Homework 8
ME5659 Spring 2024

Due: See Canvas, turn in on Gradescope

Problem 1 (13 points)

Consider a linear state-space model

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

(a) **3 points.** Design a state feedback control law $u = -Kx$ so that the closed-loop system has natural frequency $\omega_n = 1\ rad/sec$ and damping ratio $\zeta = 0.707$.

(b) **3 points.** Design an observer gain L so that the error dynamics have natural frequency $\omega_n = 10\ rad/sec$ and damping ratio $\zeta = 0.5$.

(c) **3 points.** Use your answers to design an observer-based controller that achieves both objectives in (a) and (b). Please write the full dynamic equations including $\dot{x}, \dot{\hat{x}}, u$.

(d) **4 points.** Assume the initial conditions are given by $x_0 = \begin{bmatrix} 5 & -4 \end{bmatrix}$, $\hat{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$. Let the feedforward gain $k_g = 1$. Given a reference input $r(t) = sin(t)$, please simulate the closed-loop full dynamical system (including x and $\hat{x}$) over the time horizon $t \in [0, 10]\ sec$. Generate 2 figures: one that plots $x_1(t)$ and its estimate $\hat{x}_1(t)$ over time, and one that plots $x_2(t)$ and its estimate $\hat{x}_2(t)$ over time. Hand in your code and your plots.

SOLUTIONS

a) $\omega_n = 1$ rad/sec

$\zeta = 0.707$

Second order characteristic polynomial

$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2 \cdot 0.707 \cdot 1 \cdot s + 1$

$\Rightarrow s^2 + 1.41 s + 1$ its eigenvalues are $\mu_{1,2} = -0.705 \pm 0.709i$

Desired closed loop characteristic polynomial

$\alpha(s) = (s-\mu_1)(s-\mu_2)$

$\alpha(s) = (s + 0.705 - 0.709j)(s + 0.705 + 0.709j) = \alpha(s) = s^2 + \underbrace{1.41}_{\alpha_1} s + \underbrace{1}_{\alpha_0}$

OPEN LO CHARACTERISTIC POLYNOMIAL

$A = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\quad P = \begin{bmatrix} B & AB \end{bmatrix}$

controllability matrix $P = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

$\det P = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} = 8 \neq 0$ the system is controllable

$\det(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} s & 0 \\ -2 & s \end{bmatrix} = s^2$ $\quad P_{CCF}^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad T_{CCF} = P P_{CCF}^{-1}$

$$T_{CCF} = \begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix} \quad A_{CCF} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_{CCF} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[sI - A_{CCF}] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix} = s^2$$

$$K_{CCF} = \begin{bmatrix} 1 & 1.41 \end{bmatrix}$$

BESS-GURA FORMULA

$$K = K_{CCF} (P \cdot P_{CCF}^{-1})^{-1} = \begin{bmatrix} 0.705 & 0.25 \end{bmatrix}$$

Let's if it is correct, we want to find the eigenvalues of

$$(A - BK) = \begin{bmatrix} -1.41 & -0.5 \\ 2 & 0 \end{bmatrix}$$

we obtained these values with the specifications of $\xi = 0.707$ and $\omega_n = 1$ as whished

$s^2 + 1.41s + 1$ eigenvalues are $\mu_{1,2} = -0.705 \pm 0.709i$

LET'S use ACKERMANN'S FORMULA

$$K = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot P^{-1} \alpha(A)$$

$$\alpha(A) = A^2 + \alpha_1 A + \alpha_0 I$$

$K = \begin{bmatrix} 0.705 & 0.25 \end{bmatrix}$ same result as before.

b) $\omega_n = 10 \quad \xi = 0.5$

First of all, we need to find the OBSERVABILITY matrix

$$A = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

$\det Q = -2 \neq 0$ $\operatorname{rk} Q = 2$ the system is observable and so we can write our system in OBF

$$A_{COF} = \begin{bmatrix} 0 & -a_0 \\ 1 & -a_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$|sI - A| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right| = \left| \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix} \right| = s^2 \quad \begin{matrix} a_0 = 0 \\ a_1 = 0 \end{matrix}$$

Open loop characteristic polynomial:

$a(s) = s^2 + a_1 s + a_0$

$a(s) = s^2$ $\qquad a_0 = 0$, $a_1 = 0$

Let's now find the desired characteristic polynomial.

$\alpha(s) = (s-\mu_1)(s-\mu_2)$ where μ_1 and μ_2 are the eigenvalues taking into account the following specifications. $\omega_n = 10$ rad/s $\quad \xi = 0.5$

Second order characteristic polynomial

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 2 \cdot 0.5 \cdot 10\, s + 100 = s^2 + 10s + 100$$

eigenvalues: roots of the characteristic polynomial $\mu_{1,2} = -5 \pm 5\sqrt{3}\, i = -5 \pm 8.66\, i$

$$\alpha(s) = (s+5-5\sqrt{3}j)(s+5+5\sqrt{3}j) = s^2 + \underline{10}s + \underline{100}$$

$\alpha_0 = 100$

$\alpha_1 = 10$

$L_{OCF} = [(100)\ (10)]^T$

Bass-Gura formula

$$Q_{OCF}^{-1} = \begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$L = T_{OCF} \cdot L_{OCF} = (Q_{OCF}^{-1} \cdot Q)^{-1} L_{OCF} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}\right)^{-1}\begin{bmatrix} 100 \\ 10 \end{bmatrix}$$

$$L = \begin{bmatrix} 50 \\ 10 \end{bmatrix}$$

Let's if we obtain the same result using Ackermann's formula

$$L = \alpha(A) \cdot Q^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha(A) = A^2 + \alpha_1 A + \alpha_0 I$$

$$L = \begin{bmatrix} 50 \\ 10 \end{bmatrix}$$ same result.

Let's verify that the eigenvalues of

$A - LC$ are $\mu_1, \mu_2 = -5 \pm 8.66\, i$

$|[sI - (A - LC)]| = s^2 + 10s + 100$ correct ✓

C

Controller $\quad u(t) = -K\hat{x}(t) \qquad \dot{x} = (A - BK)x$

Observer $\quad \dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$

$\dot{\hat{x}} = A\hat{x} + B(-K\hat{x}(t)) + Ly - LC\hat{x}$

$\dot{\hat{x}} = A\hat{x} - BK\hat{x}(t) + Ly - LC\hat{x}$

$\dot{\hat{x}} = (A - BK - LC)\hat{x}(t) + L(Cx + Du)$

$D = 0$

$\Rightarrow \dot{\hat{x}} = (A - BK - LC)\hat{x}(t) + LCx$

$$\dot{\hat{x}} = \left(\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} - \underbrace{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}_{2\times1} \underbrace{\begin{bmatrix} 0.705 & 0.25 \end{bmatrix}}_{1\times2} - \underbrace{\begin{bmatrix} 50 \\ 10 \end{bmatrix}}_{2\times1} \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{1\times2} \right) \hat{x}(t) + \begin{bmatrix} 50 \\ 10 \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix} x}_{Cx = y}$$

$$\dot{\hat{x}} = \left(\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1.41 & 0.5 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 50 \\ 0 & 10 \end{bmatrix} \right) \hat{x}(t) + \begin{bmatrix} 50 \\ 10 \end{bmatrix} y$$

$$\dot{\hat{x}} = \begin{bmatrix} -1.41 & -50.50 \\ 2 & -10 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} 50 \\ 10 \end{bmatrix} y$$

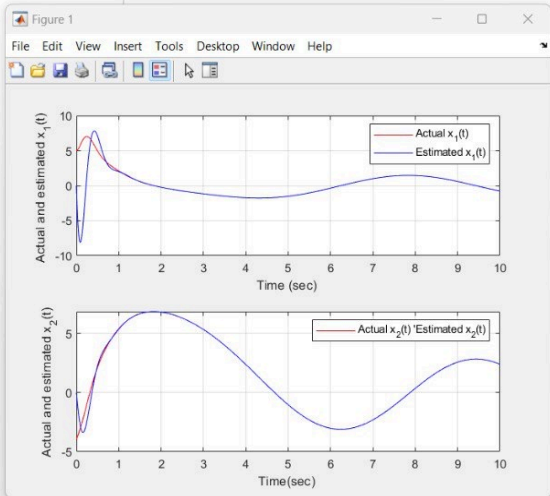
$$u = -k\hat{x}(t)$$

$$u = -\begin{bmatrix} 0.705 & 0.25 \end{bmatrix} \hat{x}(t)$$

d)

```
clear all; clc; close all;
poly = [1 1.41 1];
poly_roots=roots(poly);
syms s;
%Part A desired closed loop characteristic polynomial
alpha=(s-poly_roots(1))*(s-poly_roots(2));
simplify(alpha);
A=[0 0; 2 0];
B=[2; 0];
C=[0 1];
P=ctrb(A,B);
rank(P);
poly_open_loop=det(s*eye(2)-A);
Pccf_inv=[0 1; 1 0];
Tccf=P*Pccf_inv;
Accf=inv(Tccf)*A*Tccf;
Bccf=inv(Tccf)*B;
Kccf=[1 1.41];
K=Kccf*inv(P*Pccf_inv);
det(s*eye(2)-(A-B*K));
akerman=[0 1]*inv(P)*(A^2+1.41*A+1*eye(2));
%part b
Q=obsv(A,C);
det(Q);
rank(Q);
poly_2 = [1 10 100];
poly_roots_2=roots(poly_2);
```

```
syms s;
%Part A desired closed loop characteristic polynomial
alpha_2=(s-poly_roots_2(1))*(s-poly_roots_2(2));
simplify(alpha_2);
Locf=[100; 10];
Qocf_inv=[0 1; 1 0];
L=inv(Qocf_inv*Q)*Locf;
Akerman_2=(A^2+10*A+100*eye(2))*inv(Q)*[0;1];
det(s*eye(2)-(A-L*C));
%PART d
%initial conditions
G=1;
x0=[5; -4];
h_hat0=[0;0];
z0=[x0; h_hat0];
%define the closed-loop state space matrices
A_cl=[A -B*K; L*C (A-B*K-L*C)];
B_cl=[B*G ; B*G ];
tspan=[0 10];
%define the closed loop system
ode=@(t,z) A_cl*z+B_cl*sin(t);
[t,z]=ode45(ode,tspan,z0);
x=z(:,1:2);
x_hat=z(:,3:4);
%plotting
figure;
subplot(2,1,1);
plot(t,x(:,1),'r',t,x_hat(:,1),'b');
xlabel( 'Time (sec) ');
ylabel('Actual and estimated x_1(t) ');
legend ('Actual x_1(t)', 'Estimated x_1(t) '); grid on;
subplot (2,1,2);
plot(t,x(:,2), 'r',t,x_hat(:,2), 'b');
xlabel( 'Time(sec) ');
ylabel('Actual and estimated x_2(t)');
legend ('Actual x_2(t) ''Estimated x_2(t) '); grid on;
```



Problem 2 (12 points)

Consider a single-input single-output four-dimensional state-space model:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -650 & -180 & -90 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \quad x_0 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 90 & 15 & 10 & 0 \end{bmatrix} x$$

You may use MATLAB for this entire problem. Hand in your codes and plots.

(a) **6 points.** Design a state-feedback integral controller to obtain 2 percent overshoot and a 2 s settling time as well as a steady-state output value of 1. Compare open-loop and closed-loop responses to a unit step input.

(b) **6 points.** Design an observer-based integral controller. Let observer eigenvalues be 10 times that of the desired state-feedback control law obtained in (a). Assume the initial condition $\hat{x}(0) = 0$. Compare open-loop, closed-loop with state-feedback, and closed-loop with observer responses to a unit step input.

SOLUTIONS

a) 1.) the OL pair (A,B) is controllable iff $rk P = n$

$P = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$ $\quad P = ctrb(A,B)$

$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -650 & -180 & -90 & -6 \end{bmatrix}$ $\quad \det P = 1 \quad \Rightarrow rk = 4 \quad$ it is controllable

we could also notice that our S stem is already in CCF, it is indeed controllable

2. OL state matrix A has no eigenvalues / poles at the origin. $(s=0)$

open-loop characteristic polynomial

$a(s) = s^4 + \underbrace{6}_{a_3} s^3 + \underbrace{90}_{a_2} s^2 + \underbrace{180}_{a_1} s + \underbrace{650}_{a_0}$

OPEN LOOP EIGENVALUES.

$\lambda_1 = -0.98535 + 2.8128i$

$\lambda_2 = -0.98535 - 2.8128i$

$\lambda_3 = -2.01465 + 8.3137i$

$\lambda_4 = -2.01465 - 8.3137i$

3. the open loop system has no zero at $s=0$

$\Rightarrow$ 2 and 3 imply that

$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is non singular and (A_1, B_1) is controllable

$H(s) = C(sI - A)^{-1}B$

$$H(s) = \frac{10s^2 + 15s + 90}{s^4 + 6s^3 + 90s^2 + 180s + 650}$$

so we can choose $K_1 = [K \;\; -K_I]$ to place eigenvalues as desired.

REQUIRMENTS.

2% 0.5

$T_s = 2s$

$y_{ss} = 1$

$U(s) = 1/s$

$$\xi = -\frac{\ln(2/100)}{\sqrt{\pi^2 + \ln^2(2/100)}} = 0.78$$

$$T_s = \frac{4}{\xi \omega_n} \qquad \omega_n = \frac{4}{T_s \cdot \xi} = 2.56$$

Second order characteristic polynomial.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 2 \cdot 0.78 \cdot 2.56 s + 2.56^2 = s^2 + 4s + 6.55$$

eigenvalues are

$$\mu_1 = \frac{-20 + j\sqrt{255}}{10} \; ; \quad \mu_2 = \frac{-20 - j\sqrt{255}}{10}$$

the third and fourth eigenvalues are chosen to be 10 times further to the left half plane

$\mu_3 = -20 \qquad \mu_4 = -21$

we need to consider the augmentum system 4 eigenvalues $A\,(n=4) + 1$ state $\xi = 5$

$\mu_5 = -22$ (to use the command place on matlab we need different eigenvalues.

AUGMENTED CL system

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} A - BK & -BK_I \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix}$$

$$A_{CL} = \underbrace{\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}}_{A_1} - \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_1} \underbrace{\begin{bmatrix} K & -K_I \end{bmatrix}}_{K_1}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -650 & -180 & -90 & -6 & 0 \\ -90 & -15 & -10 & 0 & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

we already know that (A_1, B_1) is controllable. In fact:

$P_1 = \mathrm{ctrb}(A_1, B_1)$

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & -6 \\ 0 & 0 & 1 & -6 & -54 \\ 0 & 1 & -6 & -54 & 684 \\ 1 & -6 & -54 & 684 & 1186 \\ 0 & 0 & -10 & 45 & 540 \end{bmatrix}$$

$\det P_1 = -90$

$\mathrm{rk}\, P_1 = 5 \Rightarrow$ controllable.

Let's design K_1 for an equivalent system

$\dot{x} = A_1 x + B_1 u$

$K_1 = \mathrm{place}(A_1, B_1, p)$

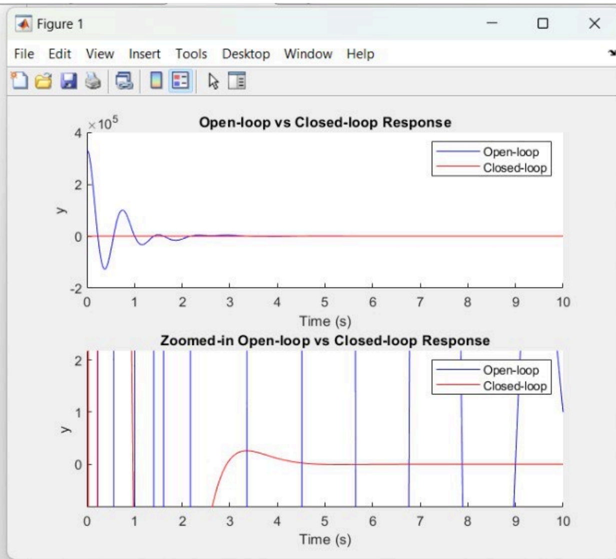
where p are the desired eigenvalues that are given

$$p = \left[\frac{-20 + j\sqrt{255}}{10}, \frac{-20 - j\sqrt{255}}{10}, -20, -21, -22 \right]$$

$$K_1 = [\underbrace{34882 \quad 8036 \quad 1491 \quad 61}_{K} \quad \underbrace{-672}_{K_I}]$$

$K_I = 672$

```
clear all; clc; close all;

A = [0 1 0 0; 0 0 1 0; 0 0 0 1; -650 -180 -90 -6];
B = [0;0;0;1];
C = [90 15 10 0];

P = ctrb(A, B);
CLEig = [(-20+1i*sqrt(255))/10 (-20-1i*sqrt(255))/10 -20 -21 -22];
rank(P);

syms s;
char_poly = det(s * eye(size(A)) - A);
H = C*inv((s*eye(4)-A))*B;
simplify(H);

% Part A : Design the state feedback servomechanism
% Computing the controller gain
A_1 = [A zeros(4,1); -C 0];
B_1 = [B;0];
P_1 = ctrb(A_1, B_1);
K_1 = place(A_1, B_1, CLEig);

t = 0:0.01:10;
x = [2; 1; 3; 0];

% Define the open-loop system dynamics function
OLFun = @(t, x) A * x;

% Define the closed-loop system dynamics function
CLFun = @(t, x) (A_1 - B_1 * K_1) * x;

% Simulating open-loop and closed-loop responses
[t_ol, x_ol] = ode45(OLFun, t, x);
[t_cl, x_cl] = ode45(CLFun, t, [x; 0]);
```




```
% Plotting responses
figure;
subplot(2, 1, 1);
hold on;
plot(t_ol, (C * x_ol') ./ (C * x_ol(end, :)'), 'b'); % Normalize open-loop response
plot(t_cl, C * x_cl(:, 1:4)', 'r');
xlabel('Time (s)');
ylabel('y');
legend('Open-loop', 'Closed-loop');
title('Open-loop vs Closed-loop Response');

% Zoom-in for better comparison
subplot(2, 1, 2);
hold on;
plot(t_ol, (C * x_ol') ./ (C * x_ol(end, :)'), 'b'); % Normalize open-loop response


plot(t_cl, C * x_cl(:, 1:4)', 'r');
xlabel('Time (s)');
ylabel('y');
legend('Open-loop', 'Closed-loop');
title('Zoomed-in Open-loop vs Closed-loop Response');
axis([0, 10, 0, 2]);
```

b) $Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$

$$Q = \begin{bmatrix} 90 & 15 & 10 & 0 \\ 0 & 90 & 15 & 10 \\ -6500 & -1800 & -810 & -45 \\ 29250 & 1600 & 2250 & -540 \end{bmatrix}$$

$\det Q \neq 0$ $\quad$ $rk\, Q = 4$ $\quad$ the system is observable

Now we choose observable gain $\hat{L} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix}$

Apply Bass gura or ACKERMAN to (A, C) or place command on matlab
place (A, C, CL eig)

$$L = \begin{bmatrix} -0.4747 \\ -2.9906 \\ 12.6587 \\ 34.5827 \end{bmatrix}$$

Check if $\sigma(A - LC) = \sigma(A_{11} - L_1 C_1) \cup \sigma(A_{22})$

```
>> A-L_1*C

ans =

   1.0e+03 *

    0.0427    0.0081    0.0047         0
    0.2692    0.0449    0.0309         0
   -1.1393   -0.1899   -0.1266    0.0010
   -3.7624   -0.6987   -0.4358   -0.0060

>> eig(A-L_1*C)

ans =

 -21.0000 + 0.0000i
 -20.0000 + 0.0000i
  -2.0000 + 1.5969i
  -2.0000 - 1.5969i
```

✓

Closed-loop system $(2n+1)$ dimensional 9x9

control law

$\dot{\xi} = r - y$

$u = -K\hat{x} + K_I \xi$

$\hat{x}$ estimate

$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly$ OBSERVER EQUATION

A_{TOT}

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} \underset{4\times4}{A} & \underset{4\times1}{BK_I} & \underset{4\times4}{-BK} \\ \underset{1\times4}{-C} & \underset{1\times1}{0} & \underset{1\times4}{0} \\ \underset{4\times4}{LC} & \underset{4\times1}{BK_I} & \underset{4\times4}{A-BK-LC} \end{bmatrix} \begin{bmatrix} x \\ \xi \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r$$

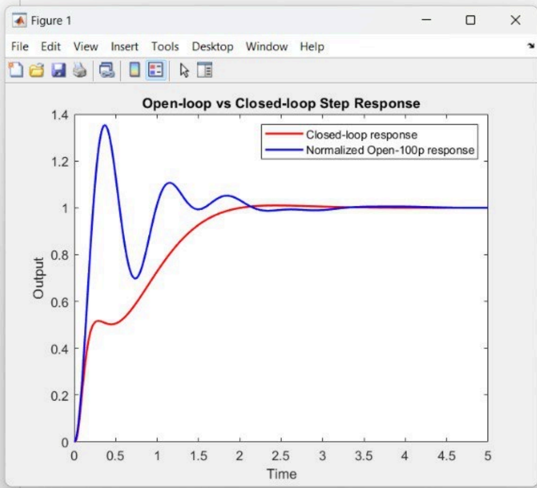
$$y = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi \\ \hat{x} \end{bmatrix}$$

$x_1\ x_2\ x_3\ x_4$
ξ
$\hat{x}_1\ \hat{x}_2\ \hat{x}_3\ \hat{x}_4$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```matlab
clear all; clc; close all;
A = [0 1 0 0; 0 0 1 0; 0 0 0 1; -650 -180 -90 -6];
B = [0;0;0;1];
C = [90 15 10 0];
% Define the state feedback gain matrix for the closed-loop system
K = [34881 8036 1491 61];
KI=672.4667;
L=[-0.4747; -2.9906; 12.6587; 34.5827];
A_cl=[A B*KI -B*K; -C 0 zeros(1,4); L*C B*KI A-B*K-L*C];
display(A_cl);
eig(A_cl)
B_cl=[0; 0; 0; 0; 1; 0; 0; 0; 0];
C_cl=[C 0 0 0 0 0];
% Create the closed-loop system
sys_cl = ss(A_cl, B_cl, C_cl, 0);
% Simulate the closed-loop system response to a unit step input
t = 0:0.01:5;
[y_closed_loop, t_closed_loop] = step(sys_cl, t);
% Simulate the open-loop system response to a unit step input
[y_open_loop, t_open_loop] = step(ss(A, B, C, 0), t);
% Normalize the open-loop output trajectory to 1 for comparison
y_open_loop_normalized = y_open_loop / y_open_loop(end);
y_closed_loop_normalized = y_closed_loop / y_closed_loop(end);
%Plot the responses
plot(t_closed_loop, y_closed_loop_normalized, 'r', 'LineWidth', 1.5);
hold on;
plot(t_open_loop, y_open_loop_normalized, 'b-', 'LineWidth', 1.5);
hold off;
xlabel('Time');
ylabel ('Output');
title( 'Open-loop vs Closed-loop Step Response');
legend ('Closed-loop response', 'Normalized Open-100p response');
```




```
A_cl =

   1.0e+04 *

         0    0.0001         0         0         0         0         0         0         0
         0         0    0.0001         0         0         0         0         0         0
         0         0         0    0.0001         0         0         0         0         0
   -0.0650   -0.0180   -0.0090   -0.0006    0.0672   -3.4881   -0.8036   -0.1491   -0.0061
   -0.0090   -0.0015   -0.0010         0         0         0         0         0         0
   -0.0043   -0.0007   -0.0005         0         0    0.0043    0.0008    0.0005         0
   -0.0269   -0.0045   -0.0030         0         0    0.0269    0.0045    0.0031         0
    0.1139    0.0190    0.0127         0         0   -0.1139   -0.0190   -0.0127    0.0001
    0.3112    0.0519    0.0346         0    0.0672   -3.8643   -0.8735   -0.1927   -0.0067
```

Indeed, if we display eig(A-cl)
the eigenvalues that I have set up.

```
>> eig(A_cl)

ans =

 -22.1658 + 1.8764i
 -22.1658 - 1.8764i
 -21.0839 + 0.0000i
 -19.9212 + 0.0000i
 -18.6682 + 0.0000i
  -1.9999 + 1.5971i
  -1.9999 - 1.5971i
  -2.0002 + 1.5972i
  -2.0002 - 1.5972i
```