DYNAMIC NEWSVENDOR MODEL FOR OPTIMISTIC AND PESSIMISTIC POLICY-BASED PROFIT FORECASTING

A PREPRINT

Advait Ramesh Iyer
*aiyer01@syr.edu

May 14, 2020

ABSTRACT

The aim of this study is to propose and implement a dynamic multi-period newsvendor model for a perishable product by incorporating forecasts for profit maximization within optimistic, neutral and pessimistic scenarios learned through partial knowledge of demand. The proposed model is an attempt to simulate the inventory management process under varying levels of risk in order to evaluate risk management strategies.

***K*eywords** Dynamic newsvendor model · Hurwicz criterion · Monte Carlo simulation · Neural network

1 Introduction

1.1 Background

Single-period newsvendor models have been extensively studied since late 1800s, and has been crucial in building the foundation of inventory control methods aimed at profit maximization and cost minimization strategies for business owners. The newsvendor problem belongs to the set of decision theory problems which deal with situations where a decision about the event has to be made before the event is realized. The vendor incurs a cost c per unit of the product, and sells them at a price p per unit. Edgeworth (1888, 1951) proposed the Central Limit Theorem to provide an estimate for the optimal ordering quantity as the $(1 - \frac{c}{p})^{th}$ quantile of the demand distribution. However, due to the risk-neutral [1] nature of the solution and an unrealistic expectation of complete knowledge of the demand distribution, the classical newsvendor model fails to provide any real insights.

1.2 Use cases of the Newsvendor model

The family of newsvendor models, i.e., scenarios where one has to take a decision before the payoff is realized is a common problem across multiple industries. Due to differences in the objective of various organizations, the problem is popularized by a different name, for e.g., the Value-at-Risk (VaR) model in finance. They are all types of stochastic optimization problem. A few cases of stochastic optimization problems from other industries are:

1. **Safety stock level estimation:** Safety stock is maintained in the inventory to avoid a stockout holding cost.
2. **Seasonal goods purchasing:** Retailers sell items which they order once a season. Decision for optimal order size is crucial because a sub-optimal leads to opportunity, or carrying costs.
3. **Product retirement:** When a product line is about to retire or shut down, the manufacturers need to estimate the optimal order size such that the customers' demands are met while no leftover inventory is leftover, as the product is about to be discontinued.
4. **Supply line planning:** The capacity of a factory is set by taking into consideration the safety levels, number of machines, and other operational factors. Slightest errors in estimation of the appropriate order size results in large capital costs.

*MS Business Analytics, Whitman School of Management, Syracuse University

5. **Customer booking:** Any business where there is high fluctuation of demand, for e.g., public transport. An airline's primary objective is to achieve zero empty seats. However, this might lead to overbooking due to last-minute cancellations, and the customers have to be compensated for the inconvenience caused to them.

An optimal solution for any function The demand and supply side constraints and variables in the newsvendor problem share certain aspects of the mathematical representation with other stochastic optimization use cases.

1. **Decision variable (Q):** The decision quantity (order quantity, safety stock level, overbooking level, plant capacity, etc.) Q^* represents the optimal value for Q.
2. **Uncertain demand (D):** A random variable which follows certain prior probability distribution. Demand may be discrete or continuous in nature. This paper focuses on discrete system states, i.e., a discrete random demand at each instance in time-period $T = T_1, T_2, T_3, .., T_N$, as this is the most common industrial setting.
3. **Unit overage cost (h):** The cost of buying one unit more than the demand during a single-period $h = c - s$, where c is the unit cost and s is the salvage value (cost required to dispose off an unsold product).
4. **Unit underage cost (b):** The cost of buying one unit less than the demand during a single-period. $b = p - c$, where p is the unit price and c is the unit cost.

Let D denote the single-period random demand, with mean $\mu = E[D]$ and variance $\sigma^2 = V[D]$. When Q units are ordered, $min(Q, D)$ units are sold, and $(Q - D)^+ = max(Q - D, 0)$ units are salvaged. The profit and the expected profit are well defined, and can be given as:

$$\pi(Q) = pEmin(Q, D) + sE(Q - D)^+ - cQ \tag{1}$$

Using $min(Q, D) = D - (D - Q)^+$, the expected profit can be re-iterated as $\pi(Q) = (p - c)\mu - G(Q)$ where $G(Q) = (c - s)E(Q - D)^+ + (p - c)E(D - Q)^+ \geq 0$. Using (1), the problem now can be viewed as maximizing $\pi(Q)$ rather than minimizing expected overage and underage cost $G(Q)$.

The newsvendor problem is not worth further investigation for the deterministic demand case. Let $G^{det}(Q) = h(\mu - Q)^+ + b(Q - \mu)^+$ representing the cost when demand D is deterministic. The problem also becomes trivial when $s = c$, as for this case, an infinitely large order could be placed, all demands could be met, and the remaining items could be returned for no extra cost.

To find the optimal order size, the objective function is to be set in a way where the marginal costs of loss and gain are equal. The expected loss $Pr(D \geq Q)b$ needs to be equated with the expected gain $Pr(D < Q)h$, which gives:

$$\begin{aligned}
Pr(D \geq Q)b &= Pr(D < Q)h \\
(1 - F(Q))b &= F(Q)h \\
\therefore b - F(Q)b &= F(Q)h \\
\therefore b &= F(Q)(b + h) \\
F(Q) \equiv Pr(D \leq Q) &= \frac{b}{b + h} = \frac{p - c}{p - s} \equiv \beta
\end{aligned} \tag{2}$$

This result shows that the optimal order quantity for the newsvendor model is always at the critical fractile (or critical ratio).

The scope of this study is limited to uncertainty in demand, and supply side constraints are constant. It is expensive and difficult to setup an infrastructure that helps the organization capture systematic errors in the market, nevertheless the model in the study does not require any additional information to provide the stochastically optimal solution.

1.3 Decision making under demand uncertainty

The Hurwicz Criterion [3] establishes a control parameter for a decision maker's risk management approach under uncertainty (Hurwicz 1951, Raiffa and Luce 1989). For a level of optimism $\lambda \in [0, 1]$, the Hurwicz criterion proposes a trade-off between pessimism and optimism as:

$$\max_{x \geq 0} \left\{ \lambda \cdot \max_{\mathbb{P} \in \zeta} \mathbb{E}_{\mathbb{P}}[\Pi(x, \tilde{u}, \tilde{v})] + (1 - \lambda) \cdot \min_{\mathbb{P} \in \zeta} \mathbb{E}_{\mathbb{P}}[\Pi(x, \tilde{u}, \tilde{v})] \right\} \tag{3}$$

If $\tau = 1$, the criterion would take the most optimistic approach, which is calculating the maximum of all maximums from a sample.

$$\max_{x \geq 0} \max_{\mathbb{P} \in \zeta} \mathbb{E}_{\mathbb{P}}[\Pi(x, \tilde{u}, \tilde{v})] \tag{4}$$

Similarly, if $\tau = 0$, the most pessimistic approach which is the maximum of all minimums from a sample.

$$\max_{x \geq 0} \min_{\mathbb{P} \in \zeta} \mathbb{E}_{\mathbb{P}}[\Pi(x, \tilde{u}, \tilde{v})] \tag{5}$$

The criterion is well known for its strong empirical and economic impact in decision theory (Schmeidler and Gilbos 1989, Ellsberg 2011). Through the usage of the criterion, a risk-neutral problem can be extended to study the near-future tendency of the decision maker in taking a pessimistic or optimistic decision policy [4]. The study uses this criterion to vary the risk management tendency for optimal profit forecasting.

2 Method

2.1 Choice of demand distribution

2.1 Poisson distributed demand:

Poisson distributed demand is used widely across management science community due to strong similarities between demand patterns, and how customers who walk into a store make purchases. If D is Poisson distributed demand with parameter $\lambda > 0$ if $P(D = k) = \exp(-\lambda)\frac{\lambda^k}{k!}$ for $k = 0, 1, 2, ...$, where sample mean $\mu = \lambda$ and standard deviation $\sigma = \lambda$. Poisson distribution is the limit condition for binomial distribution with large n and small p, where $\lambda = np$, and can be approximated as a normal distribution when λ is large [2].

2.1 Weibull distributed demand:

Previous studies conducted on the newsvendor problem with censored demand, and Bisi et al. (2009) have proved that family of Weibull distributions are the only members studied in the newsvendor problem whose solutions are highly scalable [6]. Although this study did not investigate further into the scalability factor, the Weibull distribution was chosen to compare the exploration-exploitation factors in neural networks' learning ability, trained on Poisson and Weibull distributions respectively.

2.2 Discrete Time Markov Chain (DTMC)

Decision making in inventory management requires the optimal satisfaction of competing objectives: reduction of storage and ordering costs, and avoidance of stock-outs. Therefore, the objective of the inventory manager is to identify a method through which the current environment characteristics are learned and the anticipation of the future is estimated. Since the demand in real-life is never stationary, the lack of market information is covered for as a regime-switching mechanism [7] through a Markov-modulated demand model. What that means is, we assume that the uncertain demand encodes within itself information regarding various market metrics which are unknown to us at the time being.

An S-valued stochastic sequence $X = (X_n : n \geq 0)$ has a markov property if for each $n \geq 0$ and subset $A \subseteq S$,

$$P(X_{n+1} \in A | X_0 = x_0, ..., X_{n-1} = x_{n-1}, X_n = x) = P(X_{n+1} \in A | X_n = x) \tag{6}$$

for $(x_0, x_1, ..., x_{n-1}, x) \in S^{n+1}$. The one-step transition probability associated with time $n + 1$ is denoted by $P(n + 1, x, A)$. For cases where S is discrete in nature, the one-step transition can be represented in the form of a square matrix $P(n + 1) = (P(n + 1, x, y) : x, y \in S)$, where $P(n + 1, x, y) = P(X_{n+1} = y | X_n = x)$.

Let us denote the cashflow during the demand satisfaction as a, old inventory discard as b, and inventory replenishment as c. The discount factor $\beta \in [0, 1]$ is used to specify the amount of importance given to a future outcome in comparison to the current one. For a higher value of β, a higher level of importance is given to the future outcomes. The Markov property follows:

$$v(a) = P(X_T = a + b + c | X_0 = a)$$

$$= \sum_{i=1}^{a+b+c} \beta^t_{t \in T} P(X_T = a + b + c, X_1 = i | X_0 = a)$$

$$= \sum_{i=1}^{a+b+c} \beta^t_{t \in T} P(X_T = a + b + c | X_1 = i, X_0 = a) P(X_1 = i | X_0 = a)$$

$$= \sum_{i=1}^{a+b+c} \beta_{t \in T}^{t} P(X_T = a+b+c | X_1 = i) P(X_1 = i | X_0 = a) \tag{7}$$

Through equation (7), it can be inferred that the conditional probability of current state can be computed by chaining conditional probabilities of past states. A discrete-time Markov chain is used to calculate the probability of discrete demand states at time periods $T = 0, 1, 2, ..., T_N$ for N such periods. An optimal decision policy can be identified when large enough samples are collected, or simulated using a repeated sampling strategy.

2.3 Monte Carlo Simulation using Sample Average Approximation (SAA) and the Hurwicz Criterion

Majority of industry professionals who work with the newsvendor model on a regular basis are concerned with the external market factors which are correlated with demand. As trends and seasonality play a major role in the encoding necessary latent information about the market, it strongly governs the decisions for optimal marketing strategy of the corporation, therefore adopting a non-parametric method might hinder other aspects of the business. For these reasons, a Markov Chain Monte Carlo (MCMC) approach was adopted, owing to MCMC's *Bayesian* approach of using posterior estimated demand distribution to make predictions.

Sample Average Approximation (SAA): There are multiple methods for sampling in stochastic programming (A. Shapiro 2009). Assuming that $x \neq \phi$, $\mathbb{E}[g(.,\xi)]$ is lower continuous, and $\mathbb{E} \sup_{x \in \mathbb{X}} g^2(x, \xi) < \infty$, a general representation of a family of $\{g_N(.)\}$ random approximations of the function $g(.)$ can be given as:

$$g_N(x) = \frac{1}{N} \sum_{i=1}^{N} G(x, \xi^i), \tag{8}$$

where $\{\xi^1, \xi^2, ..., \xi^N\}$ is a sample from the distribution of ξ. When $\xi^1, ..., \xi^N$ are i.i.d., $g_N(x)$ is a standard *Monte Carlo* estimator of $g(x)$. We can minimize this objective function as follows:

$$\min_{x \in \mathbb{X}} g_N(x) \tag{9}$$

For $x \in \mathbb{X}$, the quantity $g_N(x)$ is a random variable which depends on the sample $\{\xi^1, ..., \xi^N\}$. for a particular realization $\{\hat{\xi^1}, ..., \hat{\xi^N}\}$, we define:

$$\hat{g_N}(x, \hat{\xi^1}, ..., \hat{\xi^N}) = \frac{1}{N} \sum_{i=1}^{N} G(x, \hat{\xi^i}) \tag{10}$$

The Monte Carlo method uses the operator $\hat{g_N}$ to perform repeated sampling, as the objective function tries to minimize the error between the fitted function and actual demand. The paper proposes an approach for estimating optimal profit policy in neutral and optimistic [2] environments. In the compounded version of the sampling algorithm, the output of the algorithm is given as the mean of Monte Carlo and Hurwicz criterions' respective outputs at any given level of optimism.

3 Experiment Results

The supply-side constants in the experiment are: Selling Price : \$ 3, Variable Cost: \$ 0.8, Monthly Backlog: (\$ 0.5), Holding Cost: \$ 0.3, Inventory Level: 100, Lifetime: 7 Days.

Neural network architecture: Number of inputs : 6 [3], number of hidden layers : 1, number of hidden nodes : 32, Activation function: Rectified Linear Unit (ReLU), Optimizer: Adam [8]

[2] As optimism $\to 0$, pessimism $\to 1$

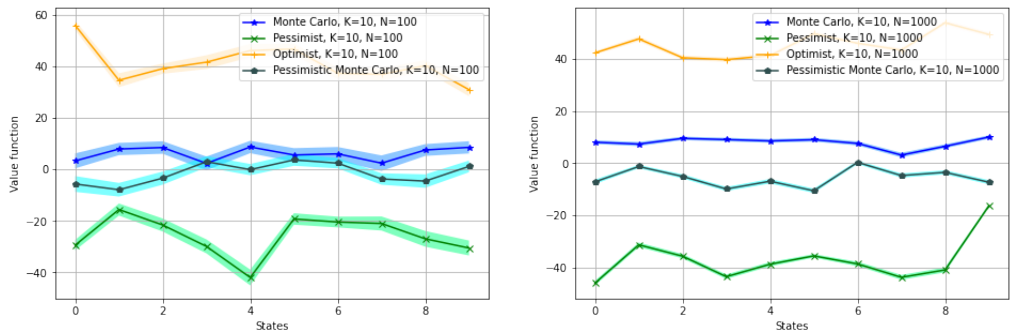
[3] Each input was chosen out of a pool of samples and standard deviation

3.1 Poisson distributed demand

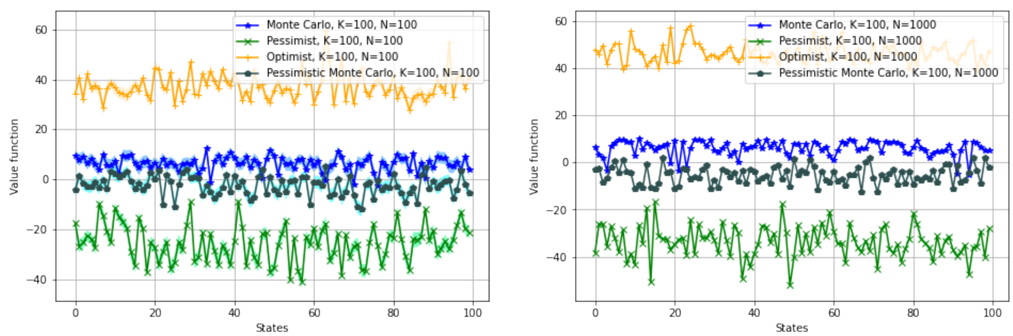
Given value of λ : 15

3.1 Approach: **Pessimistic**, Importance to future states: **Low**

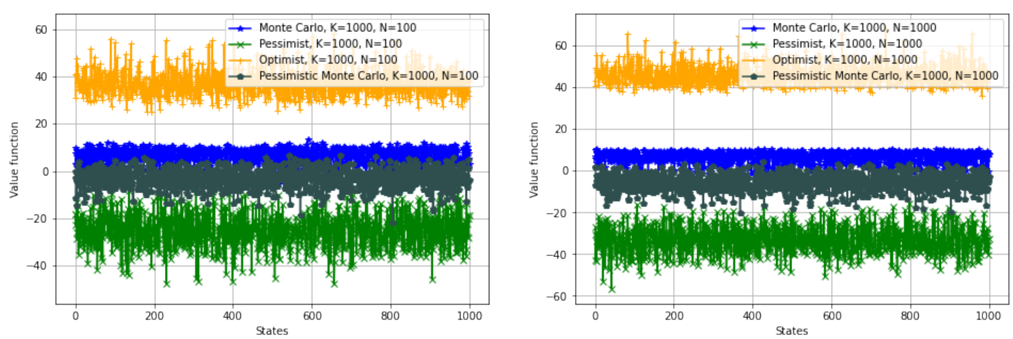
For k [4]=*10*



For *k=100*



For *k=1000*



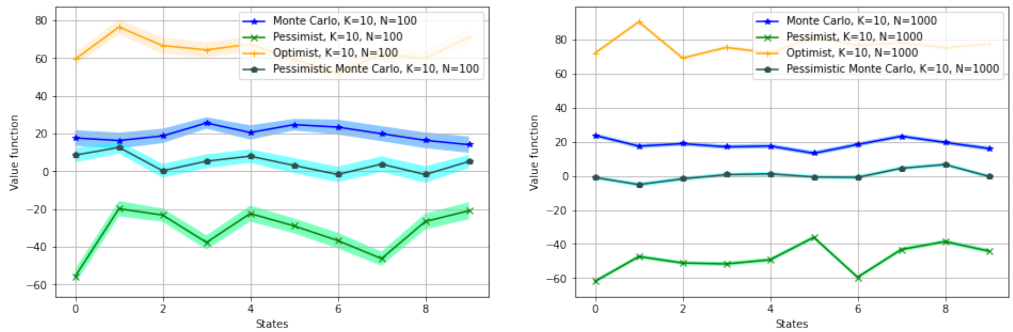
Observations:

1. Monte Carlo simulation and Pessimistic Monte Carlo (near sighted) approach seem to converge around value function = 0.
2. The confidence intervals are more clearly visible for higher values of k.

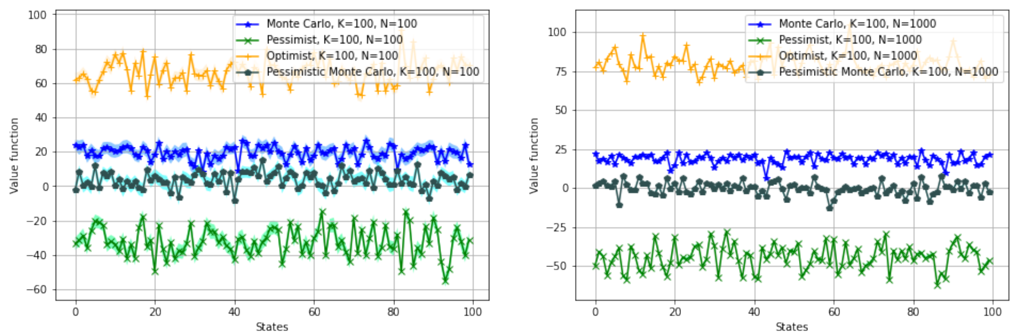
[4]k is the sample size

3.1 Approach: **Pessimistic**, Importance to future states: **High**

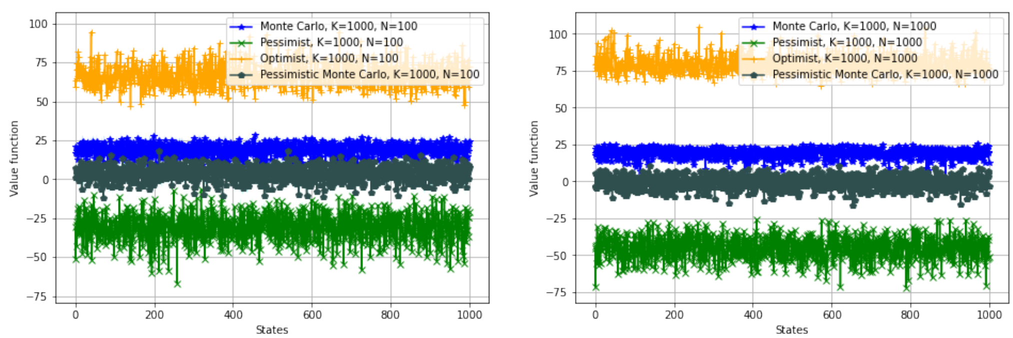
For *k=10*



For *k=100*

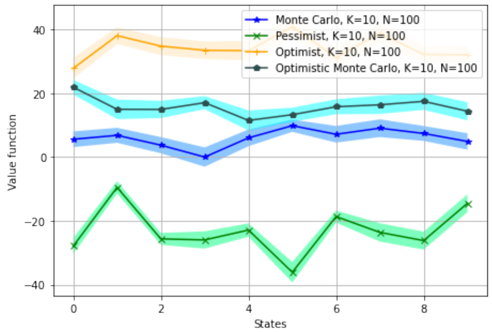


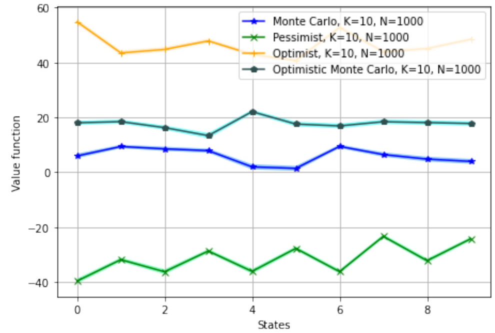
For *k=1000*



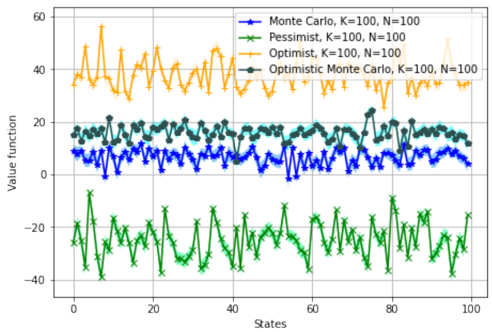
3.1 Approach: **Optimistic**, Importance to future states: **Low**

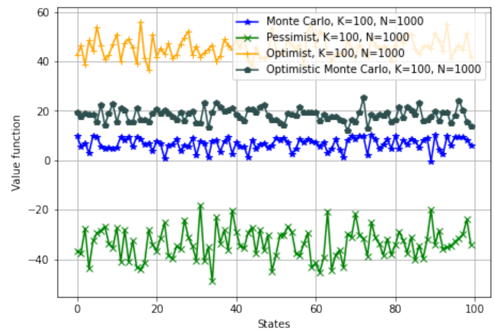
For k=*10*



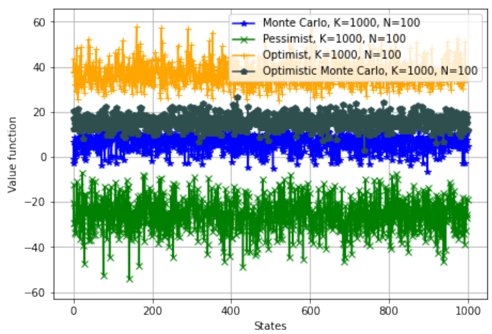


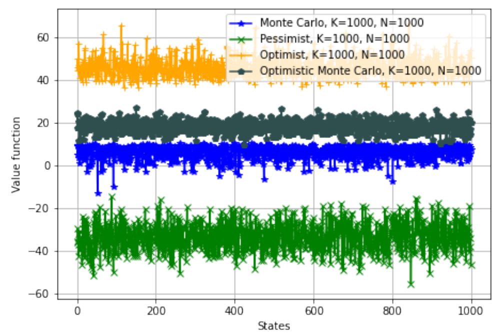
For k=*100*





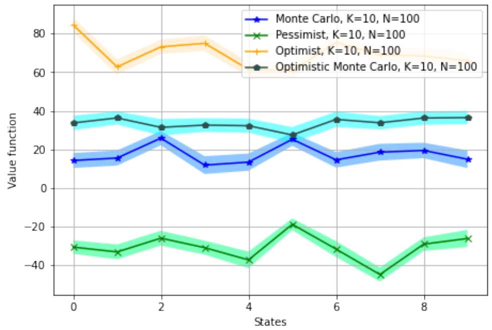
For k=*1000*

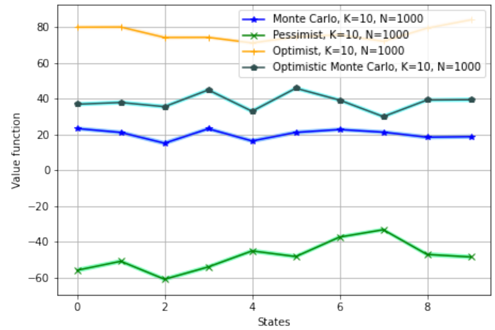




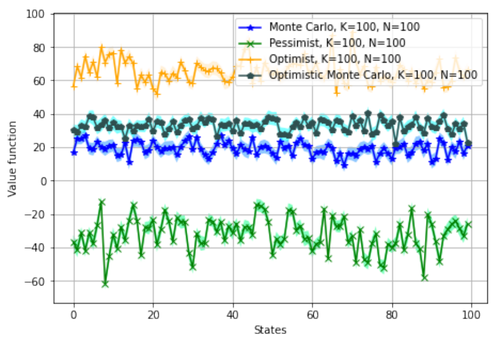
3.1 Approach: **Optimistic**, Importance to future states: **High**

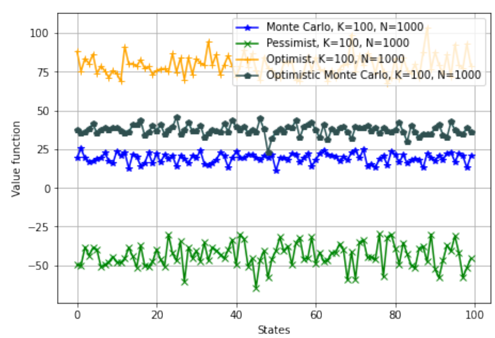
For *k=10*



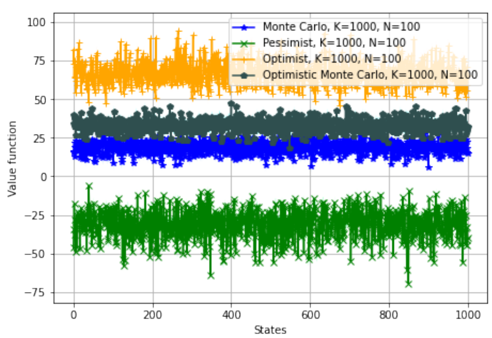


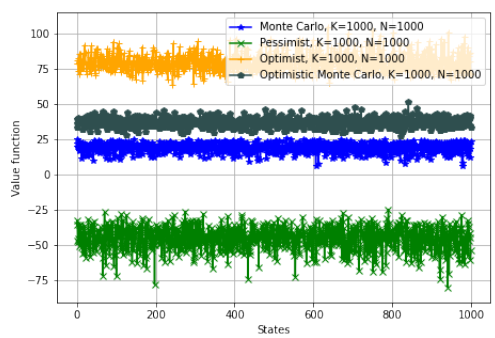
For *k=100*





For *k=1000*



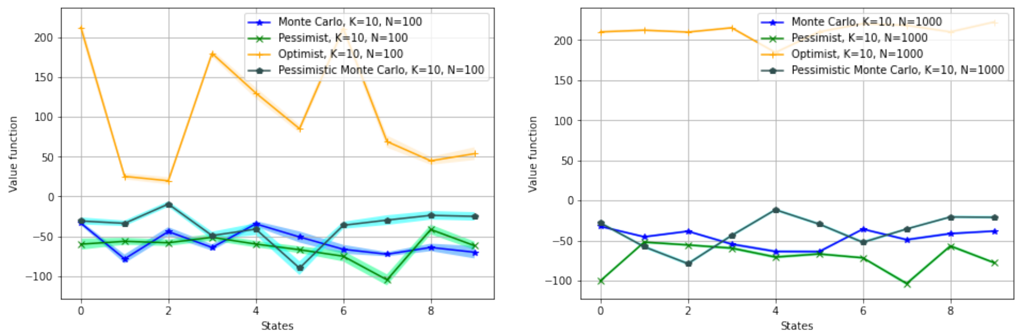


3.2 Weibull distributed demand

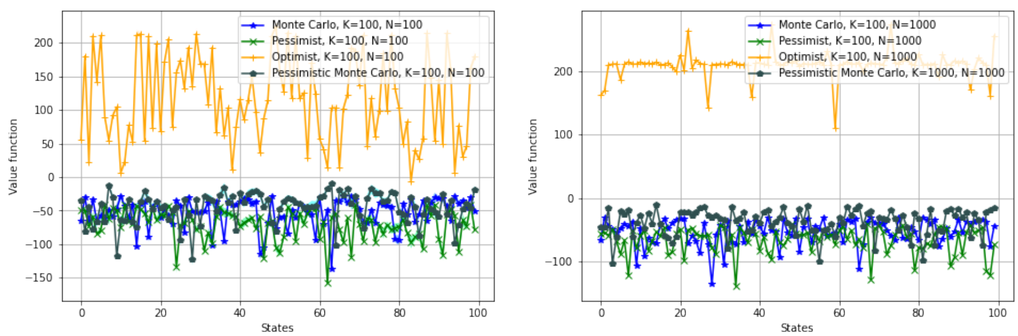
Given value of failure rate over time (β) : 0.4

3.2 Approach: **Pessimistic**, Importance to future states: **Low**

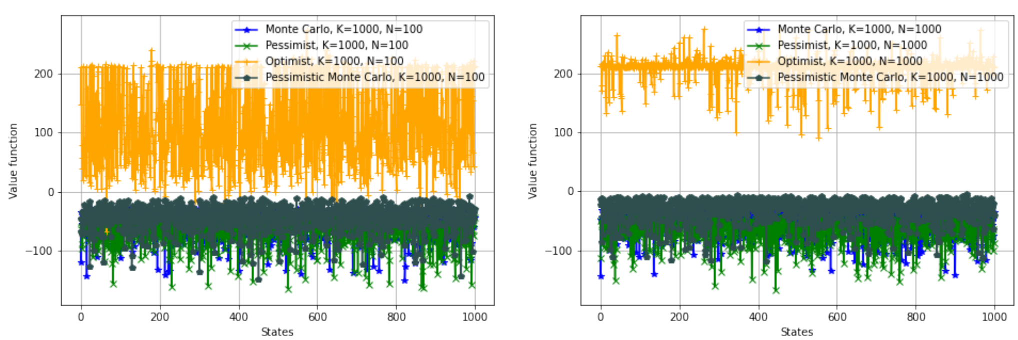
For *k*[5]=*10*



For *k=100*



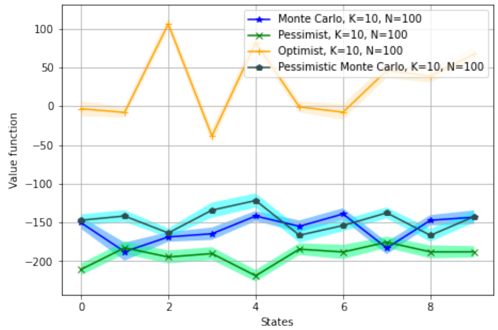
For *k=1000*

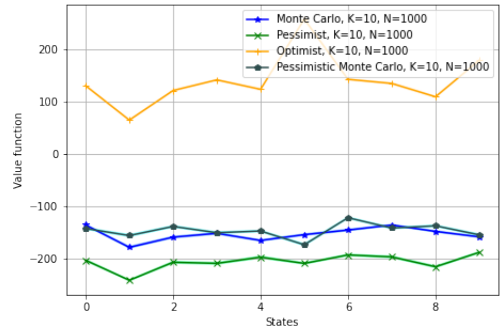


[5]k is the sample size

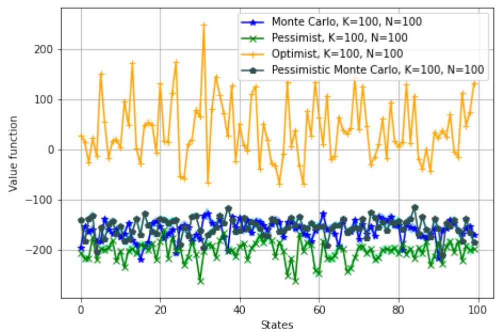
3.2 Approach: **Pessimistic**, Importance to future states: **High**

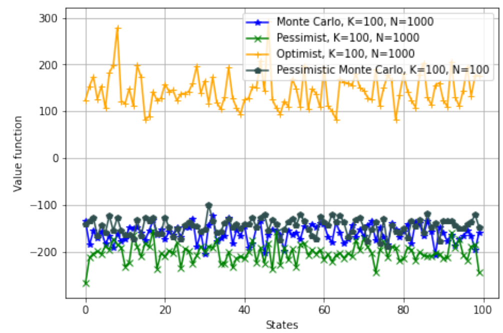
For *k=10*



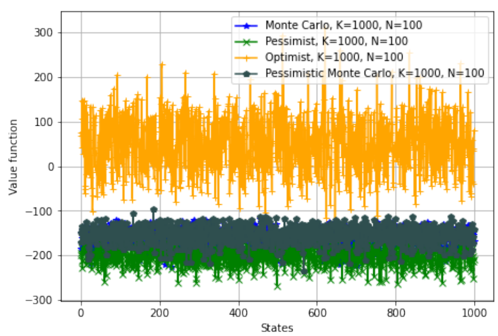


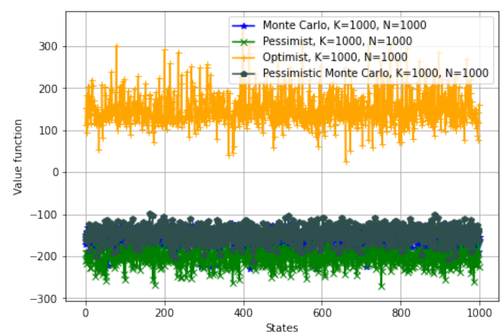
For *k=100*





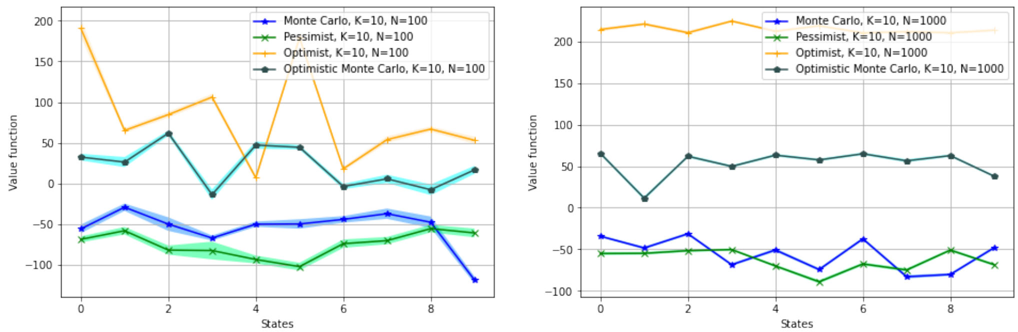
For *k=1000*



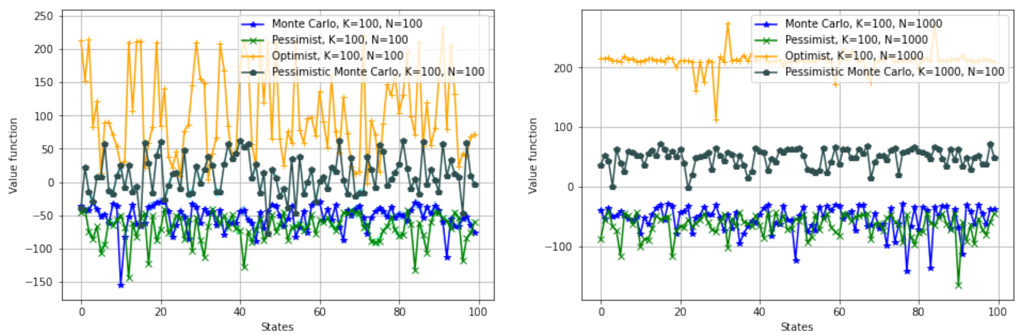


3.2 Approach: **Optimistic**, Importance to future states: **Low**

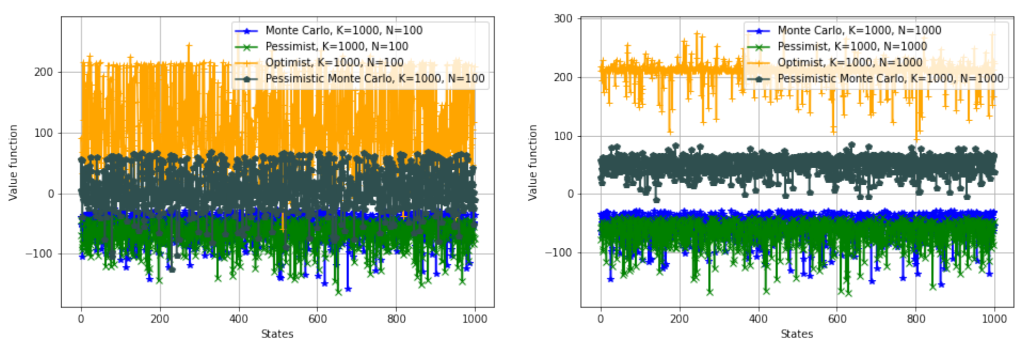
For *k=10*



For *k=100*

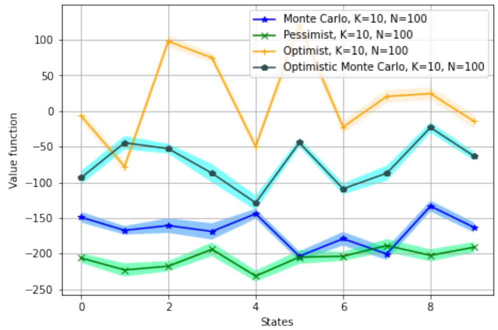


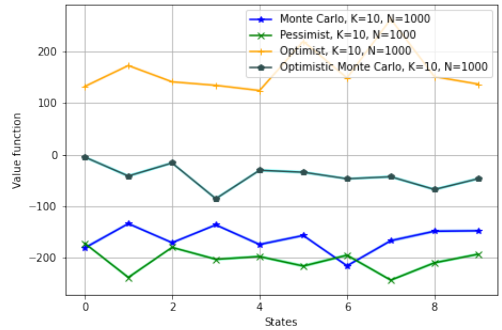
For *k=1000*



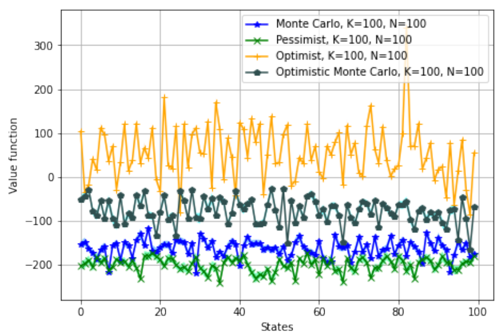
3.2 Approach: **Optimistic**, Importance to future states: **High**

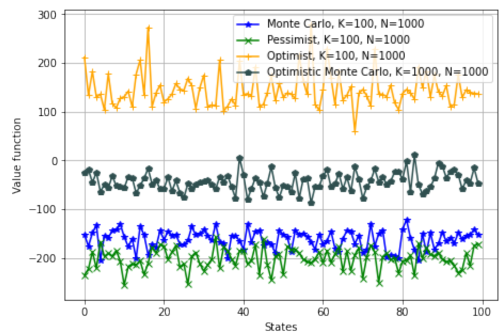
For *k=10*



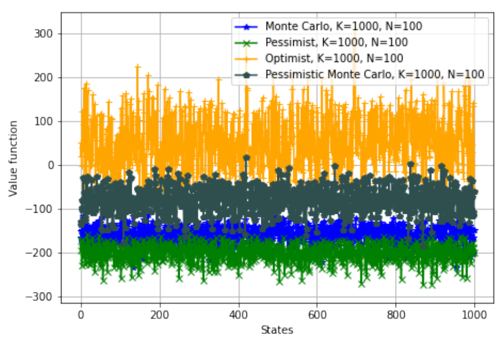


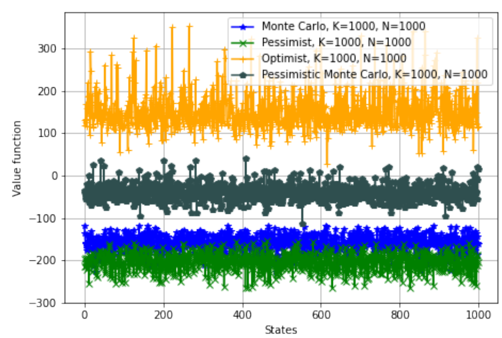
For *k=100*





For *k=1000*



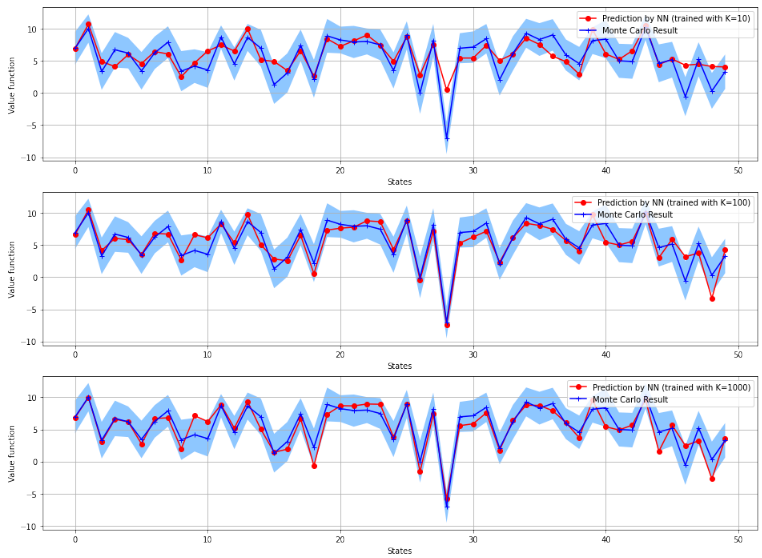


Through this exercise, it was shown that the new Hurwicz Monte Carlo hybrid has better command on the exploration-exploitation equilibrium than Monte Carlo, however further investigation needs to take place in this area. Perhaps more complex algorithms can be developed with more elaborate resampling or stratified sampling schemes.

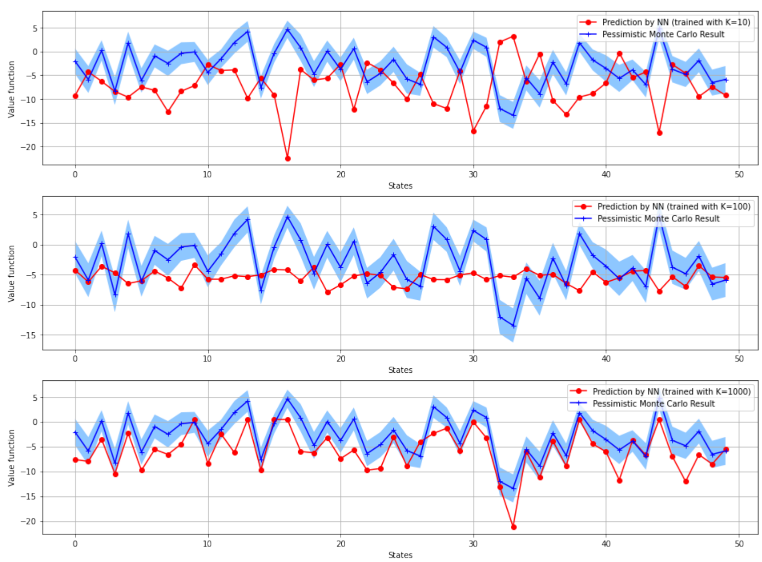
3 **Predictive performance of Neural network**

3.1 Distribution: Poisson, Approach: **Pessimistic**, Importance to future states: **Low**

For *Monte Carlo*

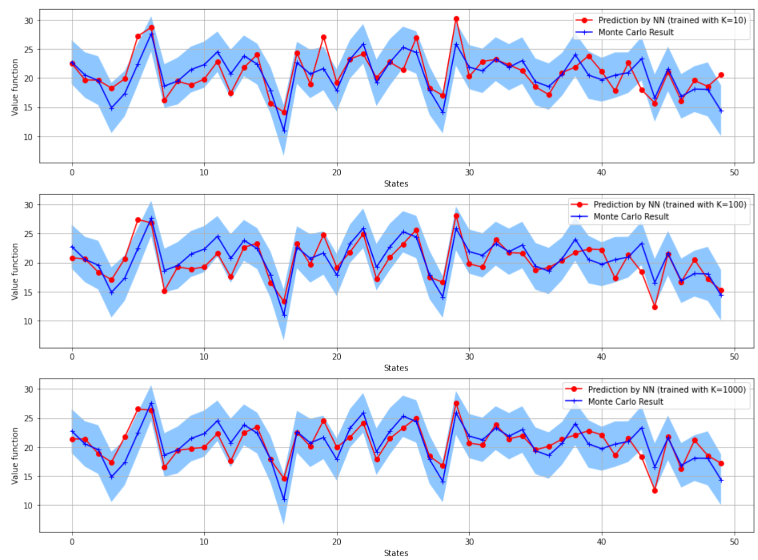


For *Pessimistic Monte Carlo*

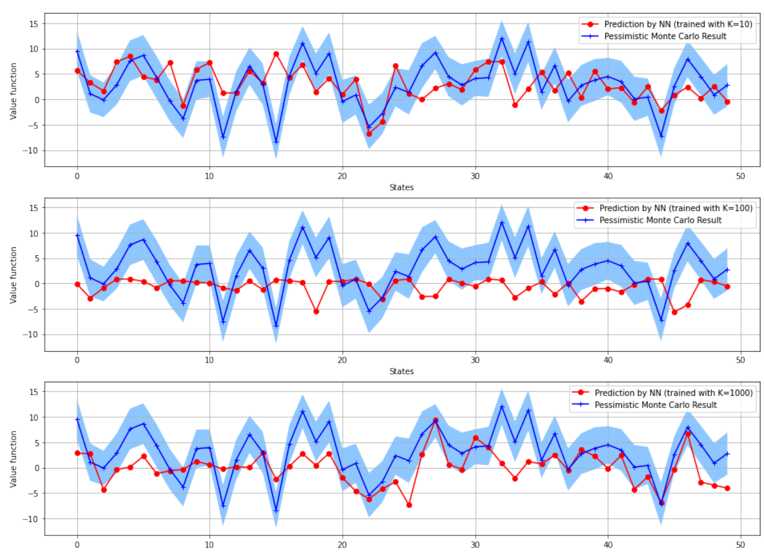


3.1 Distribution: Poisson, Approach: **Pessimistic**, Importance to future states: **High**

For *Monte Carlo*

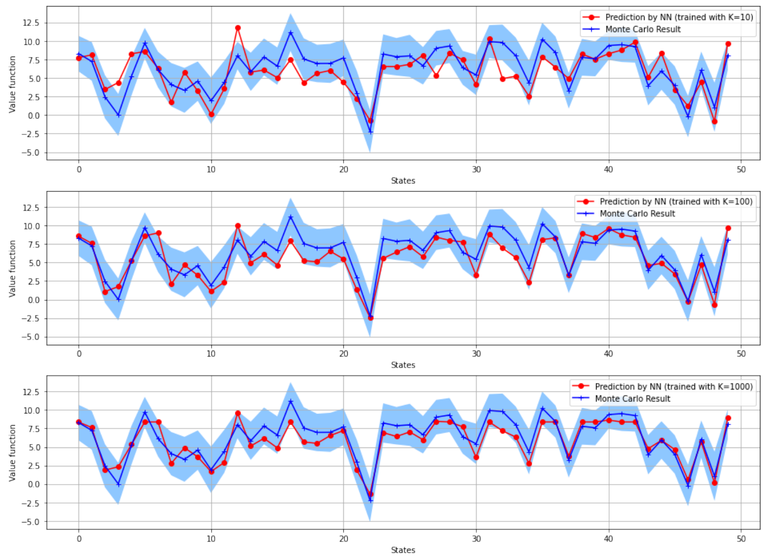


For *Pessimistic Monte Carlo*

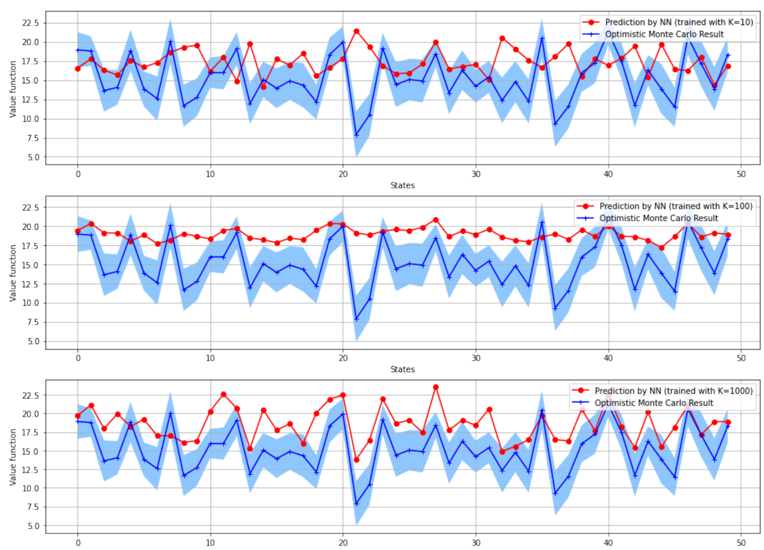


3.1 Distribution: Poisson, Approach: **Optimistic**, Importance to future states: **Low**

For *Monte Carlo*

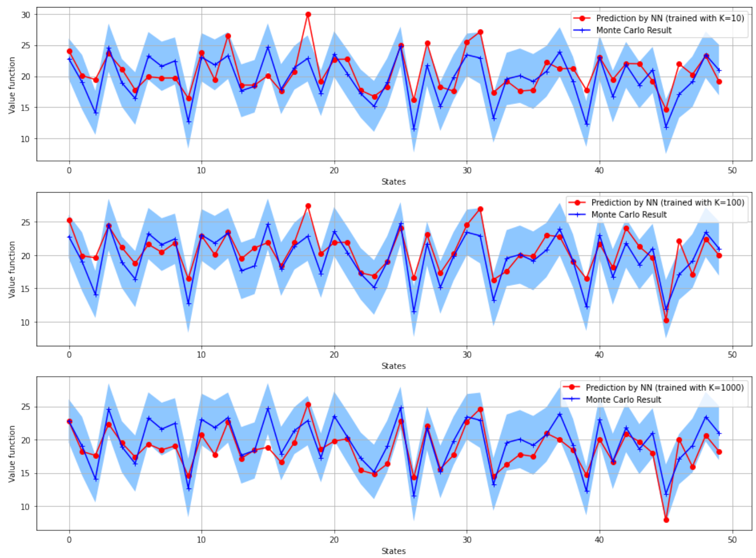


For *Optimistic Monte Carlo*

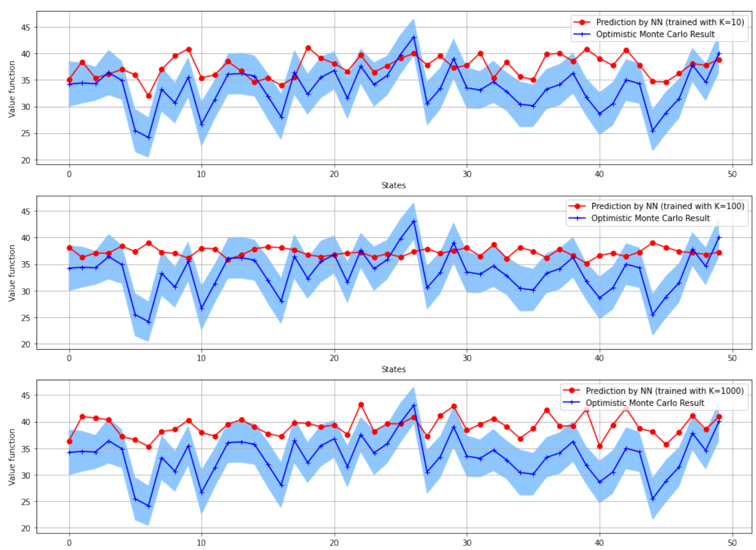


3.1 Distribution: Poisson, Approach: **Optimistic**, Importance to future states: **High**

For *Monte Carlo*

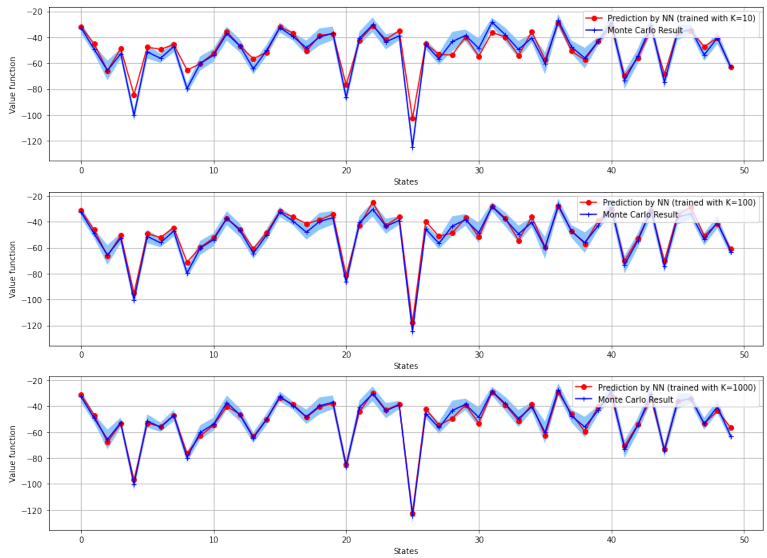


For *Optimistic Monte Carlo*

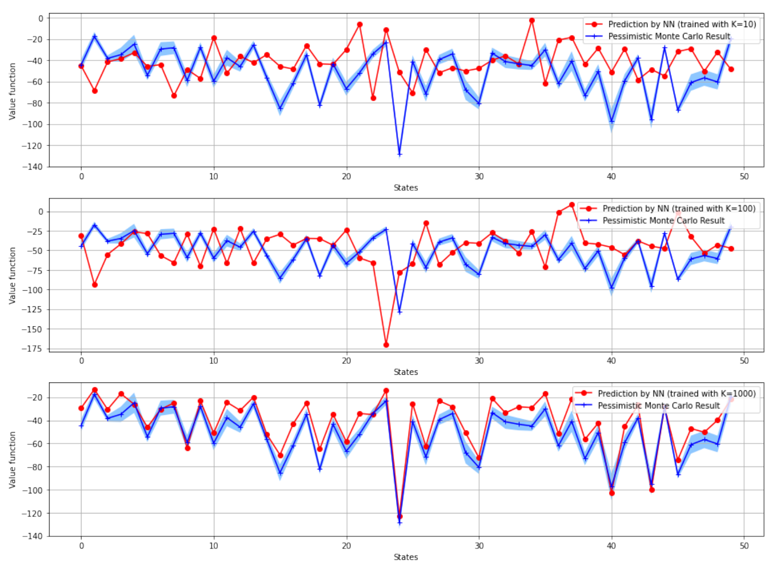


3.2 Distribution: Weibull, Approach: **Pessimistic**, Importance to future states: **Low**

For *Monte Carlo*

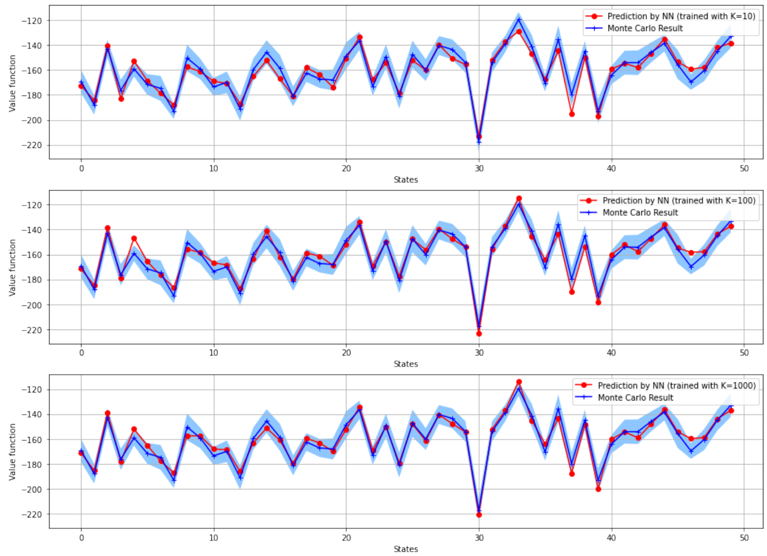


For *Pessimistic Monte Carlo*

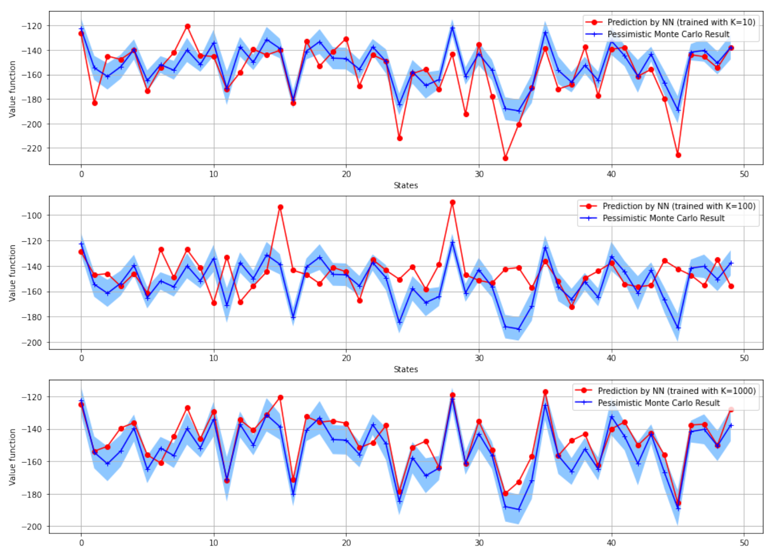


3.2 Distribution: Weibull, Approach: **Pessimistic**, Importance to future states: **High**

For *Monte Carlo*

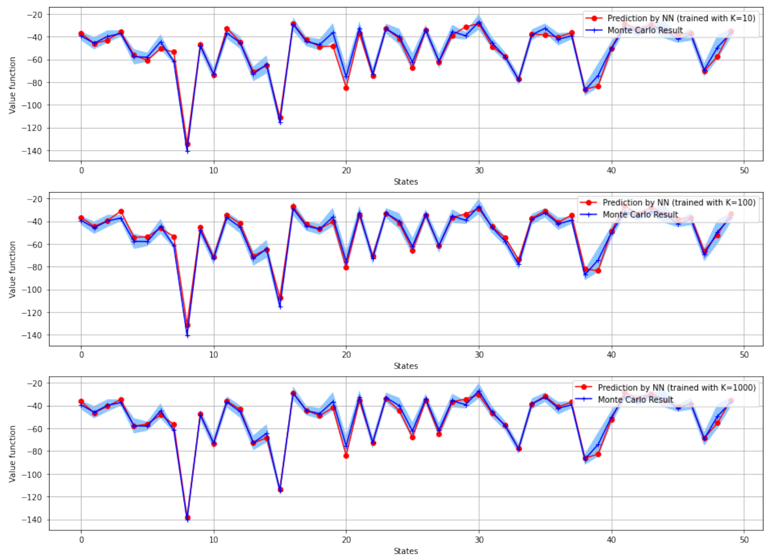


For *Pessimistic Monte Carlo*

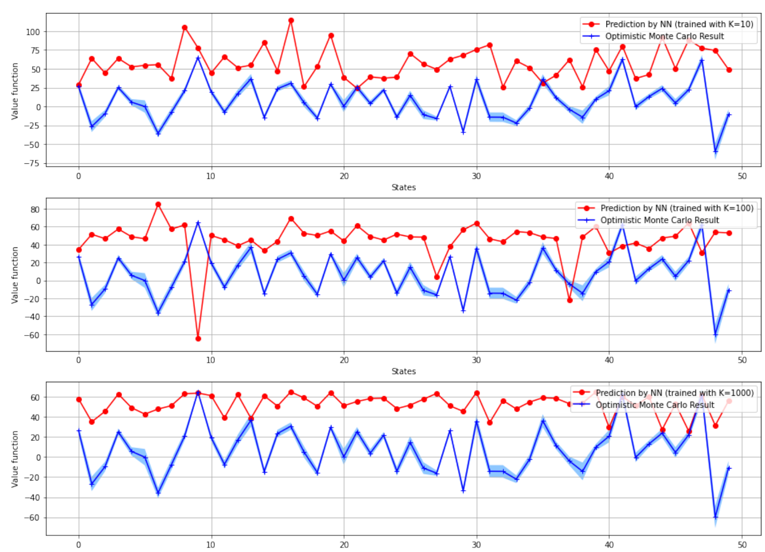


3.2 Distribution: Weibull, Approach: **Optimistic**, Importance to future states: **Low**

For *Monte Carlo*

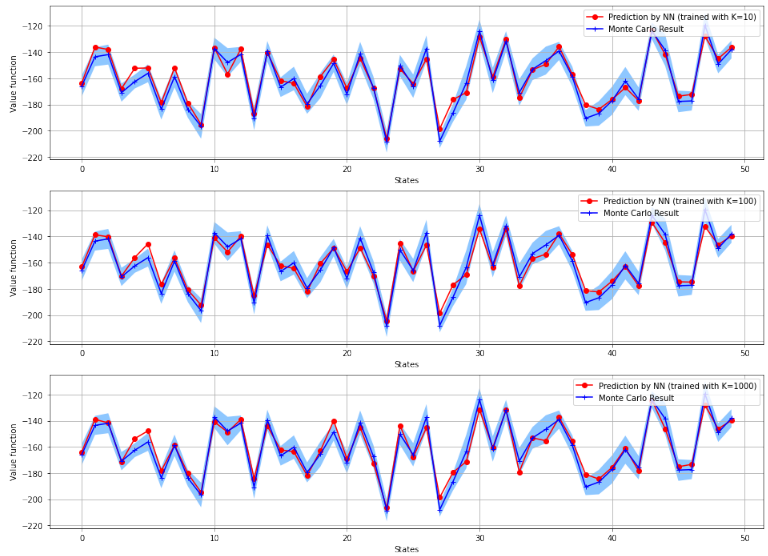


For *Optimistic Monte Carlo*

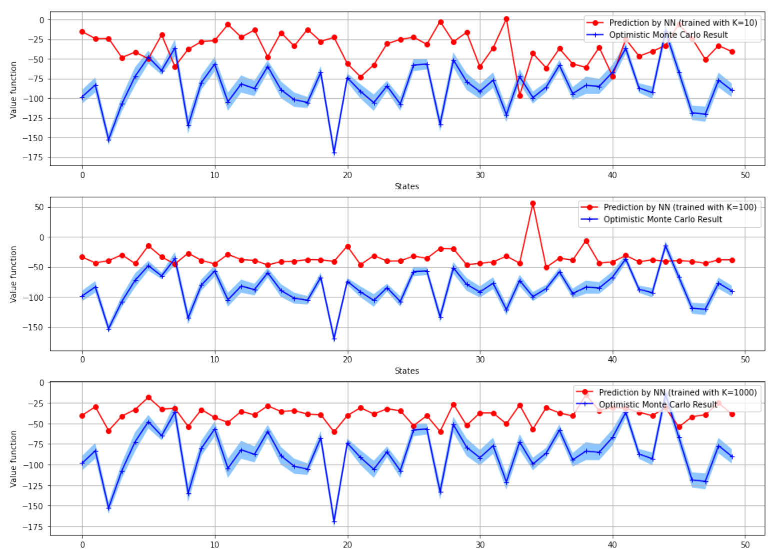


3.2 Distribution: Weibull, Approach: **Optimistic**, Importance to future states: **High**

For *Monte Carlo*



For *Optimistic Monte Carlo*



Neural networks seem to recognize stationary solutions after being trained at various sample sizes even when trained over optimistic or pessimistic data-points. Since the Monte Carlo simulation is a simulation of scenarios and not real scenarios themselves, predicting whether the neural network-based regression model is actually performing better than the variants of Monte Carlo is difficult to say. Wasserstein space analysis [9] can be conducted to learn how much the lowest cost is to turn one function into other by transp

3.3 Conclusion

The dynamic inventory management system provides the decision maker with flexibility to choose higher or lower risk solutions than what traditional Monte Carlo methods suggest, which is risk-neutral.

References

[1] Charles X. Wang, and Scott Webster. The loss-averse newsvendor problem. In *Omega 37, The International Journal of Management Science 2009*, pages 94–96. Elseiver, 2006.

[2] Arthur V. Hill. The newsvendor problem. In *University of Minnesota 2017*, pages 3-5. Elsevier, 2006.

[3] Lenoid Hurwicz. The generalized Bayes-minimax principle: a criterion for decision making under uncertainty. In *Yale University*, Cowles Commission Discussion Paper, Statistics No. 355.

[4] Karmen Pažek, and Č. Rozman. Decision making under conditions of uncertainty in agriculture: a case study of oil crops. *ISSN 1330-7142 UDK = 631.368*, 2009.

[5] Guillermo Gallego. The newsvendor problem. *IEOR 4000: Production Management*, 1995.

[6] Arnab Bisi, Maqbool Dada, and Surya Tokdar. The censored newsvendor problem with newsvendor demand distributions. *Department of Statistical Science, Duke University*, pages 6-8.

[7] Erhan Bayraktar, and Michael Ludkovski. Inventory management with partially observed nonstationary demand. *NSF, DMS-0604491*, pages 6-8.

[8] Diederik P. Kingma, and Jimmy Lei Ba. Adam: a method for stochastic optimization. *arXiv:1412.6980v9*, ICLR, 2015.

[9] Peyman Mohajerin Esfahani, and Daniel Kuhn. Data-Driven Distributionally Robust Optimization Using the Wasserstein Metric: Performance Guarantees and Tractable Reformulations. *arXiv:1505.05116v3*, 2017.