Homework 3
ME5659 Spring 2024

Due: See Canvas, turn in on Gradescope

Problem 1 (9 points)

Consider the following linear systems $\dot{x} = Ax, x(0) = x_0$, where

(i) $A = \begin{bmatrix} 0 & 1 \\ -14 & -4 \end{bmatrix}, \quad x_0 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}.$

(ii) $A = \begin{bmatrix} 0 & 1 \\ -14 & 4 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}.$

(iii) $A = \begin{bmatrix} 0 & 1 \\ -14 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$

(a) **3 points.** Characterize the stability of the equilibrium point from the eigenvalues of A.

(b) **3 points.** Use Lyapunov stability analysis to determine whether the system equilibrium state $x_{eq} = 0$ is asymptotically stable. Use $Q = I$ in the Lyapunov equation. (Do all calculations by hand.)

(c) **3 points.** Use MATLAB to plot the state trajectories $x(t)$ vs. time t with the initial condition x_0. Each plot has two trajectories $x_1(t), x_2(t)$. Hand in your plots and your code.

Problem 2 (6 points)

Consider the inverted pendulum (Figure 1) which is characterized by

$$ml^2\ddot{\theta} = mglsin\theta - b\dot{\theta} + T$$

where T denotes a torque applied at the base and g is the gravitational acceleration. We assume that $u = T$ and $y = \theta$ are its input and output, respectively.

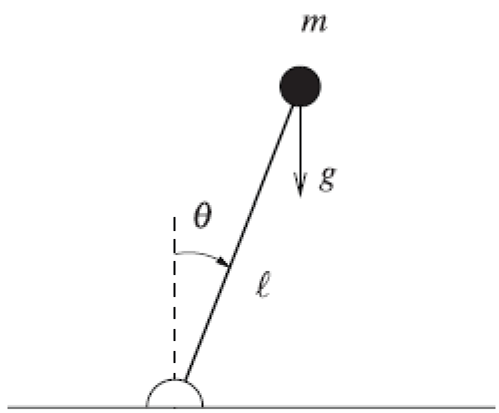

Figure 1: Simple pendulum

(a) **(3 points)** Perform local linearization of this system around the equilibrium point $\theta = \pi$, derive the linear state-space models and determine whether it is stable or not.

(b) **(3 points)** Perform local linearization of this system around the equilibrium point $\theta = 0$, derive the linear state-space models and determine whether it is stable or not.

Problem 3 (10 points)

Consider the following linear system $\dot{x} = Ax, x(0) = x_0$, where

$$A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$$

(a) **(4 points)** Under what conditions on a, b is the system equilibrium, $x_{eq} = 0$, asymptotically stable? Use Lyapunov stability analysis.

(b) **(3 points)** Write the Lyapunov function $V(x)$ and its time derivative $\dot{V}(x)$. Use the P matrix obtained in (a) to evaluate stability.

(c) **(3 points)** If $b = 0, a < 0$, show that the linear system is stable but not asymptotically stable.