Homework 5
ME5659 Spring 2024

Due: See Canvas, turn in on Gradescope

Problem 1 (12 points)

Consider the following linear systems, where

(i) $\dot{x} = \begin{bmatrix} -6 & -3 & -5 \\ 0 & -3 & 1 \\ 2 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} -2/3 & 1/3 \\ 1/3 & -2/3 \\ 1/3 & 1/3 \end{bmatrix} u, \quad y = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} x.$

(ii) $\dot{x} = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u, \quad y = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x.$

(a) **4 points.** Are these linear systems controllable? Please justify your answers.

(b) **4 points.** Find the Kalman controllable canonical form for the above linear systems.

(c) **4 points.** Are these linear systems stabilizable? Please justify your answers.

Problem 2 (13 points)

Consider the following state-space model:

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad x(0) = x_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$
$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} x$$

(a) **(3 points)** Is the system controllable? If not, please transform the state-space model into a diagonal canonical form (DCF) where the controllable subspace should be separated from the uncontrollable subspace. Please intuitively tell whether the state-space model in DCF is stabilizable or not.

(b) **(4 points)** Please use the PBH eigenvector test and the PBH rank test to justify the stabilizability of the state-space model in DCF.

(c) **(3 points)** If the system is stabilizable, please calculate the input $u(t)$ that drives $x(0)$ to 0 in 5 seconds.

(d) **(3 points)** Verify that the input achieves this by plotting the state trajectories $x(t)$ vs. time t with the initial condition $x(0)$ in Matlab. The plot should have two trajectories $x_1(t), x_2(t)$.