involving squares and cubes

(a) Expand $ab(ac + b^2)$.

(b) Simplify $x(xy + y) - y(xy + x)$.

HTML5

Solution

HTML5

(a) $ab(ac + b^2) = a^2bc + ab^3$ Distributive Law

(b) $x(xy + y) - y(xy + x) = x^2y + xy - xy^2 - yx$ Distributive Law

$= x^2y - xy^2$

Problem-solving Tip

(a) $a \times a = a^2$

$b \times b^2 = b \times b \times b$

$= b^3$

(b) $yx = xy$

$x^2y \neq xy^2$

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Practice Now 9 (textbook 2A, pg 57)

Expand each of the following expressions.

(a) Expand $xy(yz + x^2 - xy)$.

B. Expansion of quadratic expressions of the form $(px+q)(rx+s)$

e.g. $(x+2)(x+3)$

Method 1: Distributive Law

$$(x+2)(x+3) = x(x) + x(3) + 2(x) + 2(3)$$
$$= x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Method 2: Multiplication frame

$\times$	x	$+2$
x	x^2	$+2x$
$+3$	$+3x$	$+6$

Worked Example 10 (textbook 2A, pg 59)

Expanding quadratic expressions using multiplication frame

Without using algebra discs, expand $(3x + 4)(7 - 8x)$ using a multiplication frame.

Solution

Method 1:

$\times$	$-8x$	$+7$
$3x$	$-24x^2$	$+21x$
$+4$	$-32x$	$+28$

$$(3x + 4)(7 - 8x) = (3x + 4)(-8x + 7)$$
$$= -24x^2 + 21x - 32x + 28$$
$$= -24x^2 - 11x + 28$$

Method 2:

$\times$	7	$-8x$
$3x$	$21x$	$-24x^2$
$+4$	$+28$	$-32x$

$$(3x + 4)(7 - 8x) = 21x - 24x^2 + 28 - 32x$$
$$= -24x^2 - 11x + 28$$

Reflection

The region containing x^2 in the rectangular array is different for both methods. Does this affect the answer? Why or why not?

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Practice Now 10 (textbook 2A, pg 59)

Expand each of the following expressions using a multiplication frame.

(a) $(4x+7)(3x+5)$

=

$\times$	$4x$	$+7$
$3x$		
$+5$		

(b) $(9x-4)(x+6)$

=

$\times$		

(c) $(3y+11)(2-7y)$

=

$\times$		

Worked Example 11 (textbook 2A, pg 59)

Simplifying quadratic expressions

Simplify $(2x - 3)(x + 5) - 2(4x + 1)(5 - 3x)$.

Solution

$\times$	x	$+5$
$2x$	$2x^2$	$+10x$
-3	$-3x$	-15

$\times$	5	$-3x$
$4x$	$20x$	$-12x^2$
$+1$	$+5$	$-3x$

Big Idea

Equivalence

Writing $(2x - 3)(x + 5)$ and $(4x + 1)(5 - 3x)$ in their equivalent expanded forms $2x^2 + 7x - 15$ and $-12x^2 + 17x + 5$ helps us to simplify the expression.

$$
\begin{aligned}
(2x - 3)(x + 5) - 2(4x + 1)(5 - 3x) &= 2x^2 + 10x - 3x - 15 - 2(-12x^2 + 20x - 3x + 5) \\
&= 2x^2 + 7x - 15 - 2(-12x^2 + 17x + 5) \\
&= 2x^2 + 7x - 15 + 24x^2 - 34x - 10 \\
&= 26x^2 - 27x - 25
\end{aligned}
$$

$-(x - y) = -x + y$;
$-(x + y) = -x - y$

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Practice Now 11 (textbook 2A, pg 59)

Simplify each of the following expressions.

(a) $(3x - 2)(x + 4) - 5x(x - 3)$

(b) $(5y - 1)(y + 6) + 3(4y - 5)(9 - 2y)$

Worked Example 12 (textbook 2A, pg 60)

Expanding quadratic expressions in two variables

Expand $(x + 2y)(x - 3y)$.

Solution

Method 1:

$$(x + 2y)(x - 3y) = x^2 - 3xy + 2yx - 6y^2 \quad \text{Distributive Law}$$
$$= x^2 - xy - 6y^2$$

Method 2:

$\times$	x	$-3y$
x	x^2	$-3xy$
$+2y$	$+2yx$	$-6y^2$

$$\therefore (x + 2y)(x - 3y) = x^2 - 3xy + 2yx - 6y^2$$
$$= x^2 - xy - 6y^2$$

Problem-solving Tip

Recall that multiplication is commutative, therefore $yx = xy$.

Reflection

Which method do you prefer? Why?

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Practice Now 12 (textbook 2A, pg 60)

1. **Expand** $(2x - 7y)(5x + y)$.

2. **Simplify** $(3w + 5v)(2v - 5w) - 6w(w - 2v)$.

Marked Assignment:

Workbook 2A
Worksheet 3C (p.37-40): Q2(a), 5(f), 7(d), 7(f)

3.4 Factorisation of quadratic expressions

A. Factorisation by extracting common factors

In Book 1, we have learnt how to **factorise** an algebraic expression of the form $ab + ac$ by extracting the common factor a in both terms to obtain:

$$ab + ac = a(b + c).$$

Factorisation is the reverse of expansion:

$$\begin{tikzcd} a(b+c) \arrow[r, bend left, "\text{expand}"] & ab+ac \arrow[l, bend left, "\text{factorise}"] \end{tikzcd}$$

Fig. 3.6

Recall

Factorisation is the process of expressing an algebraic expression as the *product* of two or more factors.

Information

This diagram helps to depict the reverse process of expansion and factorisation.

When extracting negative common factors such as -1, we have to *change the sign* inside the brackets as shown below:

Extracting negative common factor

factor out -1

$$-x - y = -(x + y)$$

change sign

Recap!

Practice Now 13 (textbook 2A, pg 62)

Factorise each of the following expressions completely.

(a) $12x+8$

(c) $-15x-25$

(f) $-42xy-12xz$

(h) $-9z-24bz-15cz$

Worked Example 14 (textbook 2A, pg 63)

Factorising expressions involving squares and cubes

Factorise each of the following expressions completely.

(a) $6x^2 + 15x$

(b) $-a^3bc - a^2b^2$

Solution

(a) $6x^2 + 15x = 3x(2x + 5)$ HCF of 6 and 15 = 3; HCF of x^2 and $x = x$

(b) $-a^3bc - a^2b^2 = -a^2b(ac + b)$ HCF of a^3 and $a^2 = a^2$; HCF of b and $b^2 = b$

TRAINING ARENA!

Practice Now 14A (textbook 2A, pg 63)

Factorise each of the following expressions completely.

(a) $10x^2 + 8x$

(c) $-49b - 28b^2$

(d) $2\pi r^2 + 2\pi rh$

(e) $x^2yz^3 - yz^2$

B. Factorisation of quadratic expressions of the form $x^2 + bx + c$

Using multiplication frame

We have learnt previously to use the multiplication frame to help us in the expansion of $(x+2)(x+3) = x^2 + 5x + 6$.

We now use the multiplication frame to work out the factorisation of $x^2 + 5x + 6$.

Step 1: Write the term x^2 and the constant term 6 in the multiplication frame as shown in (a)

$\times$		
	x^2	
		$+6$

(a)

Step 2: Fill in the sides with possible factors as shown in (b)

Factors of x^2 is

$$x^2 = x \times x$$
$$= -x \times -x$$

Factors of 6

$$6 = 1 \times 6$$
$$= 2 \times 3$$
$$= -1 \times -6$$
$$= -2 \times -3$$

$\times$	x	$+2$
x	x^2	
$+3$		$+6$

(b)

Choose the factors where the sum of it gives $+5$ since the coefficient of x is $+5$.

Step 3: Fill in the multiplication frame and check if the expansion is valid.

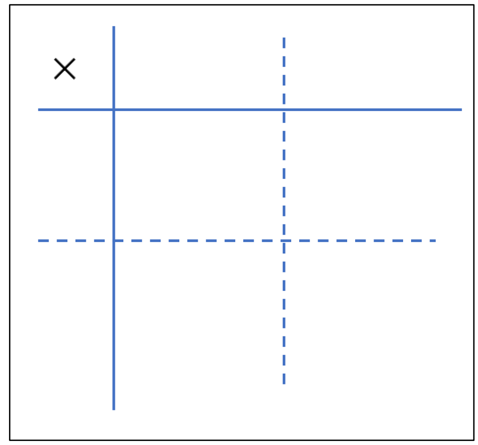
Since $+2 \times x = +2x$ and $+3 \times x = +3x$, we have $+2x + 3x = +5x$.

We write the final answer $x^2 + 5x + 6 = (x+2)(x+3)$

$\times$	x	$+2$
x	x^2	$+2x$
$+3$	$+3x$	$+6$

Try it yourself! (Follow the steps and fill in the multiplication frame)

Q1. Let us try factorising the expression $x^2 - 5x + 6$.



Step 1: Write the term x^2 and the constant term -6 in the multiplication frame.

Step 2: Fill in the sides with possible factors.

List down the factors of -6.

$$\begin{aligned} 6 &= 1 \times 6 \\ &= -1 \times -6 \\ &= 2 \times 3 \\ &= -2 \times -3 \end{aligned}$$

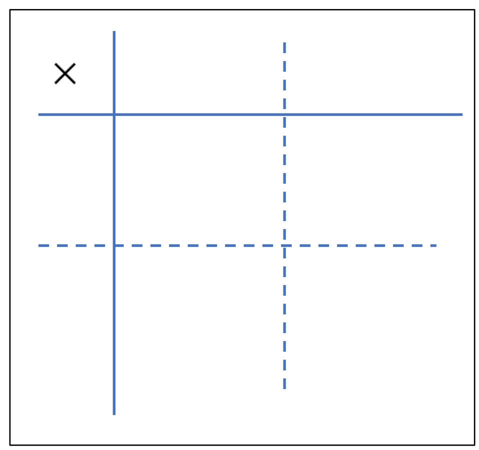
Choose the pair of factors whose sum is -5 (since coefficient of x is $-5x$).

Step 3: Fill in the remaining blanks

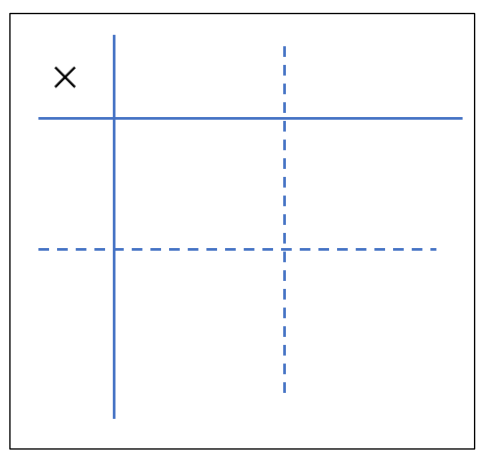
Since $-2 \times x = -2x$ and $-3 \times x = -3x$, we have $(-2x) + (-3x) = -5x$.

We write the final answer $x^2 - 5x + 6 =$ ____________________________

Q2. Factorise $x^2 + 5x - 6$.



Q3. Factorise $x^2 - 5x - 6$.



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Practice Now 14B (textbook 2A, pg 68)

Factorise each of the following quadratic expressions expressions.

(a) $x^2 + 6x + 5$

(c) $x^2 - x - 12$

(e) $y^2 - 8y + 12$

(g) $z^2 + 8z + 12$

E. Factorisation of quadratic expressions of the form ax^2+bx+c , where $a \neq 1$.

Factorise $2x^2+7x-15$**.**

Step 1: Write the term $2x^2$ and the constant term -15 in the multiplication frame.

Step 2: Fill in the sides with possible factors and complete the frame.

*Notice that $2x^2 = 2x \times x$

$$
\begin{aligned}
-15 &= 1 \times (-15) \quad \text{or} \quad (-1) \times 15 \\
&= 3 \times (-5) \quad \text{or} \quad (-3) \times 5
\end{aligned}
$$

Which pair of factors of −15 should we put in the two ? positions?

×	x	?
$2x$	$2x^2$	
?		-15

guess →

Let us try −3 and 5 in these positions.

×	x	-3
$2x$	$2x^2$	
$+5$		-15

check →

$5x - 6x = -x \neq 7x$, so this cannot work.

×	x	-3
$2x$	$2x^2$	$-6x$
$+5$	$+5x$	-15

guess ↓

We can try −3 and 5 again but in different positions.

×	x	$+5$
$2x$	$2x^2$	
-3		-15

check →

Complete the frame. Since $10x - 3x = 7x$, this is correct.

×	x	$+5$
$2x$	$2x^2$	$+10x$
-3	$-3x$	-15

$\therefore 2x^2 + 7x - 15 = (2x-3)(x+5)$

TRAINING ARENA!

Practice Now 14C (textbook 2A, pg 71)

Factorise each of the following quadratic expressions expressions completely.

(a) $2x^2 + 7x + 6$

(c) $6y^2 - 11y + 4$

(e) $-x^2 + 6y - 9$

(g) $4x^2 - 6x - 4$

Worked Example 16 (textbook 2A, pg 71)

Factorising quadratic expressions in two variables using multiplication frame

Factorise $x^2 + 5xy - 6y^2$.

Solution

Let us compare the factorisation of $x^2 + 5x - 6$ and the factorisation of $x^2 + 5xy - 6y^2$.

> **Problem-solving Tip**
>
> If you cannot factorise $x^2 + 5xy - 6y^2$ directly, you can factorise $x^2 + 5x - 6$ first and then add in the appropriate y and y^2 later.

Factorisation of $x^2 + 5x - 6$:

since $6x - x = 5x$

$\times$		
	x^2	
		-6

$\rightarrow$

$\times$		
	x^2	$+6x$
	$-x$	-6

$\rightarrow$

$\times$	x	$+6$
x	x^2	$+6x$
-1	$-x$	-6

$x^2 + 5x - 6 = (x - 1)(x + 6)$

Factorisation of $x^2 + 5xy - 6y^2$:

since $6xy - xy = 5xy$

$\times$		
	x^2	
		$-6y^2$

$\rightarrow$

$\times$		
	x^2	$+6xy$
	$-xy$	$-6y^2$

$\rightarrow$

$\times$	x	$+6y$
x	x^2	$+6xy$
$-y$	$-xy$	$-6y^2$

$x^2 + 5xy - 6y^2 = (x - y)(x + 6y)$

TRAINING ARENA!

Practice Now 16 (textbook 2A, pg 72)

Factorise each of the following expressions completely.

(a) $x^2 + 2xy - 8y^2$

(c) $6x^2 + 11xy + 5y^2$

(e) $-a^2 + 5ab - 6b^2$

Marked Assignment:

Workbook 2A
Worksheet 3D (p.41-44): Q1(f), 2(b), 4(b), 6

3.5 Factorisation of algebraic expressions into the form $(a+b)(c+d)$

A. Factorsiation into the form $(a+b)(c+d)$.

How do we factorise $ac+ad+bc+bd$?

Method 1: Factorisation using multiplication frame

		extract common factor c	extract common factor d
	$\times$	c	$+d$
extract common factor a	a	ac	$+ad$
extract common factor b	$+b$	$+bc$	$+bd$

$\therefore\ ac + ad + bc + bd = (a + b)(c + d)$

Method 2: Factorisation by grouping

In the expansion of $(a + b)(c + d)$, the intermediate step is $a(c + d) + b(c + d)$:

$$\begin{aligned}(a + b)(c + d) &= a(c + d) + b(c + d)\\ &= ac + ad + bc + bd\end{aligned}$$

To factorise an algebraic expression of the form $ac + ad + bc + bd$, we should group the four terms into two appropriate groups, where the two terms in each group have a common factor. Then, we can extract the common factor of each group:

$$\begin{aligned}ac + ad + bc + bd &= (ac + ad) + (bc + bd) && \text{group the 4 terms into 2 groups}\\ &= a(c + d) + b(c + d) && \text{extract common factor from each group}\\ &= (a + b)(c + d) && \text{extract common factor } (c + d)\end{aligned}$$

Attention

It does not help if we group $ac + ad + bc + bd$ as $(ac + bd) + (bc + ad)$. Why?

This method is called **factorisation by grouping** because we *group* the four terms into two appropriate groups first.

Worked Example 17 (textbook 2A, pg 75)

Factorising algebraic expressions into the form $(a + b)(c + d)$

Factorise each of the following expressions completely.

(a) $ab + ad + 2bc + 2cd$ **(b)** $12ax - 6by - 8bx + 9ay$

Solution

(a) Method 1:

$\times$	b	$+d$
a	ab	$+ad$
$+2c$	$+2bc$	$+2cd$

$\therefore ab + ad + 2bc + 2cd$
$= (a + 2c)(b + d)$

Method 2:

$ab + ad + 2bc + 2cd$

$= (ab + ad) + (2bc + 2cd)$ group the 4 terms into 2 groups

$= a(b + d) + 2c(b + d)$ extract common factors from each group

$= (a + 2c)(b + d)$ extract common factor $(b + d)$

> **Reflection**
>
> **(a)** Is there another way to arrange the four terms? E.g. in **Method 2**, can we arrange the terms as such: $ab + 2bc + ad + 2cd$?

(b) Method 1(i):

$\times$	$4x?$	
6	$12ax$	$-6by$
	$-8bx$	$+9ay$

$6 \times 4x \neq 12ax$
This arrangement of the four terms does not help.

Method 2(i):

$12ax - 6by - 8bx + 9ay$

$= 6(2ax - by) - 8bx + 9ay$

This arrangement of the four terms does not help.

Method 1(ii):

$\times$	$3a$	$-2b$
$4x$	$12ax$	$-8bx$
$+3y$	$+9ay$	$-6by$

If we only extract $2b$,
$4x \times 2b \neq -8bx$.
Hence, it is a good practice to always check by expansion.

$\therefore 12ax - 6by - 8bx + 9ay$
$= (4x + 3y)(3a - 2b)$

Method 2(ii):

$12ax - 6by - 8bx + 9ay$

$= (12ax - 8bx) + (9ay - 6by)$ group the 4 terms into 2 groups

$= 4x(3a - 2b) + 3y(3a - 2b)$ extract common factors from each group

$= (3a - 2b)(4x + 3y)$ extract common factor $(3a - 2b)$

> **Reflection**
>
> **(b)** Which method do you prefer? Why?

TRAINING ARENA!

Practice Now 17 (textbook 2A, pg 75)

Factorise each of the following expressions completely.

(a) $ab+ac+2bd+2cd$

(c) $6ax-20by-8bx+15ay$

Practice Now 18 (textbook 2A, pg 76)

Find the two factors of each of the following expressions.

(a) $6xy-15x+20-8y$

(b) $6ab-9ac+21c-14b$

Practice Now 19 (textbook 2A, pg 77)

Factorise each of the following expressions fully.

(a) $x^2 + xy - 3x - 3y$

(b) $15w^2 - 20w - 6wz + 8z$

Marked Assignment:

Workbook 2A
Worksheet 3E (p.45-46): Q1(e), 2(d)

We have come to the end of this chapter!

Let's see what you have learnt...

1. Addition and subtraction of like terms in x^2

To add or subtract like terms in x^2, we add or subtract their coefficients in the same way we would add or subtract real numbers.

Real numbers	Terms in x^2
$(-2) + (-3) = -5$	$(-2x^2) + (-3x^2) = -5x^2$
$5 + (-2) = 5 - 2 = 3$ $-2 + 5 = 5 - 2 = 3$	$5x^2 + (-2x^2) = 5x^2 - 2x^2 = 3x^2$ $-2x^2 + 5x^2 = 5x^2 - 2x^2 = 3x^2$
$2 - 5 = -3$	$2x^2 - 5x^2 = -3x^2$
$-5 - 2 = -7$	$-5x^2 - 2x^2 = -7x^2$
$5 - (-2) = 5 + 2 = 7$	$5x^2 - (-2x^2) = 5x^2 + 2x^2 = 7x^2$

2. The Distributive Law for $(a + b)(c + d)$

$$(a + b)(c + d) = ac + ad + bc + bd$$

3. Expansion of quadratic expressions using multiplication frame

For example, to expand $(3x + 4)(7 - 8x)$:

$\times$	7	$-8x$
$3x$		
$+4$		

$\rightarrow$

$\times$	7	$-8x$
$3x$	$21x$	$-24x^2$
$+4$	$+28$	$-32x$

$$\therefore (3x + 4)(7 - 8x) = 21x - 24x^2 + 28 - 32x$$
$$= -24x^2 - 11x + 28$$

- Expand $(2x + 5)(3 - x)$ using a multiplication frame.

4. Factorisation of quadratic expressions using multiplication frame

For example, to factorise $2x^2 + 7x - 15$:

Method 1: Guess and Check

$-15 = 1 \times (-15)$ or $(-1) \times 15$

$\quad = 3 \times (-5)$ or $(-3) \times 5$

$\times$	x	?
$2x$	$2x^2$	
?		-15

guess →

$\times$	x	$+5$
$2x$	$2x^2$	
-3		-15

check →

$\times$	x	$+5$
$2x$	$2x^2$	$+10x$
-3	$-3x$	-15

$\therefore 2x^2 + 7x - 15 = (2x - 3)(x + 5)$

Method 2: Use some reasoning

$-30 = 1 \times (-30)$ or $(-1) \times 30$

$\quad = 2 \times (-15)$ or $(-2) \times 15$

$\quad = 3 \times (-10)$ or $(-3) \times 10 \longrightarrow (-3) + 10 = 7$ (coefficient of x term)

$\quad = 5 \times (-6)$ or $(-5) \times 6$

$\times$		
	$2x^2$	$?x$
	$?x$	-15

guess →

$\times$		
	$2x^2$	$-3x$
	$+10x$	-15

check →

$\times$	$2x$	-3
x	$2x^2$	$-3x$
$+5$	$+10x$	-15

$\therefore 2x^2 + 7x - 15 = (2x - 3)(x + 5)$

- Factorise $15 + x - 2x^2$ using a multiplication frame.

5. Factorisation of algebraic expression into the form $(a + b)(c + d)$

Method 1: Multiplication frame

$\times$	c	$+d$
a	ac	$+ad$
$+b$	$+bc$	$+bd$

$\therefore ac + ad + bc + bd = (a + b)(c + d)$

Method 2: By grouping

$$\begin{aligned} ac + ad + bc + bd &= (ac + ad) + (bc + bd) \\ &= a(c + d) + b(c + d) \\ &= (a + b)(c + d) \end{aligned}$$