# Homework 7
# ME5659 Spring 2024

**Due: See Canvas, turn in on Gradescope**

## Problem 1 (7 points)

Consider a linear state-space model

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D} = 0$$

(a) **1 points.** Is it possible to arbitrarily place the closed-loop eigenvalues with state-feedback control $u = -Kx$? Why or why not?

(b) **2 points.** Can one choose the desired closed-loop eigenvalues to be $\lambda_1 = -2, \lambda_2 = -3, \lambda_3 = -4$? If possible, determine the necessary feedback gain matrix $K$ by a hand calculation.

(c) **4 points.** Consider a state-feedback control law $u = -Kx + k_g r$, where $r(t) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is the reference input. Compute the $k_g$ such that the system outputs $y_1, y_2$ will track the given reference input. Use Matlab to verify your answers by plotting two trajectories $y_1$ vs. time $t$ and $y_2$ vs. time $t$.

## Problem 2 (10 points)

Consider a single-input single-output rotational mechanical system described by:

$$J\ddot{\theta} + b\dot{\theta} + k\theta = \tau$$

where the single input is an externally applied torque $\tau(t)$, and the output is the angular displacement $\theta(t)$. $J = 1\,kgm^2$ is the system inertia, $b = 1\,Nms/rad$ is the rotational viscous damping coefficient, and $k = 2\,Nm/rad$ is the torsional spring constant. We assume that a unit step external torque is applied. The initial condition is given by: $\theta(0) = 0\,rad$ , $\dot{\theta}(0) = 0\,rad/s$. (Feel free to use MATLAB)

(a) **4 points.** Evaluate the percent overshoot and the settling time. Plot $\theta(t)$ and $\dot{\theta}(t)$ of the open-loop system (with unit step input) for $t \in [0, 4]\,s$.

(b) **6 points.** Shape the dynamic response so that the percent overshoot is 2% and the setting time $t_s = 1\,s$. The steady-state performance should be the same as the one in the open-loop response. Plot $\theta(t)$ and $\dot{\theta}(t)$ of the closed-loop system (with a state-feedback control law and a unit step input) for $t \in [0, 4]\,s$.

## Problem 3 (8 points)

Consider a system with transfer function

$$G_{op}(s) = \frac{(s-1)(s+2)}{(s+1)(s-2)(s+3)}$$

(a) **4 points.** Is it possible to change the transfer function to

$$G_{cl}(s) = \frac{(s-1)}{(s+2)(s+3)}$$

by state feedback?

(b) **2 points.** Is the resulting system BIBO stable?

(c) **2 points.** Is the resulting system asymptotically stable?