2) Necessary and sufficiency condition for DCF: Matrix A has linearly independent eigenvectors i.e $\lambda_1 \neq \lambda_2$.

2) • eigen values $|\lambda I - A| = 0$.
• eigen vectors: $(\lambda_1 I - A) V_1 = 0$. $V_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$
$(\lambda_2 I - A) V_2 = 0$ $V_2 = \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$
• Trans^n Matrix $T = \begin{bmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{bmatrix}$. $A^* = T^{-1} A T$ $\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$
$B^* = T^{-1} B$; $C^* = CT$; $D^* = D$.
Now, $x^* = T^{-1} x$ i.e $x^*(0) = T^{-1} x(0)$
DCF: $\dot{x}^* = A^* x^* + B^* u$
$y = C^* x^* + D^* u$.

2) JCF: 1) When $\lambda_1 = \lambda_2 = \lambda$, Find V_1 by $(\lambda I - A) V_1 = 0$
2) $AV_2 = V_1 + \lambda V_2 \rightarrow$ Find V_2.
3) $T = [V_1 \ V_2]^T$
4) $JCF = T^{-1} A T \rightarrow$ Find J, A^*, B^*, C^*, D^*

2) $e^{at} = 1 + at + \frac{1}{2}(at)^2 + \cdots + \frac{1}{n!}(at)^n$

$$x(t) = \underbrace{e^{At} x(0)}_{\text{Free response or zero input response}} + \underbrace{\int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau}_{\text{Forced response or zero state response}}$$

$$y(t) = C e^{At} x(0) + C \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau + D u(t)$$

2) $e^{At} = T e^{A^* t} T^{-1}$
if $A^* = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$, $e^{A^* t} = \begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{pmatrix}$

2) Jordan Form:

$$\begin{matrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{matrix} \rightarrow \begin{matrix} e^{\lambda t} & te^{\lambda t} & 0 \\ 0 & e^{\lambda t} & 0 \\ 0 & 0 & e^{\lambda t} \end{matrix}$$

$$\begin{matrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{matrix} \rightarrow \begin{matrix} e^{\lambda t} & te^{\lambda t} & \frac{t^2}{2} e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{matrix}$$

2) Laplace: For $\dot{x} = Ax + Bu$, $y = Cx + Du$ → FOR COMPARING TWO STATE MODELS
$X(s) = (sI - A)^{-1} x_0 + (sI - A)^{-1} B U(s)$
$Y(s) = C(sI - A)^{-1} x_0 + (C(sI - A)^{-1} B + D) U(s)$

2) $G(s)$ in DCF $= C^*(sI - A^*)^{-1} B^* + D^*$

3) $Re(\lambda_1, \lambda_2) < 0 \rightarrow$ Asymptotically stable
$Re(\lambda_1, \lambda_2) > 0 \rightarrow$ unstable
$Re(\lambda_1) > 0$ & $Re(\lambda_2) < 0 \rightarrow$ Marginally stable
$Re(\lambda_1, \lambda_2) = 0 \rightarrow$ STABLE.

If Matrix P is Positively definite ie $P_{11} > 0$, System is Asymp Stable. If $P_{11} < 0$, Asymp unstable.
No unique solution, System is Asym unstb.

3) $ml^2 \ddot{\theta} = mgl \sin\theta - b\dot{\theta} + T$
$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$

$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$, $B = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}$

Linearized System: $\Delta\dot{x} = A \Delta x + B \Delta u$

3) Lyapunov Stability:
$A^T P + PA = -Q$ $(Q = I)$
$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ $(P_{12} = P_{21})$
$V(x) = x^T P x$; $\dot{V}(x) = x^T Q x$

3) LYAPUNOV for Non-linear Systems:
System is Lyapunov stable if $\dot{V}(x)$ is -ve semidefinite. $(\dot{V}(x) \leq 0)$
It is asymp Stable if $\dot{V}(x)$ is -ve definite. $(\dot{V}(x) < 0, \dot{V}(0) = 0)$

3) A symmetric matrix P (2x2 or 3x3 etc) is +ve definite if all eigenvalues are >0. It is +ve definite if $P>0$ if $x^T P x > 0$. $(x \neq 0)$
+ve semidefinite: $P \geq 0$ if $x^T P x \geq 0$ (For $x \in R$)

-ve definite: $Q<0$, if $-Q>0$, $x^T Q x < 0$. $(x \neq 0, x \in R)$
-ve semidef: $Q \leq 0$, if $x^T Q x \leq 0$.

3) LYAPUNOV for LTI:
If P is -ve definite: $A^T P + PA = -Q$ its stable. If its -ve semidefinte then Asymp Stable.

3) LYAPUNOV for Linear sys:
If a solution for P: $A^T P + PA = -Q$ can't be found, then system is not asymp stable.
If $P>0$, sys is asymp stable
If P is not +ve definite, has atleast one eigen with $Re>0$, then unstable.

State Space Rep

1) $$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$z(s) = \frac{u(s)}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\rightarrow \dddot{z} + a_2 \ddot{z} + a_1 \dot{z} + a_0 z = u$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)\, z(s)$$

$$\rightarrow y(t) = b_2 \ddot{z}(t) + b_1 \dot{z}(t) + b_0 z(t)$$

$$x = \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} x$$

$$A_{CCF} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix}, \quad B_{CCF} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$C_{CCF} = \begin{bmatrix} (b_0 - a_0 b_n) & (b_1 - a_1 b_n) & \cdots & (b_{n-1} - a_{n-1} b_n) \end{bmatrix}, \quad D_{CCF} = b_n$$

1) For equilibrium:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}, \quad D = 0.$$

1) State space representation example

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} (\ ) & (\ ) & (\ ) \\ (\ ) & (\ ) & (\ ) \\ (\ ) & (\ ) & (\ ) \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ K \end{bmatrix} + \begin{bmatrix} (\ ) \\ (\ ) \\ (\ ) \end{bmatrix} u$$

1) Laplace transforming ODEs in Higher order

$$J\ddot{\theta} + b\dot{\theta} = Ki, \quad L\dot{i} + Ri + K_b\dot{\theta} = V_s$$

Sol: $(Js^2 + bs)\, H(s) = K_t\, I(s)$

$(Ls + R)\, I(s) + K_b s (H(s)) = V_s / s$

$I(s) = (Js^2 + bs)\, H(s) / K_t$

On solving, $(JLs^3 + Lbs^2 + RJs^2 + Rbs + K_b K_t s) H(s) = \frac{K_t V}{s}$

$$\Rightarrow JL\dddot{\theta} + Lb\ddot{\theta} + RJ\ddot{\theta} + Rb\dot{\theta} + K_b K_t \dot{\theta} = K_t V_s$$

$\because (u = V_s)$

use the previous equation to write $\dddot{\theta}$

take: $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = \ddot{\theta}$ so, $\dot{x}_3 = \dddot{\theta}$

write state space in $\dot{x}_3$

It will be: $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & (&) & (\;) \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix} u.$

4) controllability Matrix:

$P = [B \;\; AB \;\; A^2B \cdots]$

If rank < state space dimension then uncontrollable.

4) For linear system $\dot{x} = Ax + Bu$, $\xrightarrow{}$ input $u(t)$ for $x(0) \to 0$ to t

$$W_c(t) = \int_0^t e^{A(t-\tau)} B B^T e^{A^T(t-\tau)} d\tau$$

$W_c(t)$ is Controllability grammanian.
↳ non singular $\forall\, t > 0$.

4) PBH test:

System is controllable if matrix $[\lambda_i I - A_{DCF} \,\vdots\, B_{DCF}]$ has full rank at every λ. eigen value.

4) If all eigenvalues of A has $-$ve real parts, then unique solution of $AW_c(t) + W_c(t)A^T = -BB^T$ is +ve Definite where solution is

$$W_c = \int_0^\infty e^{A\tau} BB^T e^{A^T\tau} d\tau$$

4) System is BIBO stable if $g(t) = \mathcal{L}^{-1}\{G(s)\}$ satisfies

$$\int_0^\infty \|g(\tau)\| d\tau < \infty \text{ (finite)}$$

$$G(s) = C(sI - A)^{-1}B$$

ex: If $G(s) = \frac{4}{s+1} \Rightarrow \mathcal{L}^{-1} = 4e^{-t}$

$\int 4e^{-t} = 4 < \infty$.

4) If there is pole-zero cancellation, the realisation is minimal. Thus, Although BIBO Stable, it is not Stable.

ex: $\dfrac{\cancel{(s-1)}}{(s+1)\cancel{(s-1)}}$ (Pole zero cancellation)

0) inverse $= \dfrac{\text{Adjoint } [A]}{\text{Det } |A|}$

Adj (A) = transpose of co-factor matx.

$c_{ij} = (-1)^{i+j} \begin{vmatrix} x & y \\ z & w \end{vmatrix}$, $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ \vdots & \vdots & \vdots \end{bmatrix}$

4) CCF:

1) Find controllability matrix
2) charecteristic polynomial $|sI - A| = 0$.
   $a_3s^3 + a_2s^2 + a_1s + a_0 = 0$.
3) $P_{CCF}^{-1} = \begin{bmatrix} a_1 & a_2 & 1 \\ a_2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
4) $T = P \cdot P_{CCF}^{-1}$
5) Find CCF:

$A_{CCF} = TAT^{-1}$; $B_{CCF} = T^{-1}B$, $C_{CCF} = CT$, $D_c = D$.

4) To compute particular input $u(t)$ that acheives driving $x(0)$ from t_0 to t_1 we use: $u(t) = -B^T e^{A^Tt} W_c^{-1} x(0)$

1) output equation when input linearized equation is given:

$$Y(s) = C(sI - A)^{-1}x_0 + (C(sI - A)^{-1}B + D)\,U(s)$$

apply inverse laplace in between and put the desired t value.

1) Transfer function calculation:

$$H(s) = C(sI-A)^{-1}B + D.$$

1) rank $\begin{bmatrix} zI-A & -G \\ C & 0 \end{bmatrix} = \text{rank}\begin{bmatrix} zI-\bar{A} & -G \\ C\bar{A} & CG \end{bmatrix}$

Sub: $\bar{A} = (I-GMC)A$

$$\begin{bmatrix} zI-\bar{A} & -G \\ C\bar{A} & CG \end{bmatrix} = \begin{bmatrix} zI-A+GMCA & -G \\ CA-CGMCA & CG \end{bmatrix}$$

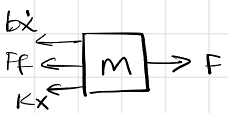
$$= \begin{bmatrix} I & 0 \\ -C & zI \end{bmatrix}\begin{bmatrix} zI-A & -G \\ C & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ -MCA & I \end{bmatrix}$$

So, both are equivalent as $\det\begin{bmatrix} I & 0 \\ -C & zI \end{bmatrix} = \det I \det zI \neq 0$.

1) eq of $ml^2\ddot{\theta} + b\dot{\theta} + mgl\sin\theta = T$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos x_1 & \frac{-b}{ml^2} \end{bmatrix}_{x_{eq}}$$

$$B = \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix}$$

1) 

$$m\ddot{x} + b\dot{x} + Kx = F - F_f$$

$$2\ddot{x} + 3\dot{x} + x = F - x^2$$

3) If $\dot{x}$ is given and x_1, x_2 asked:

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)\,d\tau$$

3) 1) Find $(sI-A)^{-1}$

2) $e^{At} = \mathcal{L}^{-1}((sI-A)^{-1})$

3) $C(sI-A)^{-1}Bu(s)$

4) $x(t) = e^{At}x_0 + \mathcal{L}^{-1}(C(sI-A)^{-1}Bu(s))$

5) Controllability test: $P = [B \; AB \; A^2B \cdots]$

if rank$(P) = n \rightarrow$ controllable ✓

KCF: Step-1: Find T

$T = [B \; 1] \rightarrow$ ex: if $B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}$, T will be:

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ $\Rightarrow$ T should be non singular i.e $\text{Det}|T| \neq 0$

Step-2:

$$\hat{A} = T^{-1}AT, \quad \hat{B} = T^{-1}B, \quad \hat{C} = TC, \quad \hat{D} = D$$

Step-3: Find $\hat{P}$

$$\hat{P} = \left[\begin{array}{c|c} B_c & A_cB_c \cdots A_c^{n-1}B_c \\ 0 & 0 \cdots\cdots 0 \end{array}\right]$$

ex: if $\hat{A} = \begin{bmatrix} -3 & 0 & -2 \\ 0 & -5 & -4 \\ 0 & 0 & -1 \end{bmatrix}$, $\hat{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \hat{P}$ will be

$\hat{P} = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix} \rightarrow$ rank will be n, making it controllable $\therefore$ KCF

Step-4: Write $\dot{z}_1, \dot{z}_2, \dot{z}_3$ equations

$$\dot{z}_1 = a_{11}z_1 + a_{12}z_2 + a_{13}z_3 + b_{11}u_1 + b_{12}u_2 + b_{13}u_3$$
$$\dot{z}_2 = a_{21}z_1 + a_{22}z_2 + a_{23}z_3 + b_{21}u_1 + b_{22}u_2 + b_{23}u_3$$
$$\dot{z}_3 = a_{31}z_1 + a_{32}z_2 + a_{33}z_3 + b_{31}u_1 + b_{32}u_2 + b_{33}u_3$$

For above ex, $\dot{z}_1, \dot{z}_2, \dot{z}_3$ will be

$\dot{z}_1 = -3z_1 - 2z_2 + u_1$, $\dot{z}_2 = -5z_2 - 4z_3 + u_2$, $\dot{z}_3 = -z_3$.

Step-5: If $Re(\bar{A}_{uc}) < 0 \rightarrow$ stable; stabilizable
(eigen values on left half plane)

{May be $Re(\bar{A}_{uc}) \rightarrow a_{33}$ of $\hat{A}$}

But if there is a non zero quantity in second row $[3-1]$, it is stabilizable.
$(n-1)$

4) to derive input $u(t)$ that drives $x(0)$ from 0 to t secs,

$$u(t) = -B^T e^{At} x(0)$$

6) If rank$(Q) = n \rightarrow$ observable!

$$Q = \begin{bmatrix} C \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

6) PBH test for observability:

If $\begin{bmatrix} \lambda I - A \\ C \end{bmatrix}$ has full rank at every λ.

Then PBH observable.

6) OCF for general SISO case:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_n s^n + b_{n-1}s^{n-1} + \dots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

6) OCF: "SYSTEM IN OCF IS OBSERVABLE!"

$$A_{OCF} = \begin{bmatrix} -a_{n-1} & 1 & 0 & 0 \cdots \\ -a_{n-2} & 0 & 1 & 0 \cdots \\ -a_{n-3} & 0 & 0 & 1 \cdots \\ \vdots & & \ddots & \vdots \\ & & & 1 \\ -a_0 & 0 & 0 \cdots & 0 \end{bmatrix}$$

$$B_{OCF} = \begin{bmatrix} b_{n-1} - a_{n-1} \cdot b_n \\ b_{n-2} - a_{n-2} b_n \\ \vdots \\ b_1 - a_1 b_n \\ b_0 - a_0 b_n \end{bmatrix}, \quad C_{OCF} = [1 \; 0 \cdots 0]$$

$D_{OCF} = b_n$

We can write in OCF only if it is observable.

6) Writing in OCF:

Step-1: $\det|sI - A| = 0 \; a_2 s^2 + a_1 s + a_0 = 0 \rightarrow \underline{a_0, a_1, a_2}$

Step-2: $r_1 = C, \; r_2 = r_1 A + a_1 r_1, \cdots r_{i+1} = r_i A + a_{n-i} r_1$

Step-3: $R = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$

Step-4: write in OCF: $\bar{A} = RAR^{-1}, \bar{B} = RB, \bar{C} = R^{-1}C.$

6) If $Re(\lambda_i(A_{22})) < 0 \Rightarrow$ detectable.

Detectability can only be checked via Kalman Controllable Form.

In $\bar{A}$, the $a_{3\times3}$ (or $a_{2\times2}$) is $Re(\lambda_i A_{22})$

If $Re(\lambda_i A_{22}) < 0 \longrightarrow$ Detectable.

4) $$G(s) = \frac{Y(s)}{U(s)} = \frac{b_n s^n + b_{n-1}s^{n-1} + \dots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

Controllable Canonical Form (CCF):

$$A_{CCF} = \begin{bmatrix} 0 & 1 & 0 & 0 \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix}, \quad B_{CCF} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad D_{CCF} = b_n$$

$C_{CCF} = [(b_0 - a_0 b_n) \; (b_1 - a_1 b_n) \cdots (b_{n-1} - a_{n-1} b_n)]$

4,6) If $Re(\lambda_i(A_{22})) < 0$

we can say it is Kalman Stable and Detectible!

6) DUALITY:

(A, B) is controllable iff (A^T, B^T) is observable

(A, C) is observable iff (A^T, C^T) is controllable

6) Ex! $G(s) = \frac{s^2 + 2s + 1}{s^3 + 6s^2 + 11s + 6}$

Step-1: put in CCF: $A_{CCF} = [\;], B_{CCF} = [\;], C_{CCF} = [\;]$

Step-2: $Q = \begin{bmatrix} C_{CCF} \\ A_{CCF} C_{CCF} \\ A^2_{CCF} C_{CCF} \end{bmatrix} \rightarrow \text{rank} \neq n \rightarrow$ not observable

Step-3: put in OCF: $A_{OCF} = [\;], B_{OCF} = [\;], C_{OCF} = [\;]$

Step-4: $P = [B \; AB \; A^2 B] \rightarrow \text{rank} \neq n \rightarrow$ not controllable

Step-5: $\frac{(s+1)^2}{(s+1)(s+2)(s+3)} \Rightarrow \frac{(s+1)}{(\;)}$

Repeat step 1–4, now new system is observable & controllable

2) When "Are these two models similar?" is asked.

$A, B, C, D \longrightarrow$ given. Then, $H(s) = C(sI - A)^{-1}B$

7) $\det(\lambda I - A) = \lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$

$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$; $t_s = \frac{4}{\zeta\omega_n}$; $\%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \cdot 100\%$

$K_d = 2\zeta\omega_n - 1, \quad K_p = \omega_n^2 - 1$

$u(t) = -Kx + K_g r$

For $G(s)$, $A = \begin{bmatrix} 0 & 1 & 0 & 0 \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

$C = [b_0 \; b_1 \cdots b_{n-1}], \quad D = d$

$\det(sI - A) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$

$\det[sI - A_{cl}] = \det(sI - (A - BK)) = s^n + c_{n-1}s^{n-1} + \cdots + c_1 s + c_0$

$K = (c_0 - a_0 \quad c_1 - a_1 \quad \cdots \quad c_{n-1} - a_{n-1})$

observer gain matrix

7) Step-1: char polynomial $s^3+2s^2+15s+18$

Step-2: Find $\lambda_1, \lambda_2, \lambda_3 \to$ eigen values

Step-3: $\% OS = e^{(\;)} \to \zeta = 0.67$

Step-4: $t_s = 3 = \frac{4}{\zeta w_n} \to w_n = 2$

Step-5: $s^2 + 2\zeta w_n + w_n^2 = 0$

Step-6: λ_1, λ_2 Find. Choose a λ_3 which is 10 times further to left

$\lambda = -1.33 \pm j1.49 \to \lambda_3 = -13.3$

Step-7: closed loop char poly: $(s^2+2\zeta w_n + w_n^2)(s+13.3)$.

Step-8: $K = [c_0 - a_0 \cdots c_{n-1} - a_{n-1}]$

7) $$K = (c_0 - a_0 \cdots c_{n-1} - a_{n-1}) \left(P \cdot \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & 1 \\ a_2 & a_3 & \cdots & & & 0 \\ a_3 & a_4 & \cdots & & & 0 \\ \vdots & & & & & \vdots \\ a_{n-1} & \cdots & & & & 0 \\ 1 & 0 & 0 & \cdots & & 0 \end{bmatrix} \right)^{-1}$$

where $P = [B \;\; AB \;\; A^2B \cdots]$

7) $$u = -\begin{bmatrix} \bar{K}_c & \bar{K}_{uc} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{uc} \end{bmatrix} + K_g r$$

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \vdots \\ \dot{\bar{x}}_{nc} \end{bmatrix} = \begin{bmatrix} \bar{A}_c - \bar{B}_c \bar{K}_c & \bar{A}_{12} - \bar{B}_c \bar{K}_{uc} \\ 0 & \bar{A}_{uc} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{uc} \end{bmatrix} + \begin{bmatrix} \bar{B}_c \\ 0 \end{bmatrix} K_g r$$

7) If system is controllable i.e $\mathrm{rank}(P) = n$, it is possible to arbitrarily place the closed loop eigen values.

7) $y = cx - CKx + K_g r$

$(I - C + CKx) = K_g \cdot r(t)$

$A' = A - BK$

7) If $J\ddot{\theta} + b\dot{\theta} + K\theta = \tau$ is given and weknow J, b, K values

$C(sI-A)^{-1}B = H(s)$

get char poly in $s^2 + 2\zeta w_n s + w_n^2$ form.

Compare and get w_n, ζ

7) When performance is asked for closed loop system,

$$\dot{x} = \underbrace{(A-BK)}_{\text{A-closed loop}} x + \underbrace{BG}_{\text{B-closed loop}} r$$

BK will be $= B\,[K_p \;\; K_d]$

$A - BK \to$ calculate

write $\dot{x} = (A-BK)x + BGr \to$ Find eigenvalue eqn compare with $s^2 + 2\zeta w_n s + w_n^2 = 0$.

Find $[K_p \;\; K_d]$.

Luenberger observer

7) For calculating observer gain L for $\lambda_{1,2,3} = \lambda_1, \lambda_2, \lambda_3 \to$ given

we get gain $L = [c_0 - a_0 \cdots c_{n-1} - a_{n-1}] = R = [r_1, r_2, r_3]^T$

$$R = \begin{bmatrix} C \\ r_1 A + a_2 r_1 \\ r_2 A + a_1 r_1 \end{bmatrix} \to \underline{\underline{\bar{L} = R^{-1} L}} \to \text{Final gain needed.}$$

8) Design K for $\lambda_1, \lambda_2, \lambda_3$ and Design L for $\lambda_1', \lambda_2', \lambda_3'$

$(s-\lambda_1)(s-\lambda_2)(s-\lambda_3) \to K$

$(s-\lambda_1')(s-\lambda_2')(s-\lambda_3') \to L$

Find gain matrix.