Control Engineering Project

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ME 5659 PROJECT

goal : To maintain the pendulum angle $\theta(t) = 0$ by using a feedback controller with some sensors and an actuator to produce an input force $F(t)$

given : cart mass = m_1, Pendulum point mass = m_2, assuming the pendulum rod is massless

<u>PROBLEM 1</u> : State space representation, Linearization and Simulation

Nonlinear EOMs of the system :

$$-m_2 L \ddot{\omega} \cos\theta + m_2 L^2 \ddot{\theta} - m_2 g L \sin\theta = 0$$
$$(m_1 + m_2)\ddot{\omega} - m_2 L \ddot{\theta} \cos\theta + m_2 L \dot{\theta}^2 \sin\theta = F(t)$$

<u>Part (a)</u> :

$$m_2 L^2 \ddot{\theta} - m_2 L \ddot{\omega} \cos\theta + m_2 g L \sin\theta$$

$$\ddot{\theta} = \frac{\ddot{\omega}}{L} \cos\theta + \frac{g}{L} \sin\theta \quad \text{—— (i)}$$

$$(m_1 + m_2)\ddot{\omega} - m_2 L \ddot{\theta} \cos\theta + m_2 L \dot{\theta}^2 \sin\theta = F(t)$$

$$(m_1 + m_2)\ddot{\omega} - m_2 \cancel{L}\left[\frac{\ddot{\omega}}{\cancel{L}} \cos\theta + \frac{g}{\cancel{L}} \sin\theta\right] \cos\theta + m_2 L \dot{\theta}^2 \sin\theta = F(t)$$

$$(m_1 + m_2)\ddot{\omega} - m_2 \ddot{\omega} \cos^2\theta - m_2 g \sin\theta \cos\theta + m_2 L \dot{\theta}^2 \sin\theta = F(t)$$

$$(m_1 + m_2 - m_2 \cos^2\theta)\ddot{\omega} - m_2 g \sin\theta \cos\theta + m_2 L \dot{\theta}^2 \sin\theta = F(t)$$

$$(m_1 + m_2 - m_2 \cos^2\theta)\ddot{\omega} = F(t) + m_2 g \sin\theta\cos\theta - m_2 L\dot{\theta}^2 \sin\theta$$

$$\ddot{\omega} = \frac{F(t) + m_2 g \sin\theta \cos\theta - m_2 L\dot{\theta}^2 \sin\theta}{m_1 + m_2 - m_2 \cos^2\theta} \quad \text{—— (ii)}$$

$$\ddot{\theta} = \left[\frac{F(t) + m_2 g \sin\theta \cos\theta - m_2 L\dot{\theta}^2 \sin\theta}{m_1 + m_2 - m_2 \cos^2\theta}\right]\left[\frac{\cos\theta}{L}\right] + \frac{g}{L}\sin\theta$$

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \\ \omega \\ \dot{\omega} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \left[\frac{F(t) + m_2 g \sin\theta \cos\theta - m_2 L\dot{\theta}^2 \sin\theta}{m_1 + m_2 - m_2 \cos^2\theta}\right]\left[\frac{\cos\theta}{L}\right] + \frac{g}{L}\sin\theta \\ \dot{\omega} \\ \frac{F(t) + m_2 g \sin\theta \cos\theta - m_2 L\dot{\theta}^2 \sin\theta}{m_1 + m_2 - m_2 \cos^2\theta} \end{bmatrix}$$

→ This is the non linear state space description for the system

Part (b):

Assume that $u = 0$; $\dot{x} = 0$ ie $\dot{\theta}, \dot{\omega} = 0$

$$\frac{f(t) + m_2 g \sin\theta \cos\theta - m_2 L\dot{\theta}^2 \sin\theta}{m_1 + m_2 - m_2 \cos^2\theta}$$

$$\sin\theta \, (m_2 g \cos\theta - m_2 L\dot{\theta}^2) = 0$$

$\sin\theta = 0$ ie. $\theta = 0, \pi$ is a solution

Linearizing

$$\Delta\dot{x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} \Delta x + \begin{bmatrix} \frac{\partial f_1}{\partial U_1} & \cdots \\ \vdots & \\ \vdots & \end{bmatrix} \Delta u$$

$f(t) = 0$

$$\frac{\partial f_4}{\partial x_1} = \frac{\partial f_4}{\partial \theta} = \frac{\partial}{\partial \theta}\left[\frac{m_1 g \sin\theta \cos\theta - m_2 L\dot{\theta}^2 \sin\theta}{m_1 + m_2 m_1 \cos^2\theta}\right]$$

$$\frac{\partial v}{\partial \theta} = \frac{d}{d\theta} \frac{1}{2} m_2 g \sin 2\theta - m_2 L\dot{\theta}^2 \sin\theta$$

$$= m_2 g \cos 2\theta - m_2 L\dot{\theta}^2 \cos\theta$$

$$\frac{\partial v}{\partial \theta} = 2m_2 \sin\theta \cos\theta = m_2 \sin 2\theta$$

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$$\frac{\partial f_4}{\partial \theta} = \left[m_1 + m_1 \cdot m_2 \cos^2\theta\right]\left[m_2 g \cos 2\theta \cdot m_2 L \dot{\theta}^2 \cos\theta\right] + \frac{\left[m_2 g \sin\theta \cos\theta - m_2 L \dot{\theta}^2 \sin\theta\right] m_2 \sin 2\theta}{\left[m_1 + m_2 - m_2 \cos^2\theta\right]}$$

$$\frac{\partial f_2}{\partial \theta} = \frac{\partial f_4}{\partial \theta} \cdot \frac{\cos\theta}{L} - f_4 \frac{\sin\theta}{L} + \frac{g}{L} \cos\theta$$

$$\left.\frac{\partial f_4}{\partial \theta}\right|_{\substack{\theta = 0 \\ \dot{\theta} = 0}} = \frac{m_1 m_2 g + 0}{m_1^2} = \frac{m_2 g}{m_1}$$

$$\left.\frac{\partial f_2}{\partial \theta}\right|_{\substack{\theta = 0 \\ \dot{\theta} = 0}} = \frac{m_2 g}{m_1} \cdot \frac{1}{L} - 0 + \frac{g}{L} = \frac{(m_1 + m_2) g}{m_1 L}$$

$$A_{\substack{\theta = 0 \\ \dot{\theta} = 0}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{m_2 g + m_1 g}{m_1 L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m_2 g}{m_1} & 0 & 0 & 1 \end{bmatrix}$$

$$\left.\frac{\partial f_2}{\partial u}\right|_{\substack{\theta = 0 \\ \dot{\theta} = 0}} = \frac{1}{m_1 L} \qquad \frac{\partial f_4}{\partial u} = \frac{1}{m_1}$$

$$B_{\substack{\theta = 0 \\ \dot{\theta} = 0}} = \begin{bmatrix} 0 \\ 1/m_1 L \\ 0 \\ 1/m_1 \end{bmatrix}$$

$$\left.\frac{\partial f_4}{\partial \theta}\right|_{\substack{\theta=\pi \\ \dot{\theta}=0}} = \frac{m_1 m_2 g}{m_1^2} = \frac{m_2 g}{m_1}$$

$$\left.\frac{\partial f_2}{\partial \theta}\right|_{\substack{\theta=\pi \\ \dot{\theta}=0}} = -\frac{m_2 g}{m_1 L} - 0 - \frac{g}{L} = -\frac{(m_1+m_2)g}{m_1 L}$$

$$\left.\frac{\partial f_2}{\partial u}\right|_{\substack{\theta=\pi \\ \dot{\theta}=0}} = \frac{1}{m_1 L}$$

$$\left.\frac{\partial f_4}{\partial u}\right|_{\substack{\theta=\pi \\ \dot{\theta}=0}} = \frac{1}{m_1}$$

$$A_{\substack{\theta=\pi \\ \dot{\theta}=0}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(m_1+m_2)g}{m_1 L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m_2 g}{m_1} & 0 & 0 & 1 \end{bmatrix}$$

$$B_{\substack{\theta=\pi \\ \dot{\theta}=0}} = \begin{bmatrix} 0 \\ 1/m_1 L \\ 0 \\ 1/m_1 \end{bmatrix}$$

→ These are the two linear state space Models

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Part (c)

→ Solved using MATLAB

C●DE and ●utput at the end of writeup.

Part (d)

→ Solved using MATLAB

PROBLEM 2 : Controllability Analysis

Part (a)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(m_1+m_2)g}{m_1 L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m_2 g}{m_1} & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1 L} \\ 0 \\ \frac{1}{m_1} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} x$$

$$P = \begin{bmatrix} B & AB & A^2 B & A^3 B \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 0 & 1/Lm_1 & 0 & g(m_1+m_2)/m_1^2 \\ 1/Lm_1 & 0 & g(m_1+m_2)/m_1^2 & 0 \\ 0 & 1/m_2 & 0 & gm_2/Lm_1^2 \\ 1/m_1 & 0 & gm_2/Lm_1^2 & 0 \end{bmatrix}$$

Rank $(P) = 4$

→ ∴ This system is controllable

Part (b)

→ Since the system is fully controllable, by choosing an appropriate value for u we can change the values of state variables to any desired value. We can keep $\theta = 0$, $\dot{\theta} = 0$, $\omega = x$ (desired position) with the intended ω

Part (c)

Assuming $g = 10$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$P_{CCF} = \begin{bmatrix} 0 & 2/3 & 0 & 80/3 \\ 2/3 & 0 & 80/3 & 0 \\ 0 & 1/2 & 0 & 10/3 \\ 1/2 & 0 & 10/3 & 0 \end{bmatrix}$$

$$\det|A - sI| = s^4 - 40s^2 \longrightarrow s^4 + \underset{a_3}{\underset{\downarrow}{0}}s^3 - \underset{a_2}{\underset{\downarrow}{40}}s^2 + \underset{a_1}{\underset{\downarrow}{0}}s + \underset{a_0}{\underset{\downarrow}{0}}$$

$$P^{-1} = \begin{bmatrix} 0 & -40 & 0 & 1 \\ -40 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$T = P_{CCF} P^{-1} = \begin{bmatrix} 0 & 0 & 0.667 & 0 \\ 0 & 0 & 0 & 0.667 \\ -16.667 & 0 & 0.5 & 0 \\ 0 & -16.667 & 0 & 0.5 \end{bmatrix}$$

$$A_{CCF} = T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 40 & 0 \end{bmatrix}$$

$$B_{CCF} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad C_{CCF} = \begin{bmatrix} -16.667 & 0 & 1.167 & 0 \end{bmatrix}$$

Part (d)

Eigen values of A : $\lambda_1 = 0$ $\lambda_2 = 0$ $\lambda_3 = 6.3246$
$\lambda_4 = -6.3246$

For $\lambda = 0$, algebraic multiplicity $= 2$

$$\text{Rank}\,(\lambda I - A)_{\lambda=0} = 3$$

$$(\lambda I - A) = 1 \neq \text{algebraic multiplicity}$$

$\therefore$ The state space representation cannot be transformed to Diagonal cannonical form

PROBLEM 3 : Observability Analysis

Part (a)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(m_1+m_2)g}{m_1 L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m_2 g}{m_1} & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1 L} \\ 0 \\ \frac{1}{m_1} \end{bmatrix} u$$

$$\dot{y} = Cx$$

(i) → if we can only measure $\theta(t)$ $C = [1\ 0\ 1\ 0]$

$$\text{then} \quad Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{Lg(m_1+m_2)}{m_1} & 0 & 0 & 0 \\ 0 & \frac{Lg(m_1+m_2)}{m_1} & 0 & 0 \end{bmatrix}$$

$$\text{Rank}(Q) = 2 < 4$$

→ ∴ The system is unobservable

(ii) → if we can only measure $\omega(t)$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ gm_2/m_1 & 0 & 0 & 0 \\ 0 & gm_2/m_1 & 0 & 0 \end{bmatrix}$$

Rank $(Q) = 4$

→ ∴ The system is observable

(iii) → if we can measure the pendulum angle $\theta(t)$ and $\omega(t)$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \frac{gm_2}{m_1} + \frac{Lg(m_1+m_2)}{m_1} & 0 & 0 & 0 \\ 0 & \frac{gm_2}{m_1} + \frac{Lg(m_1+m_2)}{m_1} & 0 & 0 \end{bmatrix}$$

Rank $(Q) = 4$

→ ∴ The system is observable

Part (b)

→ When the state variable $\omega(t)$ is measured alone or $\theta(t)$ and $\omega(t)$ can both be measured, the system is fully observable. Therefore, for any unkown inital state $x(0)$ there will be a finite $t > 0$ such that the knowledge of the input u and the output y over $[0, t]$ suffices to find uniquely the inital state $x(0)$

Part (c)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 45 & 0 & 0 & 0 \\ 0 & 45 & 0 & 0 \end{bmatrix}$$

$r_1 = C = [1 \; 0 \; 1 \; 0]$

From the characteristic Polynomial, $\det|SI - A| = 0$

$a_0 = 0, \quad a_1 = 0, \quad a_2 = 40, \quad a_3 = 0$

$r_2 = r_1 A + a_3 r = [0 \; 1 \; 0 \; 1]$

$r_3 = r_2 A + a_2 r_1 = [5 \; 0 \; -40 \; 0]$

$r_4 = r_3 A + a_1 r_1 = [0 \; 5 \; 0 \; -40]$

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -40 & 0 \\ 0 & 5 & 0 & -40 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}$$

$$A_{OCF} = R^{-1} A R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{OCF} = \begin{bmatrix} 0 \\ 1.1667 \\ 0 \\ -16.667 \end{bmatrix} \qquad C_{OCF} = [1 \; 0 \; 0 \; 0]$$

Part (d)

$$A_{CCF} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 40 & 0 \end{bmatrix} \qquad B_{CCF} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_{CCF} = \begin{bmatrix} -16.667 & 0 & 1.667 & 0 \end{bmatrix}$$

$$A_{OCF} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_{OCF} = \begin{bmatrix} 0 \\ 1.1667 \\ 0 \\ -16.667 \end{bmatrix}$$

$$C_{OCF} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

By observation trans(Aocf, Bocf, Cocf) = (Accf, Cccf, Bccf). Hence they show duality. trans(Cinv(SI-A)B) = trans(B)inv(SI-trans(A))trans(C)

$$\text{Rank}\,(P(A^T_{obs}, C^T_{OCF})) = \text{Rank}\,(P(A_{CCF}, B_{CCF})) = 4$$

$P \rightarrow$ ctrb

$$\text{Rank}\,(Q(A^T_{CCF}, B^T_{CCF})) = \text{Rank}\,(Q(A_{OCF}, C_{OCF})) = 4$$

$Q \rightarrow$ obs

$$\operatorname{ctrb}(A_{OCF}^T, C_{OCF}^T) = \begin{bmatrix} 1 & 0 & 40 & 0 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\operatorname{ctrb}(A_{CCF}, B_{CCF}) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 40 \\ 1 & 0 & 40 & 0 \end{bmatrix}$$

$$\operatorname{ctrb}(A_{OCF}^T, C_{OCF}^T) = \operatorname{ctrb}(A_{CCF}, B_{CCF}) - A$$

$$\operatorname{obsv}(A_{CCF}^T, B_{CCF}^T) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \cancel{0}1 & 0 \\ 0 & 1 & 0 & 40 \\ 1 & 0 & 40 & 0 \end{bmatrix}$$

$$\operatorname{obsv}(A_{OCF}, \cancel{B}\cdot C_{OCF}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 40 & 0 & 1 & 0 \\ 0 & 40 & 0 & 1 \end{bmatrix}$$

$$\operatorname{obsv}(A_{CCF}^T, B_{CCF}^T) = \operatorname{obsv}(A_{OCF}, C_{OCF}) - B$$

$\rightarrow$ $\therefore$ The duality relationship is proved

PROBLEM 4 : Stability Analysis

Part (a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\text{Let } Q = I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}$$

$$A^T P + PA = -Q$$

$$\left[\begin{array}{l:l:l:l} 40P_{12} + 5P_{14} + 40P_{21} + 5P_{41} & P_{11} + 40P_{22} + 5P_{42} & 40P_{23} + 5P_{43} & 40P_{24} + P_{3} \ldots + 5P_{4} \\ P_{11} + 40P_{22} + 5P_{14} & P_{12} + P_{21} & P_{13} & P_{14} + P_{33} \\ 40P_{22} + 5P_{14} & P_{31} & 0 & P_{33} \\ P_{31} + 40P_{42} + P_{44} & P_{32} + P_{41} & P_{33} & P_{34} + \end{array}\right.$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

0 != 1 — from highlights

→ There does not exist a solution for P

→ $\therefore$ A is unstable

$$Eig(A) = \begin{bmatrix} 0 \\ 0 \\ 6.32 \\ -6.32 \end{bmatrix}$$

$\lambda = 6.32 > 0$

→ The System is not asymptotically stable

Part (b)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C_1 = [1\ 0\ 0\ 0]$$

$$G(s) = C(sI-A)^{-1}B + D$$

$$= \frac{2s^2}{3(s^2-40)s^2} = \frac{2}{3(s+2\sqrt{10})(s-2\sqrt{10})}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} s/s^2-40 & s^2/s(s^2-40) & 0 & 0 \\ 40/s^2-40 & s/s^2-40 & 0 & 0 \\ 5/s(s^2-40) & 5/s^2(s^2-40) & 1/s & 1/s^2 \\ 5/s^2-40 & 5/s(s^2-40) & 0 & 1/s \end{bmatrix} \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix}$$

$$= \frac{2/3\, s^2}{(s+2\sqrt{10})(s-2\sqrt{10})s^2}$$

$$= \frac{2}{3}\left[\frac{A}{s+2\sqrt{10}} + \frac{B}{s-2\sqrt{10}}\right]$$

$$G(s) = \frac{\sqrt{10}}{60}\left[\frac{1}{s-2\sqrt{10}} - \frac{1}{s+2\sqrt{10}}\right]$$

$$g(t) = \frac{\sqrt{10}}{60}\left[e^{2\sqrt{10}t} - e^{-2\sqrt{10}t}\right]$$

$\int_0^\infty g(t)\, dt \longrightarrow \infty$ becomes unbounded term

$\therefore$ BIBO unstable for $C = [1\ 0\ 0\ 0]$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$G(s) = \frac{-1}{12}\left[\frac{1}{4\sqrt{10}}\left[\frac{1}{s+2\sqrt{10}} - \frac{1}{s-2\sqrt{10}}\right]\right] - \frac{7}{12s^2}$$

$$g(t) = \frac{\sqrt{10}}{480}\left[e^{2\sqrt{10}t} - e^{-2\sqrt{10}t}\right] - \frac{7}{12\cancel{s^2}}t$$

$$\int_0^{\infty} g(t) = \int_0^{\infty} \frac{\sqrt{10}}{480}\left[e^{2\sqrt{10}t} - e^{-2\sqrt{10}t}\right] - \frac{7}{12}t \, dt$$

Goes to infinity

$\rightarrow$ $\therefore$ The output is unbounded and BIBO Unstable

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$G(s) = \frac{3}{4(s^2 - 40)} + \frac{1.667}{4s^2}$$

$$= \frac{3}{4}\left[\frac{1}{(s+2\sqrt{10})(s-2\sqrt{10})}\right] + \frac{1.667}{4s^2}$$

$$= \frac{3}{4 \cdot 4\sqrt{10}}\left[\frac{1}{s-2\sqrt{10}} - \frac{1}{s+2\sqrt{10}}\right] + \frac{1.667}{4s^2}$$

$$g(t) = \frac{3\sqrt{10}}{160}\left[e^{2\sqrt{10}t} - e^{-2\sqrt{10}t}\right] + \frac{1.667t}{4}$$

$$\int_0^\infty g(t)\,dt = \int_0^\infty \left[\frac{3\sqrt{10}}{160}\left[\underbrace{e^{2\sqrt{10}t}}_{\infty} - e^{-2\sqrt{10}t}\right] + \underbrace{\frac{1.667t}{4}}_{\infty}\right]$$

$\longrightarrow$ $\therefore$ The system is not BIBO stable

PROBLEM 5 : State - feedback Control design

Part (a)

$t_s = 2 = \frac{4}{\zeta \omega_n}$

$\therefore \zeta \omega_n = 2$

$OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$

$\frac{\zeta^2}{1-\zeta^2} = 1.5506$

$\frac{1-\zeta^2}{\zeta^2} = \frac{1}{1.5506}$; $\frac{1}{\zeta^2} = \frac{2.5506}{1.5506}$

$\zeta = 0.7797$

$\omega_n = \frac{2}{0.7797} = 2.5651$

Eigen Value $= -\zeta\omega_n \pm \omega_n \sqrt{1-\zeta^2}\, j = -2 \pm 1.606j$

→ Let the other two eigen values $< -22 \rightarrow -30, -40$

Part (b)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

→ From 2C this system is Controllable

$$T = \begin{bmatrix} 0 & 0 & 0.667 & 0 \\ 0 & 0 & 0 & 0.667 \\ 16.67 & 0 & 0.5 & 0 \\ 0 & 16.67 & 0 & 0.5 \end{bmatrix}$$

open loop characteristic equation :

$$|A - sI| = \begin{vmatrix} -s & 1 & 0 & 0 \\ 40 & -s & 0 & 0 \\ 0 & 0 & -s & 0 \\ 5 & 0 & 0 & -s \end{vmatrix}$$

$$\to s^4 + \underset{a_3}{0s^3} - \underset{a_2}{40s^2} + \underset{a_1}{0s} + \underset{a_0}{0}$$

Desired characteristic Polynomial :

$$s^4 + \underset{c_3}{74s^3} + \underset{c_2}{1486.58s^2} + \underset{c_1}{5260.57s} + \underset{c_0}{1310.03}$$

$$K = \begin{bmatrix} c_0 - a_0 & c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \end{bmatrix} T^{-1}$$

$$= \begin{bmatrix} 2643.8 & 347.5 & -473.7 & -315.6 \end{bmatrix}$$

<u>PROBLEM6</u> : observer and observer based control design

Part (a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\underset{a_3}{s^4 + 0s^3} + \underset{a_2}{40s^2} + \underset{a_1}{0s} + \underset{a_0}{0} =$$

$$r_1 = C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$r_2 = r_1 A + a_3 r_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_3 = r_2 A + a_2 r_1 = \begin{bmatrix} 5 & 0 & -40 & 0 \end{bmatrix}$$

$$r_4 = r_3 A + a_1 r_1 = \begin{bmatrix} 0 & 5 & 0 & -40 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & -40 & 0 \\ 0 & 5 & 0 & -40 \end{bmatrix}$$

Desired eigen values for observer error dynamics

$(s+20-16.06j)(s+20+16.06j)(s+300)(s+400)$

$$\underbrace{s^4}\; + \underbrace{740s^3}_{c_3} + \underbrace{148657.96s^2}_{c_2} + \underbrace{5260573.9s}_{c_1} + \underbrace{78955525.62}_{c_0}$$

$L = R^{-1}\left[c_3 - a_3 \quad c_2 - a_2 \quad c_1 - a_1 \quad c_0 - a_0\right]$

$$= \begin{bmatrix} 1058034.78 \\ 16980688.804 \\ 740.0 \\ 148697.96 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi \\ 5 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & -4 & 0 \\ 0 & 5 & 0 & -40 \end{bmatrix}$$

$$L = R^{-1}\begin{bmatrix} c_3 - a_3 & c_2 - a_2 & c_1 - a_1 & c_0 - a_0 \end{bmatrix}$$

$$= \begin{bmatrix} 117559.42 \\ 1886743.2 \\ -116819.42 \\ -1738095.29 \end{bmatrix}$$

Full state dynamics for state ii

state space :

$\dot{x} = Ax + Bu$

$y = Cx$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} u \qquad y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

observer :

$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly$

$\hat{y} = C\hat{x}$

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & -1058034.78 & 0 \\ 40 & 0 & -16980688.8 & 0 \\ 0 & 0 & -740 & 1 \\ 5 & 0 & -148697.9 & 0 \end{bmatrix}\hat{x} + \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} u$$

$$+ \begin{bmatrix} 1058034.78 \\ 16980688.8 \\ 740 \\ 148697.96 \end{bmatrix} y \; ; \quad \hat{y} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\hat{x}$$

$$u = \begin{bmatrix} -2643.8 & -347.5 & 473.7 & 315.6 \end{bmatrix} \hat{x} + K g r$$

Controller : $u = -K\hat{x} + K g r$

Full state dynamics for case (iii) :

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} x$$

$$\dot{\hat{x}} = \begin{bmatrix} -117559.42 & 1 & -117559.42 & 0 \\ -1886703.2 & 0 & -1886743.2 & 0 \\ 116819.42 & 0 & 116819.42 & 1 \\ 1738050.24 & 0 & 1738045.24 & 0 \end{bmatrix} \hat{x}$$

$$+ \begin{bmatrix} 0 \\ 2/3 \\ 0 \\ 1/2 \end{bmatrix} u + \begin{bmatrix} 117559.42 \\ 1886743.2 \\ 116819.42 \\ 1738045.24 \end{bmatrix} y$$

$$\hat{y} = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \hat{x}$$

$$u = \begin{bmatrix} -2643.8 & -347.5 & 4737 & 315.6 \end{bmatrix} \hat{x} + K g r$$

PB7 is done in MATLAB attached at the end.

MATLAB and code for problems-

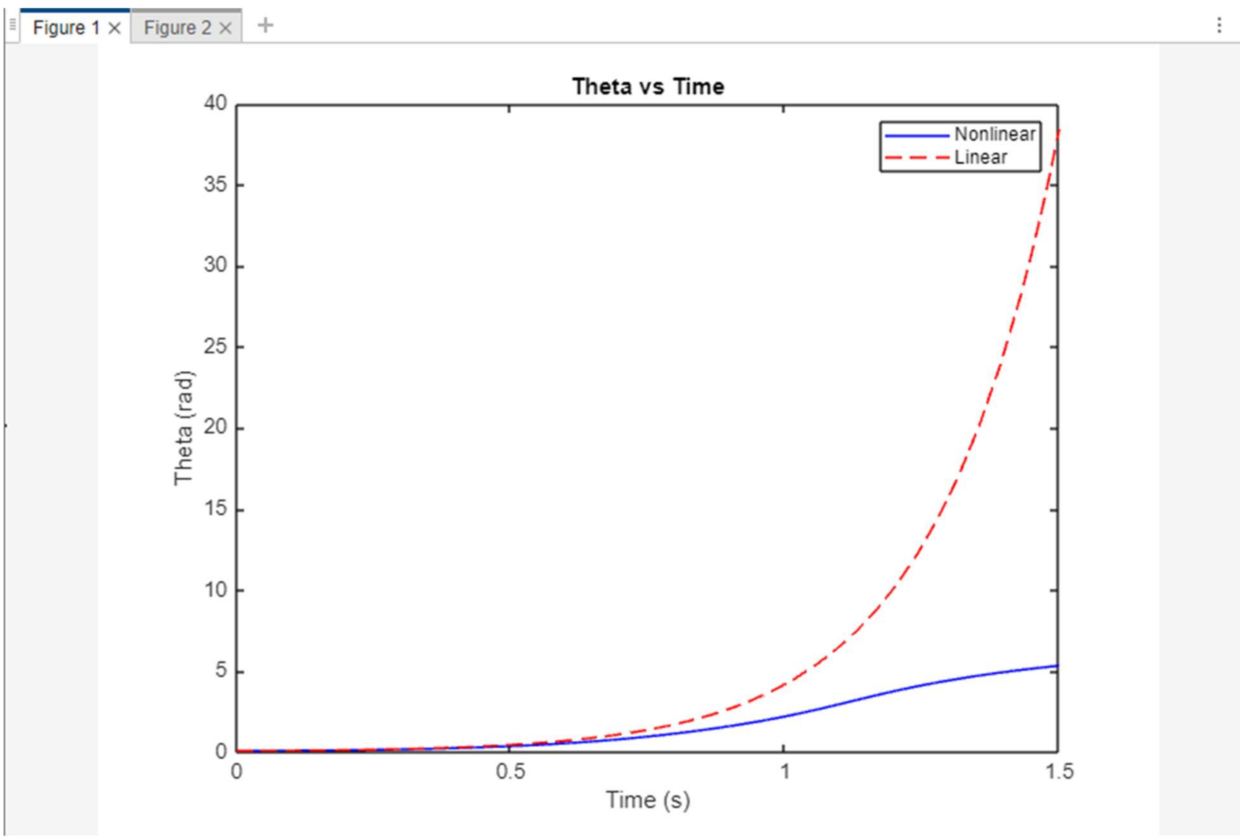
PB1.c

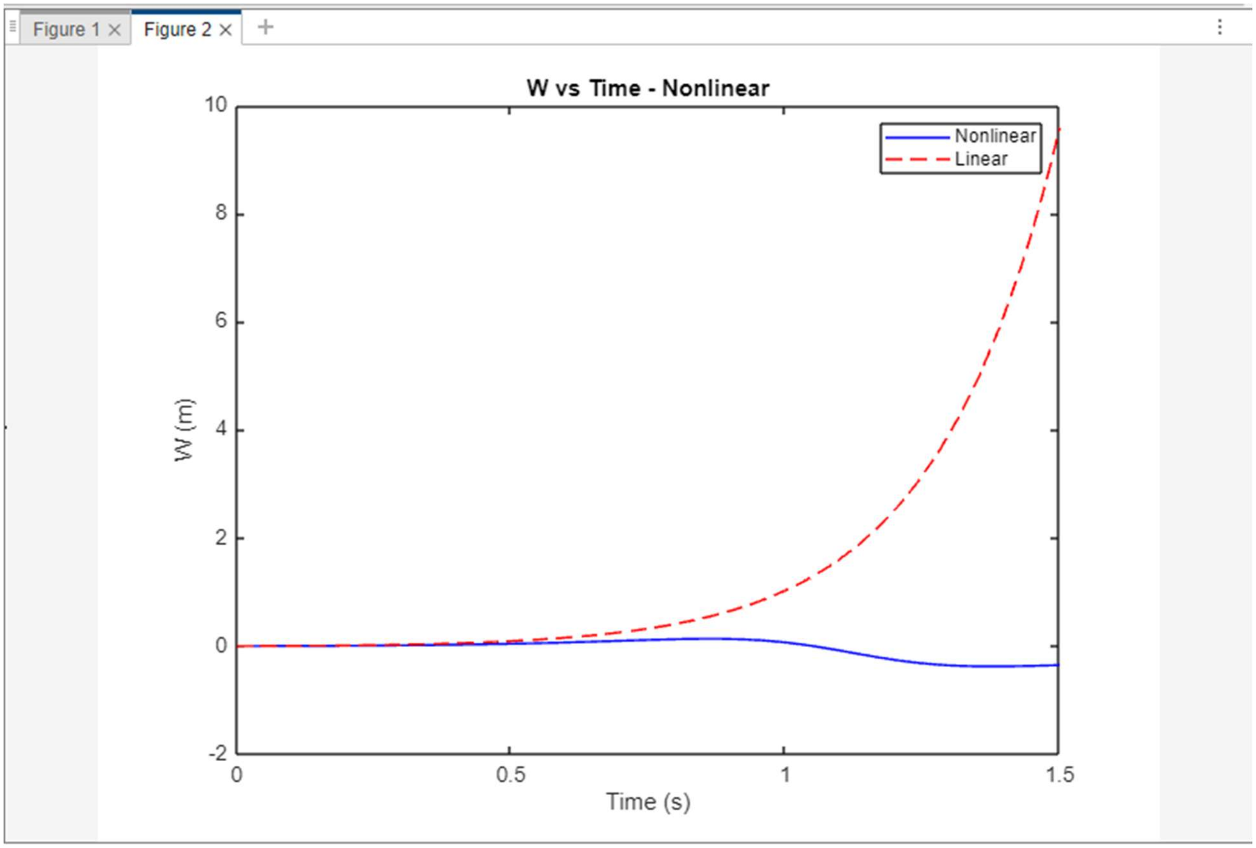
Code –

```
clear;clc;
% Define constants
m1 = 2;  % kg
m2 = 1;  % kg
L = 0.75;  % m
g = 9.81; % m/s^2

% Define the time span
tspan = [0 1.5];  % seconds

% Define the initial conditions
x0 = [0.1; 0; 0; 0];  % [theta; theta_dot; w; w_dot]

% Define the nonlinear function for the state derivatives
dxdt_nonlinear = @(t, x) [
    x(2); % dx1/dt = x2
    (0 + m2*g*sin(x(1))*cos(x(1)) - m2*L*x(2)^2*sin(x(1)))*cos(x(1))/((m1 + m2 +
m2*cos(x(1))^2)*L) + g*sin(x(1))/L; % dx2/dt
    x(4); % dx3/dt = x4
    (0 + m2*g*sin(x(1))*cos(x(1)) - m2*L*x(2)^2*sin(x(1)))/(m1 + m2 +
m2*cos(x(1))^2); % dx4/dt
];

% Define the linearized state space matrices
A = [0 1 0 0; (m2+m1)*g/m1/L 0 0 0; 0 0 0 1; m2*g/m1 0 0 0];
B = [0; 1/m1/L; 0; 1/m2];
C = [1 0 1 0];
D = 0;

% Define the linear function for the state derivatives
dxdt_linear = @(t, x) A * x + B * 0; % No input (f(t) = 0)

% Solve the nonlinear differential equations
[t_nonlinear, X_nonlinear] = ode45(dxdt_nonlinear, tspan, x0);

% Extract theta and w from the nonlinear solution
theta_nonlinear = X_nonlinear(:, 1);
w_nonlinear = X_nonlinear(:, 3);

% Solve the linearized differential equations
[t_linear, X_linear] = ode45(dxdt_linear, tspan, x0);

% Extract theta and w from the linear solution
theta_linear = X_linear(:, 1);
w_linear = X_linear(:, 3);

% Plot theta (rad) for linear and nonlinear state space in one graph
figure;
plot(t_nonlinear, theta_nonlinear, 'b-', t_linear, theta_linear, 'r--');
```

```
xlabel('Time (s)');
ylabel('Theta (rad)');
legend('Nonlinear', 'Linear');
title('Theta vs Time');

% Plot w (m) for linear and nonlinear state space in separate graphs
figure;
plot(t_nonlinear, w_nonlinear, 'b-',t_linear, w_linear, 'r--');
xlabel('Time (s)');
ylabel('W (m)');
legend('Nonlinear', 'Linear');
title('W vs Time - Nonlinear');
```

Graphs –



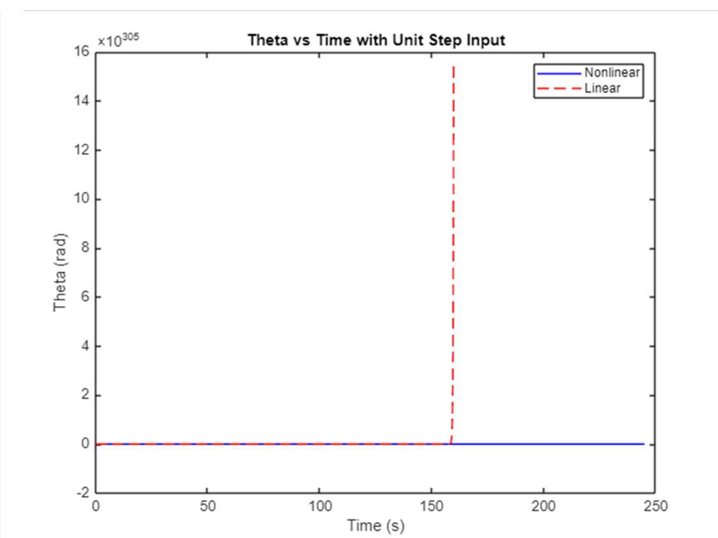


PB1.d

Code –

```
clear;clc;
% Define constants
m1 = 2;  % kg
m2 = 1;  % kg
L = 0.75;  % m
g = 9.81; % m/s^2

% Define the time span
tspan = [0 10000];  % seconds

% Define the initial conditions
x0 = [0.1; 0; 0; 0];  % [theta; theta_dot; w; w_dot]

% Define the nonlinear function for the state derivatives
dxdt_nonlinear = @(t, x) [
    x(2); % dx1/dt = x2
    (1 + m2*g*sin(x(1))*cos(x(1)) - m2*L*x(2)^2*sin(x(1)))*cos(x(1))/((m1 + m2 +
m2*cos(x(1)))*L) + g*sin(x(1))/L; % dx2/dt with f(t) = 1
    x(4); % dx3/dt = x4
    (1 + m2*g*sin(x(1))*cos(x(1)) - m2*L*x(2)^2*sin(x(1)))/(m1 + m2 + m2*cos(x(1)));
% dx4/dt with f(t) = 1
```

```
];

% Define the linearized state space matrices
A = [0 1 0 0; (m2+m1)*g/m1/L 0 0 0; 0 0 0 1; m2*g/m1 0 0 0];
B = [0; 1/m1/L; 0; 1/m2];
C = [1 0 1 0];
D = 0;

% Define the linear function for the state derivatives with f(t) = 1
dxdt_linear = @(t, x) A * x + B * 1; % Input is a unit step function

% Solve the nonlinear differential equations
[t_nonlinear, X_nonlinear] = ode45(dxdt_nonlinear, tspan, x0);

% Extract theta from the nonlinear solution
theta_nonlinear = X_nonlinear(:, 1);

% Solve the linearized differential equations
[t_linear, X_linear] = ode45(dxdt_linear, tspan, x0);

% Extract theta from the linear solution
theta_linear = X_linear(:, 1);

% Plot theta (rad) for linear and nonlinear state space in one graph
figure;
plot(t_nonlinear, theta_nonlinear, 'b-', t_linear, theta_linear, 'r--');
xlabel('Time (s)');
ylabel('Theta (rad)');
legend('Nonlinear', 'Linear');
title('Theta vs Time with Unit Step Input');
```



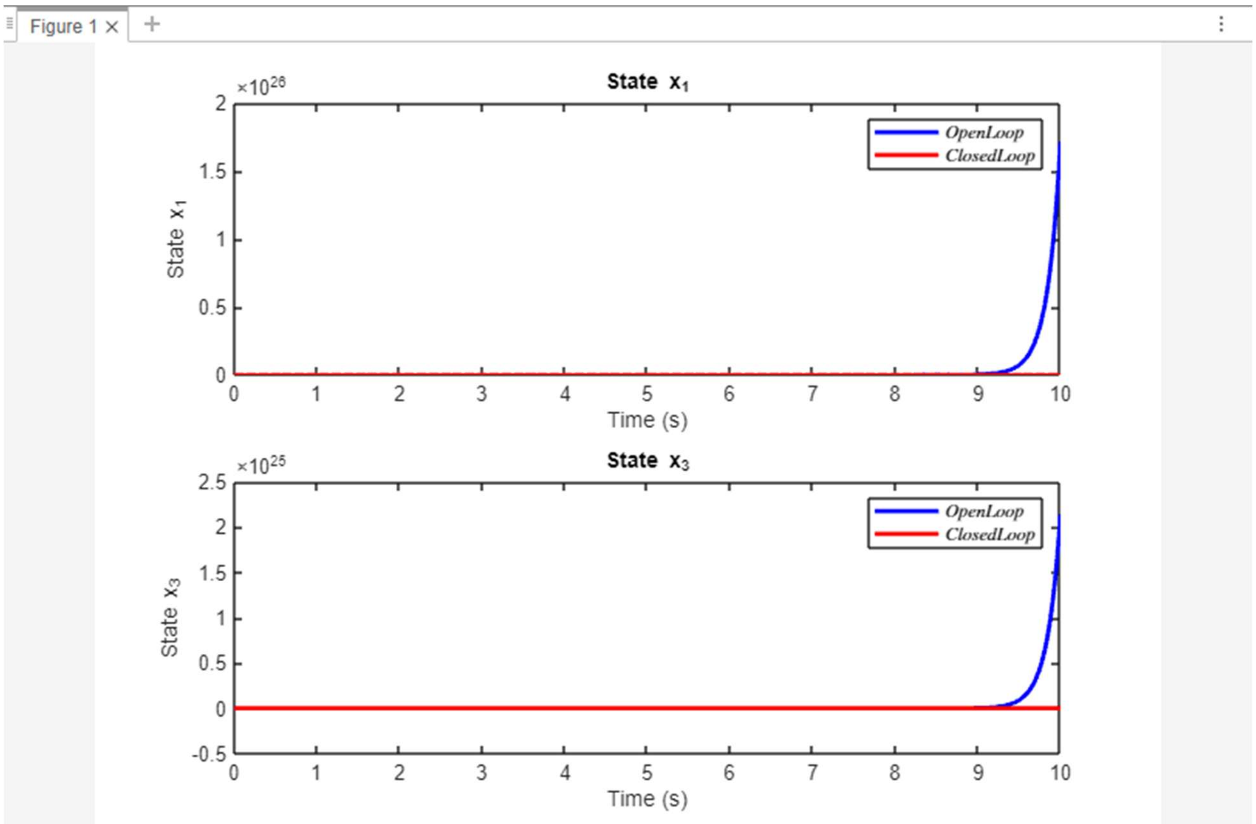
Clearly from the graph we can see when unit step function is applied to the systems, the linear one is unstable whereas the non-linear system is stable.

PB5.b

Code –

```
clear;clc;
syms s;
% Design Specifications
zeta = -log(0.02)/sqrt(pi^2 + log(0.02)^2);
wn = 4/(zeta*2);  % Ts ~ 4/(zeta*wn)

% Desired closed-loop poles
p_a = -zeta*wn + wn*sqrt(1-zeta^2)*1i;
p_conj = conj(p_a);
p_ex = [-30 -40];
p = [p_a p_conj p_ex];

des_poly = expand((s - p_a)*(s-p_conj)*(s-p_ex(1))*(s-p_ex(2)));

% Expand the polynomial
expanded_poly = expand(des_poly);

% Extract coefficients
coeffs = coeffs(expanded_poly, s, 'All');

% Extract and round the constant term
rounded_constant = round(coeffs, 2);
display(rounded_constant);
a = [0 0 -40 0];
c = [7895.55 5260.57 1486.58 74];

A = [0 1 0 0;40 0 0 0;0 0 0 1;5 0 0 0]; B = [0; 2/3; 0; 1/2]; C = [1 0 1 0];
T = [0 0 0.667 0;0 0 0 0.667;-16.667 0 0.5 0;0 -16.667 0 0.5];
x_0 = [0.1 0 0 0]';
k_p = place(A,B,p);
k_calc = (c - a)*inv(T);
display(k_p);
display(k_calc)
kg = -inv(C*inv(A-B*k_calc)*B);

sys_op = ss(A,B,C,0);
sys_cl = ss(A-B*k_calc, B*kg, C, 0);

% Define the time vector and input
t = 0:0.01:10; % Time from 0 to 10 seconds with step size 0.01
u = ones(size(t)); % Unit step input

% Simulate the response
[y_o, t, x_o] = lsim(sys_op, u, t,x_0);

% Simulate the response
[y_cl, t, x_cl] = lsim(sys_cl, u, t,x_0);

% Plot x1 and x3 separately
figure;
subplot(2,1,1); % Subplot for x1
```

```
plot(t, x_o(:,1),'b',t, x_cl(:,1),'r','linewidth',2); % x1 is the first column of x
legend({'$Open Loop$','$Closed Loop$'},'Interpreter', 'latex');
title('State x_1');
xlabel('Time (s)');
ylabel('State x_1');

subplot(2,1,2); % Subplot for x3
plot(t, x_o(:,3),'b',t, x_cl(:,3),'r','linewidth',2); % x1 is the first column of x
legend({'$Open Loop$','$Closed Loop$'},'Interpreter', 'latex');
title('State x_3');
xlabel('Time (s)');
ylabel('State x_3')
```



PB6.b

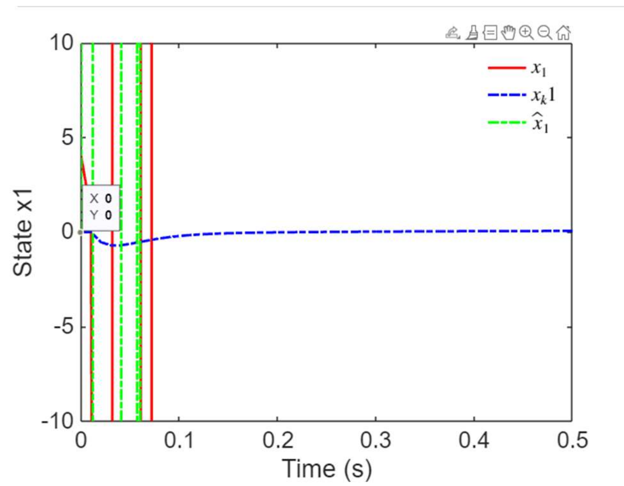
Case ii.

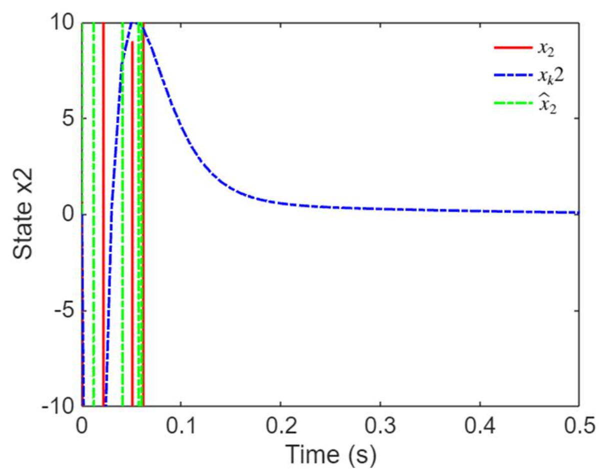
```
clear all;
clc;
% Design Specifications
zeta = -log(0.02)/sqrt(pi^2 + log(0.02)^2);
wn = 4/(zeta*2);  % Ts ~ 4/(zeta*wn)

% Desired closed-loop poles
p_a = -zeta*wn + wn*sqrt(1-zeta^2)*1i;
```

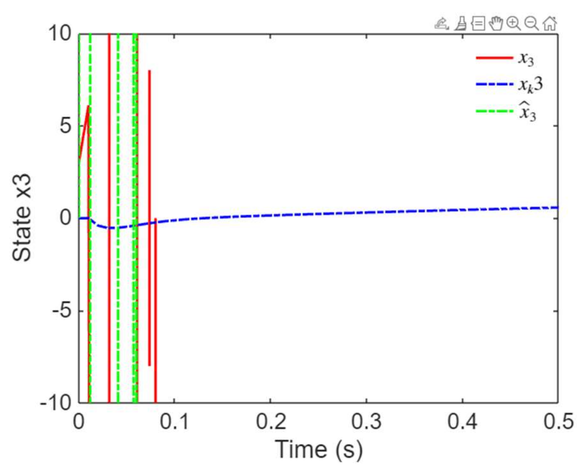
```
p_conj = conj(p_a);
p_ex = [-30 -40];
p = [p_a p_conj p_ex];

% System Matrices
A = [0 1 0 0; 40 0 0 0; 0 0 0 1; 5 0 0 0];
B = [0; 2/3; 0; 1/2];
C = [0 0 1 0];
D = 0;

% State-feedback gain (provided)
K = place(A,B,p);
kg = -inv(C*inv(A-B*K)*B);

% Observer gain (provided)
L = place(A',C',10*p);
L = L';

% Simulation parameters
t = 0:0.01:0.5;
nt = length(t);
dt = t(2) - t(1);

% Initial states
x = zeros(4, nt);
x_hat = zeros(4, nt);
x_k = zeros(4, nt);

x(:,1) = [4; 1; 3; 6];       % No initial state
x_hat(:,1) = [0; 0; 0; 0]; % Initial observer error
x_k(:,1) = [0; 0; 0; 0];       % No initial state

% Initialize outputs
y = zeros(1, nt);
y_hat = zeros(1, nt);
y_k = zeros(1, nt);

y(:,1) = C * x(:,1);
y_hat(:,1) = C * x_hat(:,1);
y_k(:,1) = C * x_k(:,1);

x_obs = x;
y_obs = y;

% Reference input (Unit step)
for i = 1:nt
    r(:,i) = 1;
end

u(:,1) = -K*x(:,1) + kg*r(1);
u_hat(:,1) = -K*x_hat(:,1) + kg*r(1);

% Simulation
for i = 1:nt-1
    % Open-loop
```

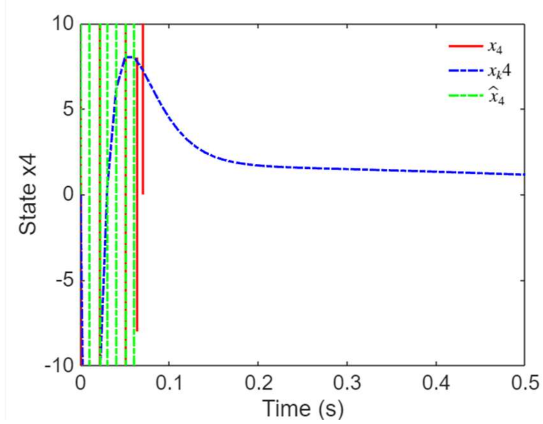
```
    x_dot(:,i) = A*x(:,i) + B*r(i); %step input
    x(:,i+1) = x(:,i) + x_dot(:,i)*dt;
    y(:,i+1) = C*x(:,i+1);

    % Closed-loop with state feedback
    x_k_dot(:,i) = A*x_k(:,i) + B*u(:,i);
    x_k(:,i+1) = x_k(:,i) + x_k_dot(:,i)*dt;
    y_k(:,i+1) = C*x_k(:,i+1);
    u(i+1) = -K*x_k(:,i + 1) + kg*r(:,i+1);  % Control input based on actual state

    % Observer update
    x_obs_dot(:,i) = A*x_obs(:,i)+ B*u_hat(:,i);
    x_obs(:,i+1) = x_obs_dot(:,i) + x_obs_dot(:,i)*dt;
    y_obs(:,i+1) = C*x_obs(:,i+1);
    x_hat_dot(:,i) = A*x_hat(:,i)+ B*u_hat(:,i)+ L*(y_obs(:,i)- C*x_hat(:,i));
    x_hat(:,i+1) = x_hat(:,i) + x_hat_dot(:,i)*dt;
    y_hat(:,i+1) = C*x_hat(:,i+1);
    u_hat(:,i+1) = -K*x_hat(:,i+1) + kg*r(:,i);

end

% Plotting state x1 and estimated state x1_hat
figure;
plot(t, x_obs(1,:), 'r', t, x_k(1,:), 'b-.',t, x_hat(1,:), 'g-.', 'LineWidth', 2);
set(gca, 'FontSize', 18);
ylim([-10 10]); % Set y-axis limits for x1
legend({'$x_1$','$x_k1$', '${\hat x}_1$'}, 'Interpreter', 'latex');
legend boxoff;
xlabel('Time (s)');
ylabel('State x1');

% Plotting state x2 and estimated state x2_hat
figure;
plot(t, x_obs(2,:), 'r', t, x_k(2,:), 'b-.',t, x_hat(1,:), 'g-.', 'LineWidth', 2);
set(gca, 'FontSize', 18);
ylim([-10 10]); % Set y-axis limits for x2
legend({'$x_2$','$x_k2$', '${\hat x}_2$'}, 'Interpreter', 'latex');
legend boxoff;
xlabel('Time (s)');
ylabel('State x2');

% Plotting state x3 and estimated state x3_hat
figure;
plot(t, x_obs(3,:), 'r', t, x_k(3,:), 'b-.',t, x_hat(1,:), 'g-.', 'LineWidth', 2);
set(gca, 'FontSize', 18);
ylim([-10 10]); % Set y-axis limits for x3
legend({'$x_3$','$x_k3$', '${\hat x}_3$'}, 'Interpreter', 'latex');
legend boxoff;
xlabel('Time (s)');
ylabel('State x3');

% Plotting state x4 and estimated state x4_hat
figure;
plot(t, x_obs(4,:), 'r', t, x_k(4,:), 'b-.',t, x_hat(4,:), 'g-.', 'LineWidth', 2);
set(gca, 'FontSize', 18);
```

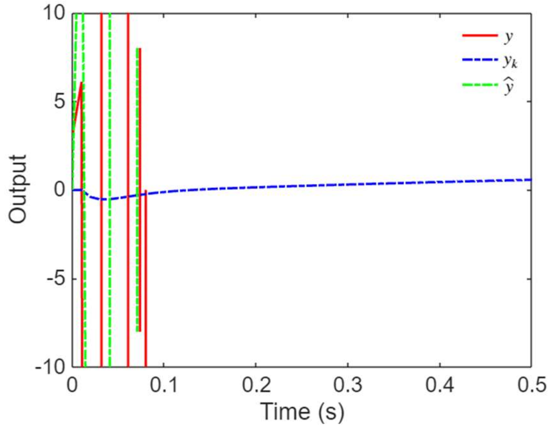
```
ylim([-10 10]); % Set y-axis limits for x4
legend({'$x_4$','$x_k4$', '${\hat x}_4$'}, 'Interpreter', 'latex');
legend boxoff;
xlabel('Time (s)');
ylabel('State x4');

% Plotting output y and estimated output y_hat
figure;
plot(t, y_obs, 'r', t, y_k, 'b-.',t, y_hat, 'g-.', 'LineWidth', 2);
set(gca, 'FontSize', 18);
ylim([-10 10]); % Set y-axis limits for output
legend({'$y$','$y_k$', '${\hat y}$'}, 'Interpreter', 'latex');
legend boxoff;
xlabel('Time (s)');
ylabel('Output');
```



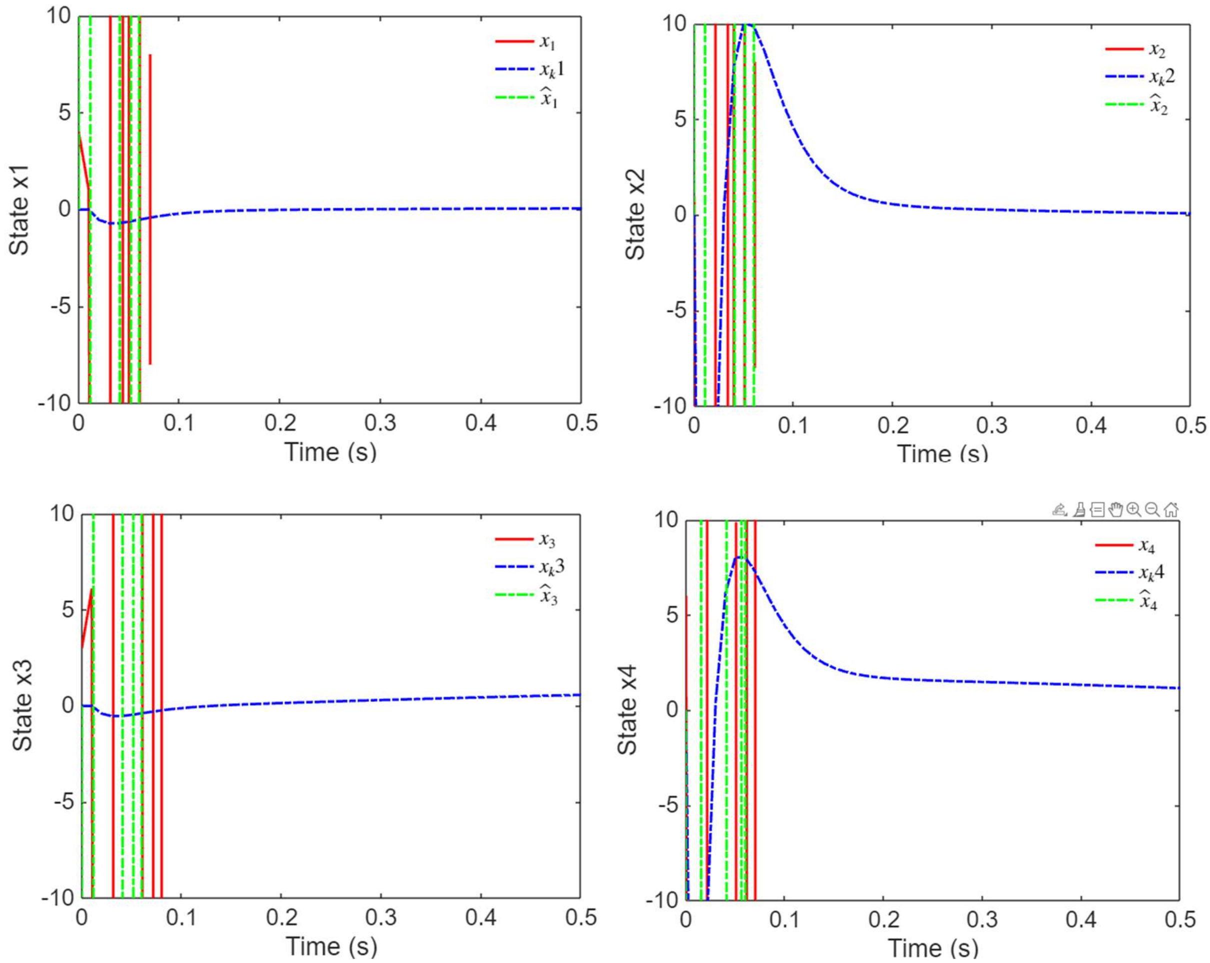


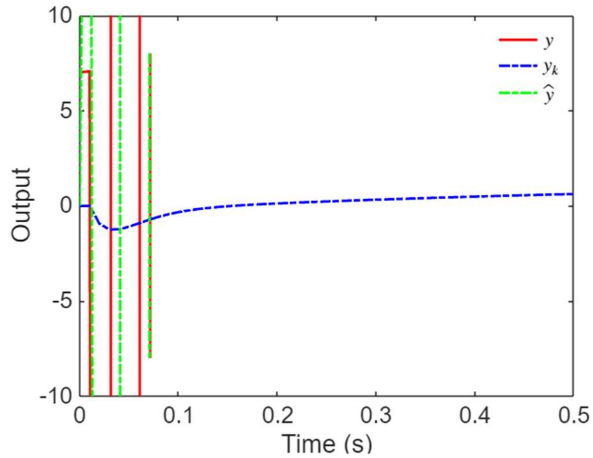






Case iii. C = [1 0 1 0]



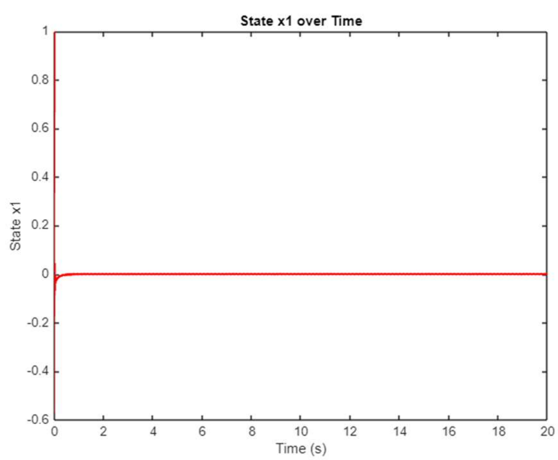


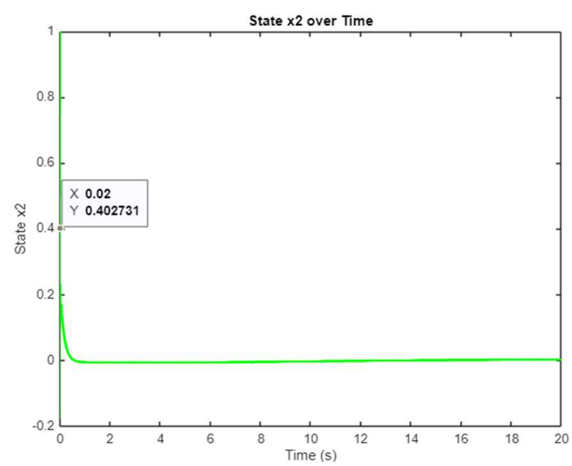
PB 7.a.

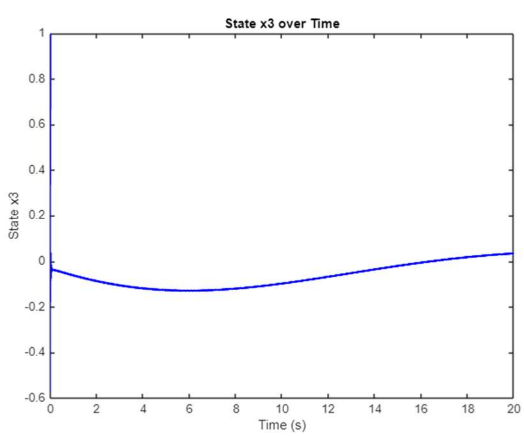
Code –

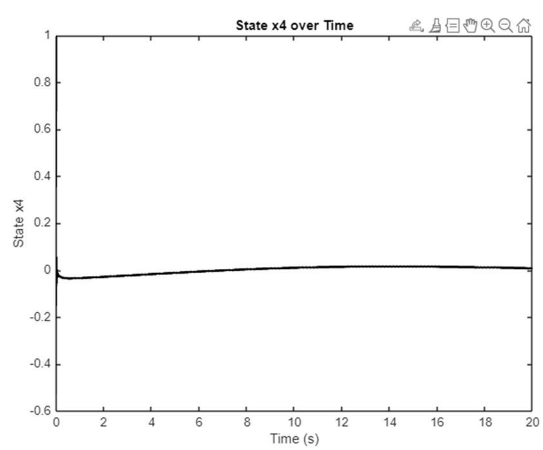
```
clear all;clc;
% System Matrices
A = [0 1 0 0; 40 0 0 0; 0 0 0 1; 5 0 0 0];
B = [0; 2/3; 0; 1/2];
C = [1 0 0 0;0 0 1 0];
D = 0;
Q = C'*C; R = 1;

[P, d, K] = care(A,B,Q,1);
Acl = A - inv(R)*B'*P;

t = 0:0.01:20; %0.01 time span of interest
nt = length(t); % number of time steps
dt = t(2) - t(1);

x(:,1) = [1;1;1;1];
y(:,1) = C*x(:,1);
u(:,1) = -inv(R)*B'*P*x(:,1);

for i = 1:nt-1
    x_dot(:,i+1) = Acl*x(:,i);
    x(:,i+1) = x(:,i) + x_dot(:,i+1)*dt;
    y(:,i+1) = C*x(:,i+1);
    u(:,i+1) = -inv(R)*B'*P*x(:,i);
end

% Plotting
figure;
plot(t, x(1,:), 'r', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('State x1');
title('State x1 over Time');
```

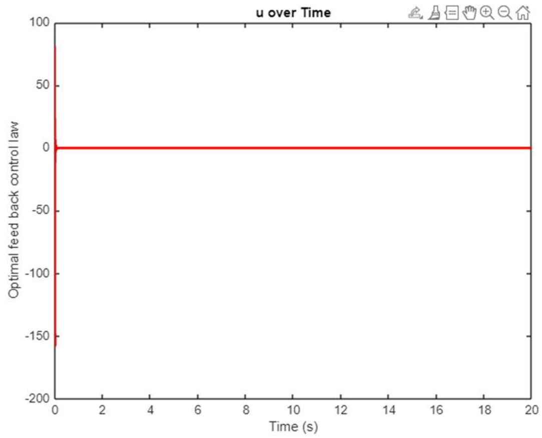
```
figure;
plot(t, x(2,:), 'g', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('State x2');
title('State x2 over Time');

figure;
plot(t, x(3,:), 'b', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('State x3');
title('State x3 over Time');

figure;
plot(t, x(4,:), 'k', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('State x4');
title('State x4 over Time');

figure;
plot(t, u, 'r', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Optimal feed back control law');
title('u over Time');
```



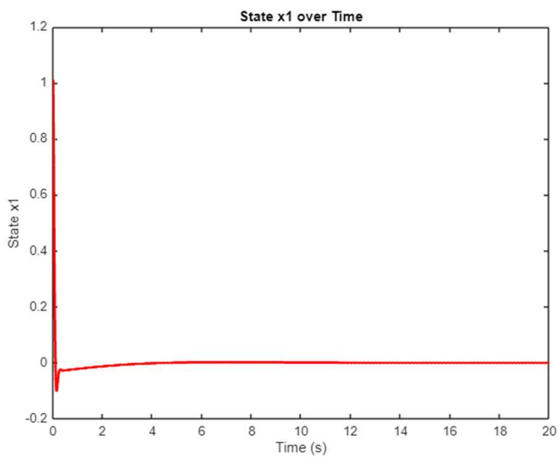


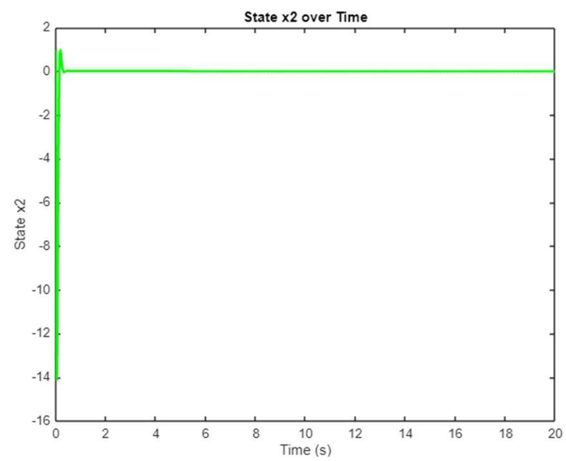


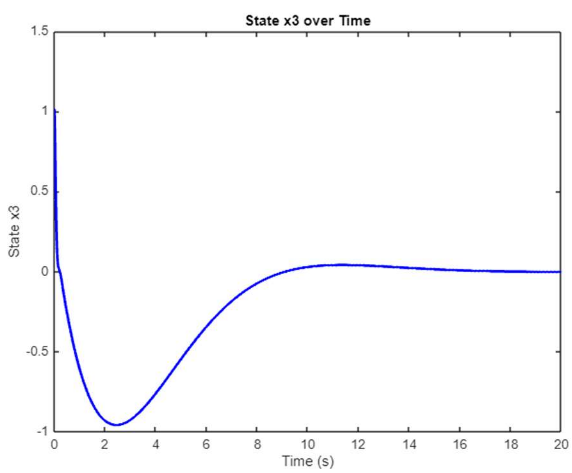


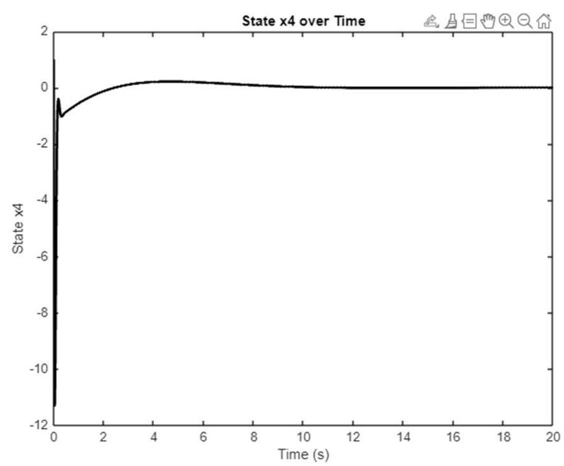


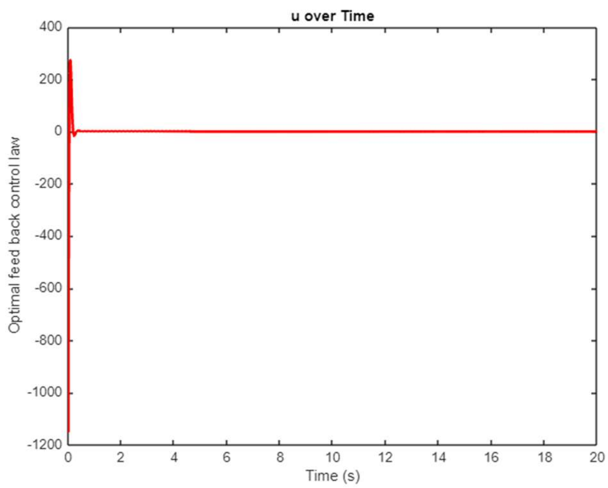
7.b. `C = [1000 0 0 0;0 0 10 0];`











The differences in the plots between the two scenarios in your question, where different weighting matrices Q are used in the LQR formulation, stem from how the relative importance of each state variable and control effort is quantified in the system's performance index. This performance index is minimized to determine the optimal control strategy.

For case 1, equal emphasis is placed on minimizing the deviations Θ over w.

For case 2, high penalty on Θ leads to more aggressive control over Θ than w.