3] $E[(Y-\hat{Y})^2] = E[(f(X)+\varepsilon-\hat{f}(X))^2] = E[((f(X)-\hat{f}(X))+\varepsilon)^2]$

$= E[(f(X)-\hat{f}(x))^2] + E[\varepsilon^2] + E[2(f(X)-\hat{f}(X))\varepsilon]$

$= E[(f(X)-\hat{f}(X))^2] + E[\varepsilon^2]$

So, $E[(\hat{Y}-Y)^2] = E[(\hat{f}(X)-f(X))^2] + E[\varepsilon^2] = E[(\hat{f}(x)-f(x))^2] + 0$

Adding and subtracting $E[\hat{f}(x)]$, we get:-

$E[(\hat{f}(X)-f(X))^2] = E[(\hat{f}(X)-f(X)+E[\hat{f}(X)]-E[\hat{f}(X)])^2]$

$= E[((\hat{f}(X)-E[\hat{f}(X)]) + (E[\hat{f}(X)]-f(X)))^2]$

$= E[(\hat{f}(X)-E[\hat{f}(X)])^2 + (E[\hat{f}(X)]-f(X))^2 + 2(\hat{f}(X)-E[\hat{f}(X)]).(E[\hat{f}(X)]-f(X))]$

$= E[(\hat{f}(X)-E[\hat{f}(X)])^2 + (E[\hat{f}(X)]-f(X))^2 + 2\{\hat{f}(X).(E[\hat{f}(X)]-f(X)) - E[\hat{f}(X)].(E[\hat{f}(x)]-f(x))\}]$

$= E[(\hat{f}(x)-E[\hat{f}(x)])^2 + (E[\hat{f}(x)]-f(x))^2 + 2\{\hat{f}(x).E[\hat{f}(x)] - \hat{f}(x).f(x) - (E[\hat{f}(x)])^2 + E[\hat{f}(x)].f(x)\}$

We know: $E[X+Y] = E[X] + E[Y]$ so,

$= E[(\hat{f}(X)-E[\hat{f}(X)])^2] + E[(E[\hat{f}(X)]-f(X))^2]$

$+ 2E[\hat{f}(X).E[\hat{f}(x)] - \hat{f}(x).f(x) - E[\hat{f}(x)].E[\hat{f}(x)] + E[\hat{f}(X)].f(X)]$