Homework 8
ME5659 Spring 2024

Due: See Canvas, turn in <u>on Gradescope</u>

Problem 1 (13 points)

Consider a linear state-space model

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

(a) **3 points.** Design a state feedback control law $u = -Kx$ so that the closed-loop system has natural frequency $\omega_n = 1\ rad/sec$ and damping ratio $\zeta = 0.707$.

(b) **3 points.** Design an observer gain L so that the error dynamics have natural frequency $\omega_n = 10\ rad/sec$ and damping ratio $\zeta = 0.5$.

(c) **3 points.** Use your answers to design an observer-based controller that achieves both objectives in (a) and (b). Please write the full dynamic equations including $\dot{x}, \dot{\hat{x}}, u$.

(d) **4 points.** Assume the initial conditions are given by $x_0 = \begin{bmatrix} 5 & -4 \end{bmatrix}$, $\hat{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$. Let the feedforward gain $k_g = 1$. Given a reference input $r(t) = sin(t)$, please simulate the closed-loop full dynamical system (including x and $\hat{x}$) over the time horizon $t \in [0, 10]\ sec$. Generate 2 figures: one that plots $x_1(t)$ and its estimate $\hat{x}_1(t)$ over time, and one that plots $x_2(t)$ and its estimate $\hat{x}_2(t)$ over time. Hand in your code and your plots.

Problem 2 (12 points)

Consider a single-input single-output four-dimensional state-space model:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -650 & -180 & -90 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \quad x_0 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 90 & 15 & 10 & 0 \end{bmatrix} x$$

You may use MATLAB for this entire problem. Hand in your codes and plots.

(a) **6 points.** Design a state-feedback integral controller to obtain 2 percent overshoot and a 2 s settling time as well as a steady-state output value of 1. Compare open-loop and closed-loop responses to a unit step input.

(b) **6 points.** Design an observer-based integral controller. Let observer eigenvalues be 10 times that of the desired state-feedback control law obtained in (a). Assume the initial condition $\hat{x}(0) = 0$. Compare open-loop, closed-loop with state-feedback, and closed-loop with observer responses to a unit step input.