

Homework 9

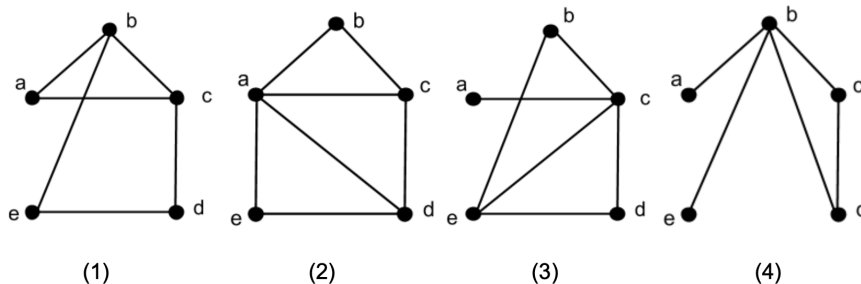
Due **Wednesday, April 22:** on CourseSite, at 1:35pm

CSE 140: Spring 2020

Please show details of your work to allow for possible partial credit.

1. (20 points) Prove or disprove each of the following statements.
 - (a) If an undirected graph G is 3-regular, then G must have an Euler circuit.
 - (b) If an undirected graph G is 4-regular, then G must have an Euler circuit.
 - (c) $K_{4,3}$ has an Euler circuit
 - (d) $K_{2,3}$ has an Euler trail
 - (e) K_7 has an Euler circuit
 - (f) A connected graph with a degree sequence $\{2, 2, 3, 4, 4, 4, 5\}$ has an Euler circuit.
 - (g) A graph with a degree sequence $\{2, 2, 2, 2, 2, 2\}$ has an Euler circuit.

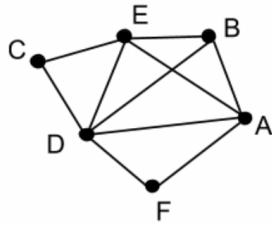
2. (15 points) For each of the following graphs:



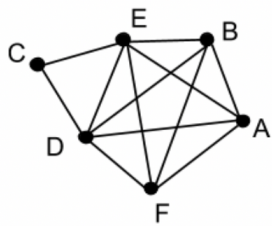
- (a) Either write one Hamiltonian cycle it has, or (informally) justify why there no such Hamiltonian cycle
 - (b) Either write one Hamiltonian path it has, or (informally) justify why there is no such Hamiltonian cycle
3. (10 points) A planar graph G has 7 vertices. One of the vertices has degree 4, two vertices have degree 3, and four vertices have degree 2. Determine the number of regions G has, making sure to justify your response.

4. (15 points) Determine whether each of the following graphs are planar or not. Make sure to justify your response.

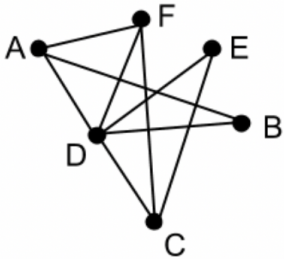
(a)



(b)



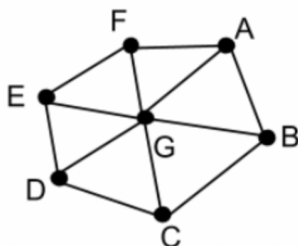
(c)



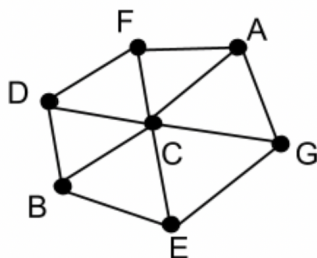
5. (10 points) Prove or disprove the following statements:

- (a) $K_{2,3}$ is nonplanar.
- (b) Every embedding of a planar graph has no edge crossings.
- (c) C_{20} is a planar graph.
- (d) K_5 has a planar embedding

6. (5 points) Determine the chromatic number of the graph shown below.



7. (10 points) The greedy algorithm is used to color the graph shown below.



The vertices are colored in lexicographical order, and the colors are ordered as:

- C_1 - Red
- C_2 - Blue
- C_3 - Green
- C_4 - Yellow
- C_5 - Purple

Determine the color assigned to each vertex.

8. (10 points) Determine whether each of the following statements is true or false. No justification is required.
- (a) The chromatic number of a graph is always less than or equal to the clique number of the graph.
 - (b) The chromatic number of a graph is always greater than the clique number of the graph.
 - (c) If the graph has a vertex of degree d , then the chromatic number must be greater than or equal to d .
 - (d) If every vertex in the graph has d or fewer neighbors, then the chromatic number must be less than or equal to $d+1$.
9. (5 points) Determine the clique number of the graph shown below.

