Homework 4
ME5659 Spring 2024

Due: See Canvas, turn in on Gradescope

Problem 1 (4 points)

Consider the following LTI system

$$\dot{x} = \begin{bmatrix} -1 & 10 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} -2 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} -2 & 3 \end{bmatrix} x - 2u$$

(a) **1 points.** Is the system asymptotically stable?

(b) **3 points.** Is the system BIBO stable? Does BIBO stability imply asymptotic stability for this system? Why or why not?

Problem 2 (9 points)

Controllability test. Consider the following linear systems, where

(i) $A = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$

(ii) $A = \begin{bmatrix} 0 & 1 \\ -10 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix}.$

(iii) $A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$

(a) **3 points.** Assess the controllability using the controllability matrix.

(b) **3 points.** Use the Popov-Belevitch-Hautus tests for controllability assessment.

(c) **3 points.** Compute the controllable canonical form.

Problem 3 (4 points)

The equations of motion of a satellite, linearized around a steadystate solution, are given by $\dot{x} = Ax + Bu$, where x_1 and x_2 denote the perturbations in the radius and the radial velocity, respectively, x_3 and x_4 denote the perturbations in the angle and the angular velocity, and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The input vector consists of a radial thruster u_1 and a tangential thruster u_2.

(a) **(2 points)** Show that the system is controllable from u.

(b) **(2 points)** Can the system still be controlled if the radial thruster fails? What if the tangential thruster fails?

Problem 4 (8 points)

Consider the following the state equation

$$\dot{x} = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 4 \\ 2 \end{bmatrix} u, \quad x(0) = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$$

(a) **(2 points)** Show that there exists an input $u(t)$ that drives $x(0)$ to 0 in finite time.

(b) **(2 points)** Compute (by hand) the particular input $u(t)$ that achieves this in 4 seconds.

(c) **(2 points)** Verify that the input achieves this by plotting the state trajectories $x(t)$ vs. time t with the initial condition $x(0)$ in Matlab. The plot should have two trajectories $x_1(t), x_2(t)$.

(d) **(2 points)** If possible, compute the controllable canonical form (CCF) of the above state equation. If you cannot, why not?