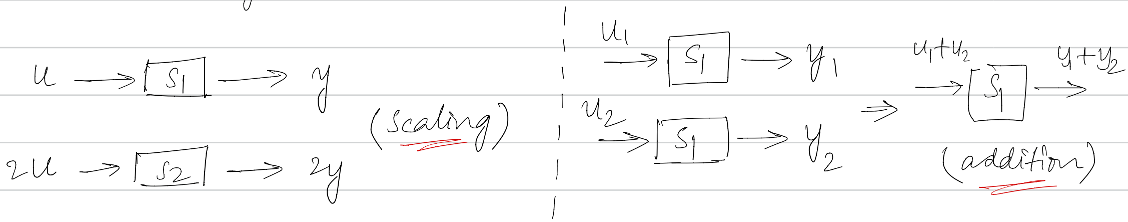
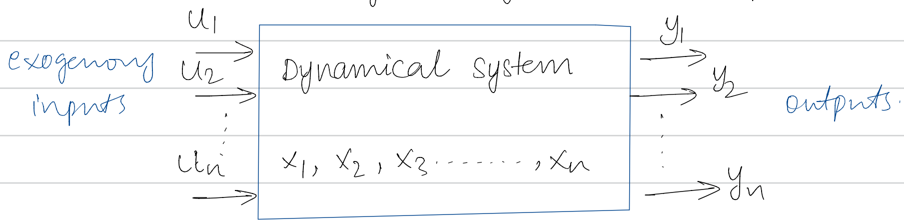
Linear systems have ADDITIVE and HOMOGENITY properties unlike Non linear systems.



STATE SPACE MODEL

It is a standard way to organize a set of ODE's



Ex: General lumped systems.

For the lumped systems, we have r input signals $u_1, u_2, \dots u_r$; m output signals as $y_1, y_2 \dots y_m$. We have n states as $x_1, x_2 \dots x_n$; these States are changing w.r.t inputs. Inside the dynamice system box, we have n functions for all states, (these functions can be linear or non linear).

$$\dot{x}_1 = f_1(x_1, x_2 \dots, x_n, u_1, u_2 \dots u_r, t)$$

$$\vdots$$

$$\dot{x}_n = f_n(x_1, x_2 \dots x_n, u_1, u_2 \dots u_r, t)$$

Time varying systems

we vectorize them as:

$$x = [x_1, x_2 \cdots x_n]^T, \quad y = [y_1, y_2 \cdots y_m]^T, \quad u = [u_1, u_2 \cdots u_r]^T$$

Then we can reorganize them:

$$\dot{x} = f(x, u, t)$$
$$y = h(x, u, t) \qquad \text{— eq. 1}$$

Convert it to matrix representation

$$\dot{x} = \underset{n\times n}{A}x + \underset{n\times r}{B}u$$
(LTI)
$$y = \underset{m\times n}{C}x + \underset{m\times r}{D}u$$

A = State Matrix
B = input Matrix
C = output Matrix

→ The order of Dynamic Systems = Number of state variables req to uniquely describe the system. ★

→ like DOF*

→ For a given Dynamic System, the order is fixed.

The state variables can be chosen by looking at the ODEs and finding variables that need initial conditions to solve the ODEs.

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_2\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_0(y) = n(t) \qquad \text{— eq. 2}$$

If we choose 'y' and 'its derivatives' as state variables: $x_1 = y$, $x_2 = \frac{dy}{dt}$, ..., $x_n = \frac{d^{n-1}y}{dt^{n-1}}$

RS5 project?
Controls assignments
Textbooks?

$$\dot{x}_1 = \dot{y} = x_2,$$

$$\dot{x}_2 = \ddot{y} = x_3,$$

$$\vdots$$

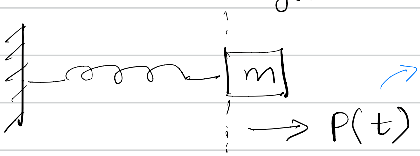
$$\dot{x}_n = \frac{d^n y}{dt^n} = -a_0 x_1 - a_1 x_2 + \cdots + -a_{n-1} x_n + u(t).$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

MATLAB

DYNAMIC SYSTEMS and DES:

how the system is changing = f(current state)



→ Position

← Acceleration

$$M\ddot{P}(t) = -K P(t)$$

$$\Rightarrow \ddot{P}(t) = -\frac{K}{m} P(t)$$

derivatives states

$$\underbrace{\dot{x} = f(x)}_{\text{defines stability}}$$

$$\underbrace{\dot{x} = f(x,u)}_{\text{Defines how a system changes}}$$

Continuous, linear, time invariant systems:

((State space representation is built around the state vector))
$x(t)$ which is a vector of all of the state variables.
((How the state vector changes / derivative of the state vector))
is a ((linear combination of the current state + linear combination of external inputs))

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

output $\rightarrow$ $y(t) = C\,x(t) + D\,u(t)$ linear combination of states & inputs

What are state variable? The minimum set of variables that fully describe the system.
(enough information to predict future behavior)

Intro to Control Systems

2 Double sided sheets of cheat sheet.

State space Rep^n

Linearity – Linearization

Laplace Tranf^n – Transfer fn^n

State space – Transfer fnc^n Transf^n

Cannonical – similarity

inverse Laplace

State space sol^{n}s

stability defin^n

Lyapunov Stability

More stability analysis

Controllability

Similarity transf^n

uncontrollable subspace

DOUBTS

Assignment – 1

2) b.

4) a, b, c, d

Assignment – 2

1) a, b

Assignment_1

a)

1) multiply Numerator and Denominator with $X(s)$

2) $X(s) = x$
$sX(s) = \dot{x}$
$s^2 X(s) = \ddot{x}$
$s^4 X(s) = x^{IV}$

3) write $x, \ddot{x}, x^{IV}$ etc in x state variable vector form.
$x_4 = \dddot{x}$
$x_3 = \ddot{x}$
$x_2 = \dot{x}$
$x_1 = x$
So, x^{IV} will be $\dot{x}_4$

4) write $\dot{x}_4$ in terms of u and substitute in y.

5) write $\dot{x} \rightarrow [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4]^T$
as $AX + BU$ form.

6) write y as $CX + DU$.

b) when $y_1, y_2, \dot{y}_1, \dot{y}_2, \ddot{y}_1, \ddot{y}_2$

1) write the equation w.r.t $\ddot{y}$ i.e $\ddot{y}_1$ & $\ddot{y}_2$ in terms of u.

2) Now write $\dot{x}$ as $y_1, \dot{y}_1, y_2, \dot{y}_2$ terms with also u.

(2)

a) $ml^2\ddot{\theta} + b\dot{\theta} + mgl\sin\theta = T$

T input, x output $\rightarrow \theta$.

$$x = [x_1 \ x_2]^T \Rightarrow [\theta \ \dot{\theta}]^T$$

$$\dot{x} = [\dot{\theta} \ \ddot{\theta}]^T$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} \qquad \rightarrow \quad \dot{x}_1 = x_2.$$

$$\dot{x}_2 = \ddot{\theta} = \frac{T - mgl\sin\theta - b\dot{\theta}}{ml^2}$$

b) ASK SHREEJIT / MANI

3)

a) $x_1(t) = P_x\cos\theta + (P_y - 1)\sin\theta$

$\dot{x}_1(t) = \dot{P}_x\cos\theta - P_x\sin\theta + \dot{P}_y\sin\theta + P_y\cos\theta - \cos\theta$

$x_2(t) = -P_x\sin\theta + (P_y - 1)\cos\theta$

$\dot{x}_2(t) = -\dot{P}_x\sin\theta - P_x\cos\theta - P_y\sin\theta + \dot{P}_y\cos\theta + \sin\theta$

$x_3(t) = \theta \longrightarrow \dot{x}_3(t) = \dot{\theta} = \omega$.

Put $P_x = v\cos\theta$, $P_y = v\sin\theta$, $\dot{\theta} = w$ above

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} (\quad) \\ (\quad) \\ \omega \end{bmatrix}$$

$\Delta\dot{x} = A\Delta x + B\Delta u$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_n} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \cdots \end{bmatrix}$$

$D = 0$

Assignment 2

$$\dot{x} = Ax + Bu \qquad \Longrightarrow \qquad \frac{Y(s)}{U(s)} = G(s) = C(sI - A)^{-1}B + D.$$
$$y = Cx + Du$$

$$(sI - A)^{-1} = \frac{\text{Adj}\,(sI - A)}{\text{Det}\,|sI - A|}$$

$\text{Adj} = C^T$

$C =$ cofactor of Matrix M.

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \qquad C_{11} = (-1)^{1+1}(\det C_{11}) \quad C_{12} = (-1)^{1+2}(\det C_{12}) \cdots$$

$$\vdots \qquad\qquad \vdots$$

$$C_{31} = (-1)^{3+1}(\det C_{31}) \cdots$$

ex: $M\ddot{x} + b\dot{x} + Kx = F \quad \longrightarrow \quad G(s) = \dfrac{X(s)}{F(s)}$

$$C(sI - A)^{-1}B + D = G(s) = \frac{X(s)}{F(s)} \qquad x_1 = x \quad x_2 = \dot{x} \quad y = x$$

$$\ddot{x} = \dot{x}_2 \qquad \dot{x}_1 = \dot{x}$$

$$M\dot{x}_2 + b\dot{x}_1 + Kx_1 = F$$

write $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = Ax + Bu$

$y = Cx + Du$

$\Big\} \quad G(s) = C(sI - A)^{-1}B + D.$

1) $x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -12 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

ASK Shreejit

2) b) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$|\lambda I - A| = 0 \rightarrow \begin{vmatrix} \lambda-1 & -2 \\ 0 & \lambda-1 \end{vmatrix} = 0 \rightarrow (\lambda-1)^2 = 0 \rightarrow \lambda_{1,2} = 1$

$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$A_1 V_1 = \lambda V_2$

$AV_1 = \lambda V_2 \rightarrow AV_1 = V_2$

$AV_2 = V_1 + \lambda V_2$

$(A - \lambda I) V_2 = V_1$

$\dot{x} = Ax + Bu$

$y = Cx$

$\begin{pmatrix} 4 & 3 \\ 0 & -2 \end{pmatrix}$

$|\lambda I - A| = 0$

$\begin{pmatrix} \lambda - 4 & -3 \\ 0 & \lambda + 2 \end{pmatrix} = 0 \Rightarrow (\lambda - 4)(\lambda + 2) = 0$

$\lambda = 4, \ \lambda = -2$ DCF ✓

$(4I - A) V_1 = 0 \Rightarrow \begin{pmatrix} 0 & -3 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = 0$

$V_1 = (x, y)$

$-3V_{12} + 6V_{12} = 0$

$V_{12} = 0$

$(-2I - A) V_2 = 0 \Rightarrow$

$\begin{pmatrix} -6 & -3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_{21} \\ V_{22} \end{pmatrix} = 0$

$-6V_{21} - 3V_{22} = 0$

$V_{21} = 1$

$V_{22} = -2$

$V_1 = \begin{pmatrix} x \\ 0 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$

$\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \right] V_1 = 0$

$\begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$0x - 2y = 0$

$y = 0$

$x = ? = 1$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow V_1$

$AV_2 = V_1 + V_2 \Rightarrow (A - 1) V_2 = V_1$

$V_2 =$ ✓