Homework 6
ME5659 Spring 2024

Due: See Canvas, turn in on Gradescope

Problem 1 (9 points)

Consider the following linear systems, where

(i) $A = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$

(ii) $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$

(iii) $A = \begin{bmatrix} 3 & 6 & 4 \\ 9 & 6 & 10 \\ -7 & -7 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} 1/3 & 4/3 \\ 4/3 & 1/3 \\ -2/3 & 1/3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 6 \end{bmatrix}.$

(a) **3 points.** Use the PBH test for observability assessment.

(b) **6 points.** If possible, compute the observer canonical form of these systems. If you cannot, put the systems into Kalman observable canonical form and evaluate their detectability.

Problem 2 (4 points)

An approximate linear model of the lateral dynamics of an aircraft, for a particular set of flight conditions, has the state and control vectors in the perturbation quantities

$$x = \begin{bmatrix} p & r & \beta & \phi \end{bmatrix}^T, \quad u = \begin{bmatrix} \delta_a & \delta_r \end{bmatrix}^T$$

where p and r are incremental roll and yaw rates, β is an incremental sideslip change, and ϕ is an incremental roll angle. The control inputs are the incremental changes in the aileron angle δ_a and in the rudder angle δ_r, respectively. This linearized model has

$$A = \begin{bmatrix} -10 & 0 & -10 & 0 \\ 0 & -0.7 & 9 & 0 \\ 0 & -1 & -0.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

If you could have either a rate gyro that measures the incremental roll rate or a bank indicator that measures incremental roll angle, which would you choose and why?

Problem 3 (12 points)

Given

$$G(s) = \frac{s-1}{s^3 + 2s^2 - s - 2}$$

(a) **3 points.** Find a three-dimensional controllable realization. Is it observable? Is it detectable?

(b) **3 points.** Find a three-dimensional observable realization. Is it controllable? Is it stabilizable?

(c) **2 points.** What causes the apparent difference in controllability and observability of the same system? Derive an irreducible transfer function for $G(s)$.

(d) **4 points.** Please derive a realization that is both controllable and observable, represent it in both controllable canonical form and observable canonial form