Triangle study

Trigonometry & right triangles

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Triangle study

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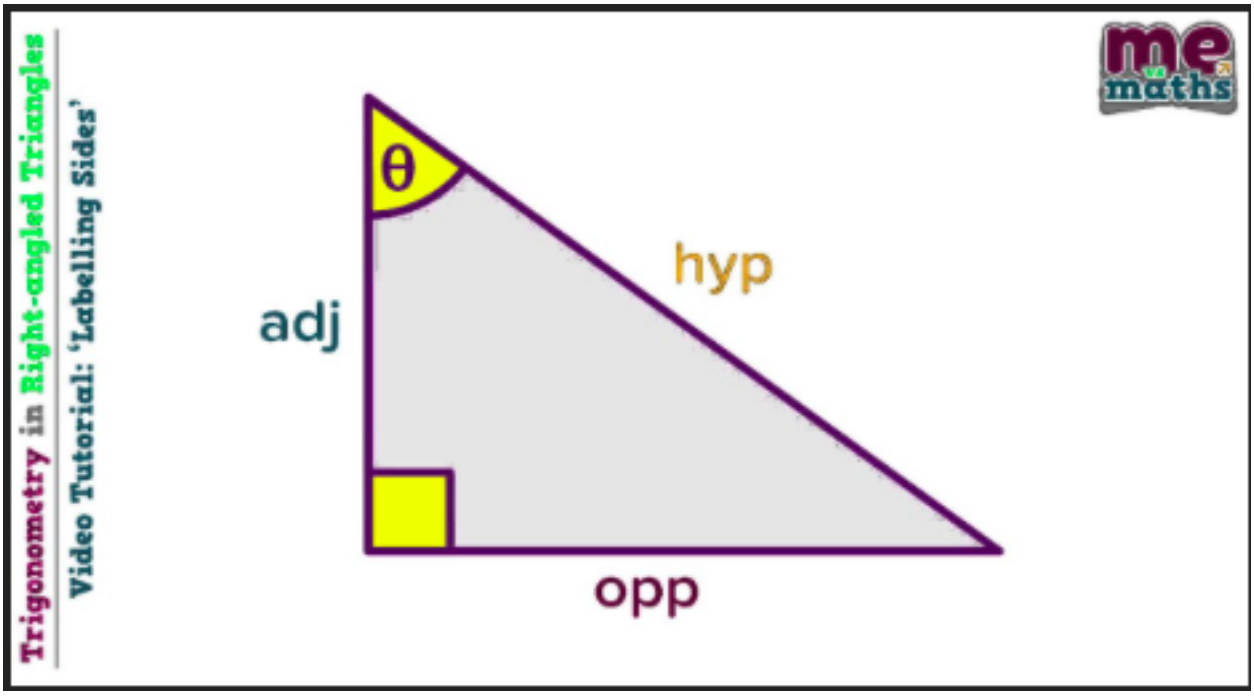
Right-angled triangles

What are right-angled triangles?

Right-angled triangles are types of triangles that have right angles in them. Using trigonometric ratios, it is possible to find the length of one of the sides of a triangle if you know 1 side and two angles, and it is possible to find an angle if you know 1 angle (the right angle) and if you know two sides. To find anything in a right-angled triangle, you have to know 3 variables[i]. To find anything in a right-angled triangle, you need to know the sides of the triangle. When you are given a question on finding the length, you will be given the right angle, and you will also be given an acute angle. Using this acute angle, you can find the names of the sides.

[ii]



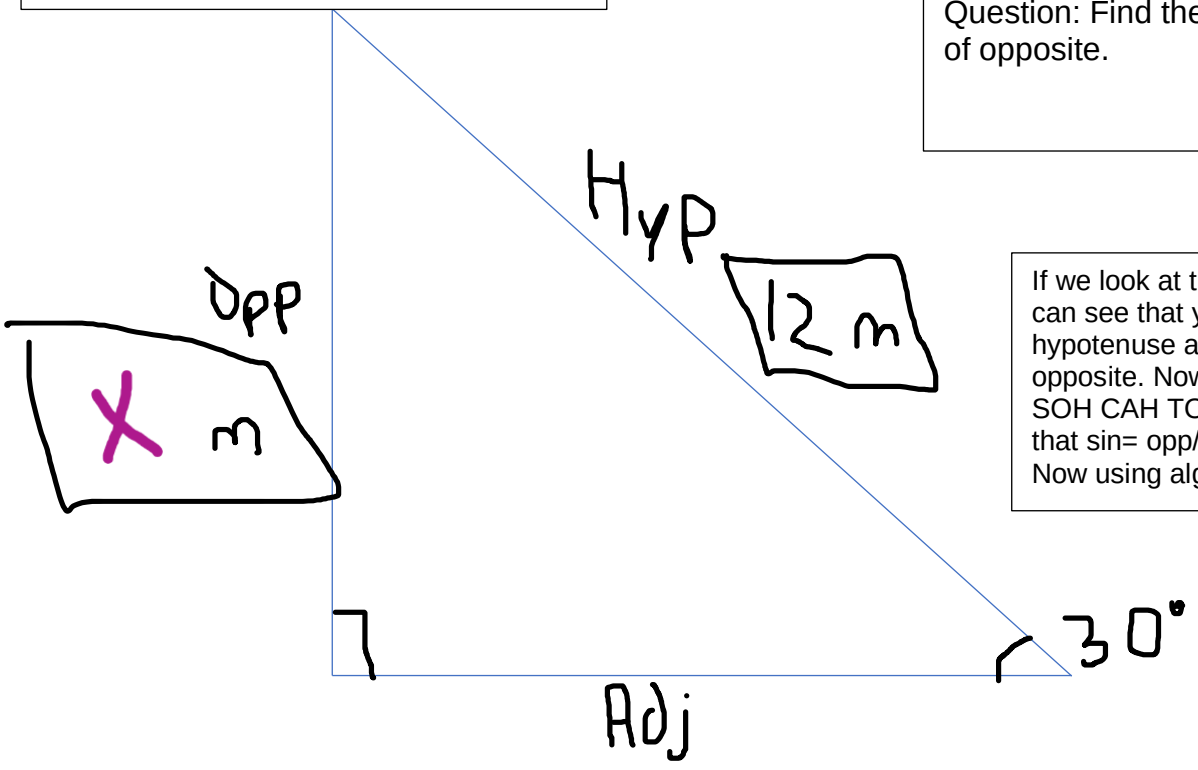
As you can see in this image, the sides are labeled. 'adj' stands for adjacent, 'opp' stands for opposite, and 'hyp' stands for hypotenuse. The hypotenuse is the easiest to find. It is the longest side of the triangle. The opposite is the side that is directly opposite the angle θ that is shown in the image.

To find the adjacent, just look for the side that constructs the angle but is not the hypotenuse. It is also the side adjacent to the angle. Using

the side names, you can also find the length of one of the other sides of the triangle if you have two. When you do this, you use the Pythagoras theorem, which says that $A_2+B_2=C_2$. A and B are the two legs of the triangle, and C is always hypotenuse. A and B can be either opposite or adjacent, because we need to find C. If you have opposite and hypotenuse, then using algebra, you can do: $C_2 - B_2= A_2$ and vice versa. To find the side, you have to find the square root of A. Now that you know the names of the sides and the Pythagoras theorem, you can find the length of them. To find sides when two sides are not given or to find an angle when two sides and one angle are given, we use ratios called sine, cosine, and tangent. The mnemonic SOH CAH TOA is useful. This means sin= opp/hyp, cos=adj/hyp, and tan= opp/adj.

In this triangle, you might be wondering, “What are sin, cos, and tan?” These are short forms for sine, cosine, and tangent, which are ratios used to find lengths.

Question: Find the length of opposite.



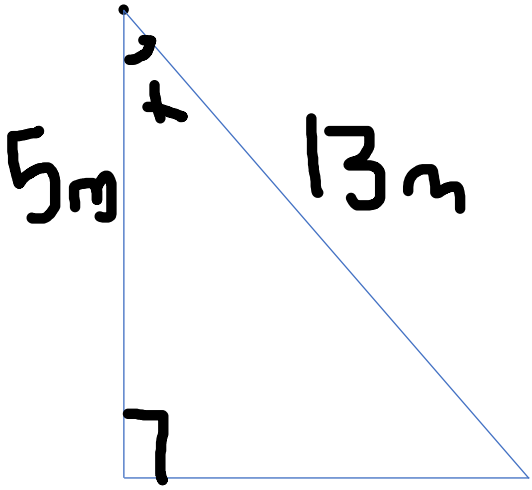
If we look at this triangle, you can see that you know the hypotenuse and you have to find opposite. Now if you think of SOH CAH TOA, you can see that sin= opp/hyp, or sin= x/12. Now using algebra-

$\sin 30 = \frac{x}{12}$

$12(\sin 30) = x$

As shown, sine has been put before the angle. So now when you have this, you have to enter this equation into a calculator, and you will get the value of x. For this example, x=6 m.

The next thing is finding angles. To find the angle of the triangle, it is required to use algebra.

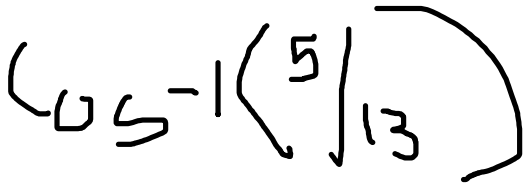


$\cos x = \frac{5}{13}$

$x = \cos^{-1} = \frac{5}{13}$

When any ratio is moved to another side, it becomes inverse. An inverse ratio looks like this: COS^{-1}

If you enter the value into a calculator[iii], it will look like this:



X=67.38013505

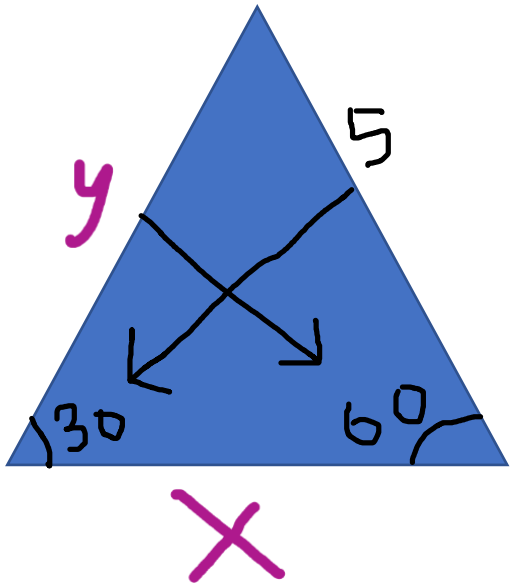
It is also possible to round the number off, so that it can be written easily.

Finding sides & angles on non-right-angled triangles: the law of Sines and Cosines

The sine rule

An important thing to understand is where to use the sine rule. The sine rule can only be used when two angles and one side are given, or two sides and one angle. There is, however, another condition:

The sine rule only works when the given sides and angles are OPPOSITE to each other.



Formula-

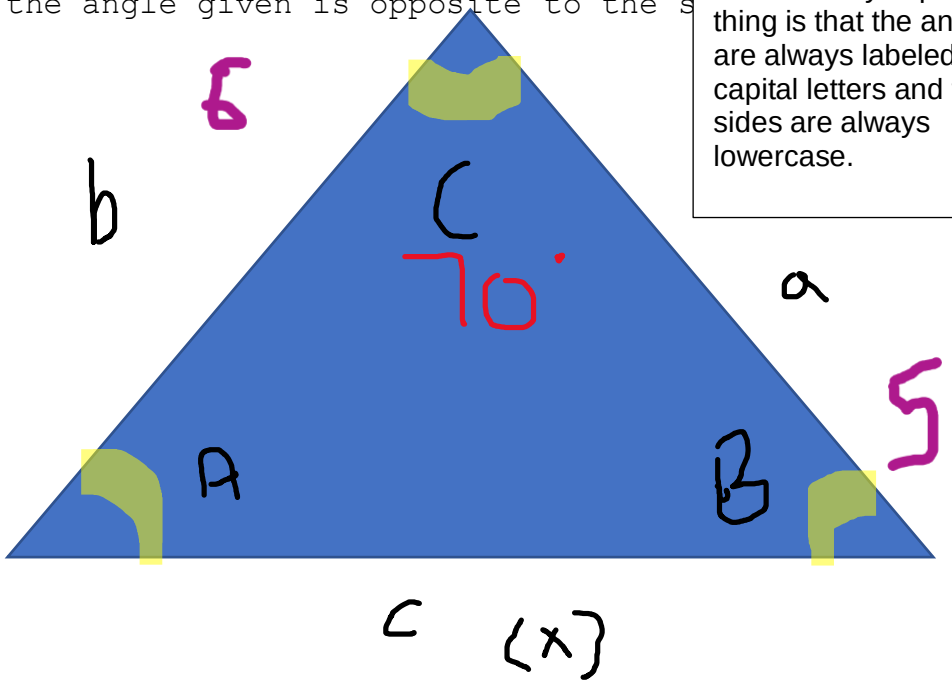
Side 1/ Sine(opposite angle)
Side 2/ Sine(opposite angle)
Side 3/ Sine(opposite angle)
If we use this in this example, it will look like this-

$$\frac{5}{\sin 30} = \frac{x}{\sin 50} = \frac{y}{\sin 60}$$

You may notice that X/Sine 50 and Y/sine 60 are shown. To find X, you have to do: 5 (sin 50)/sin 30. Notice that the angle has been moved up onto the next angle.

Next, let's use the Cosine rule[iv]. The cosine rule is somewhat similar to the Pythagoras theorem, but we'll get to how it is similar later. The Cosine rule can be used when there is 1 angle and two sides, or three angles and one side given. Remember- these rules are only used when you have to calculate the sides or angles of NON-RIGHT-ANGLED triangles. Another thing to remember when working with the Cos rule is that all the sides are labeled a, b, and c, and c is the unknown side, or x. The sides a, b, and c are opposite to the angles A, B, and C in the diagram that is given below. It is also important to know that the C[illegible] used when the angle given is opposite to the s[illegible] found.

Another very important thing is that the angles are always labeled in capital letters and the sides are always lowercase.



This triangle was just drawn, so the formula may not work on this, but when questions are given, you can use this.
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(C)$ I had also written that this is somewhat like the Pythagoras theorem- as you can see, it has a, b, and c squared.

How is trigonometry used in architecture? [iii]

Architects use trigonometry to calculate many things. Trigonometry helps in calculating the slopes on a roof or the height of structures, or scaled models (when they are drawn). The simplest example of trigonometry in architecture is the height of a building when you are standing a certain distance from it, a common math problem. But triangles in general are also used in architecture. Triangles have been used in architecture for a very long time: they were even used in ancient work like the pyramids of Giza. They are also used to construct interesting designs such as Le Louvre in France, or even in houses in the taiga biome to stop snow from being collected on the roof.

[v]

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[i] Variables can be any value. In the context of knowing 3 variables means that 3 values have to be known to find anything in the triangle.

ii https://i.ytimg.com/vi/1ALLrv2dQxc/maxresdefault.jpg

iii https://www.calculator.net

iv https://www.reference.com/world-view/architects-use-trigonometry-f5d1cf5e9de131cb

v https://www.mathsisfun.com/algebra/trig-cosine-law.html

vi https://sites.google.com/site/idelise66/museums/le-louvre-paris-france