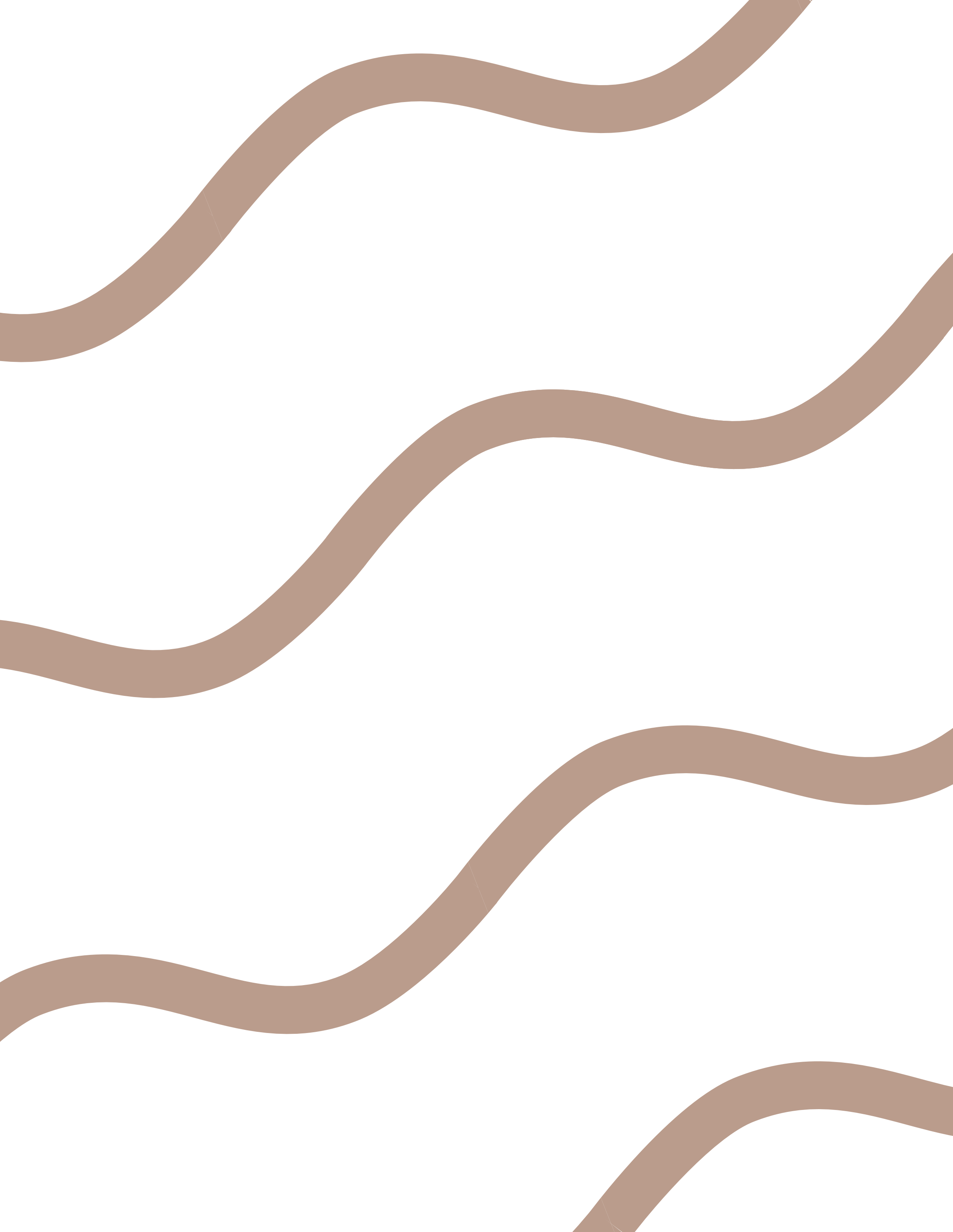

1) (a)

(i)
$$x = \begin{bmatrix} -6 & -3 & -5 \\ 0 & -3 & 1 \\ 2 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} -2/3 & 1/3 \\ 1/3 & -2/3 \\ 1/3 & 1/3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} x$$

$$P = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$P = \begin{bmatrix} -0.67 & 0.33 & 1.33 & -1.67 & -2.67 & 1.33 \\ 0.33 & -0.67 & -0.67 & 2.33 & 1.33 & -2.67 \\ 0.33 & 0.33 & -0.67 & -1.67 & 1.33 & 1.33 \end{bmatrix}$$

rank of $P = 2 < 3$

$\therefore$ Not Controllable

(ii)
$$x = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x$$

$$P = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -3 & -5 & 9 & 25 \\ 1 & -1 & -3 & 5 & 9 & -25 \\ 1 & 0 & -3 & 0 & 9 & 0 \end{bmatrix}$$

rank of $P = 2 < 3$

$\therefore$ Not Controllable

(b)

(i) Let $T = \begin{bmatrix} -0.67 & 0.33 & 1 \\ 0.33 & -0.67 & 0 \\ 0.33 & 0.33 & 0 \end{bmatrix}$

$$\bar{A} = T^{-1}AT$$
$$= \begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 2 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\bar{B} = T^{-1}B$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\bar{C} = CT \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

Kalman controllable Form.

(ii) Let $T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\bar{A} = T^{-1}AT$$

$$= \begin{bmatrix} -3 & 0 & -2 \\ 0 & 5 & -4 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\bar{B} = T^{-1}B$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\bar{C} = CT = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Kalman Controllab Form.

(c) $\bar{A}_{uc}$ for $i \Rightarrow -4$,
i.e it has eigenvalues on the left half plane,
∴ system is stabilized.

similarly for $ii \Rightarrow -1$,
eigenvalus on the left half plane,
∴ System is stabilized

2)

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$x(0) = x_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} x$$

a) If we observe, we can see that B matrix i.e $[0 \; 1]^T$ i.e x_1 cannot be manipulated by u, hence the sysistem is

NOT CONTROLLABLE.

Finding eigen values,

$$\begin{vmatrix} -1-\lambda & 0 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(1+\lambda)(4-\lambda) = 0$$

$$\lambda = 4 \text{ or } -1$$

eigen vectors

$$\begin{bmatrix} -5 & 0 \\ 1 & 0 \end{bmatrix} v_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 5 \end{bmatrix} v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & -5 \\ 1 & 1 \end{bmatrix}$$

$$A^* = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B^* = [1 \quad 0]^T$$

$$C^* = [1 \quad -9]^T$$

From $A^*_{\bar{c}} = -1$ we can say that System is stabilized.

As we can push the left hand side values to any value since the A matrix is uncontrollable subspace is exponentially stable, System is stabilized.

b) eigen vector for $\lambda_2 = -1$,

$(A^* - 2I)v = 0$ i.e

$$\begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

then $w^T B = [0 \; 1]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$

Stabilized by PBH eigenvector.

$$\operatorname{rank}[\lambda_1 I - A^* B] = \operatorname{rank}\begin{pmatrix} -5 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$= 1 < 2$

and $\lambda < 0$.

Stabilized by PBH ranktest.

(c) $u(t) = -B^T e^{At} x_0$.

$B = [0 \quad 1]^T$

$x_0 = [2 \; -1]^T$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 4 \end{bmatrix}$$

upon solving:

$$u(t) = \begin{bmatrix} -1.7357 \\ 0.2949 \end{bmatrix}$$