

$$\begin{aligned}
 3] \quad E[(Y - \hat{Y})^2] &= E[(f(x) + \varepsilon - \hat{f}(x))^2] = E[(f(x) - \hat{f}(x) + \varepsilon)^2] \\
 &= E[(f(x) - \hat{f}(x))^2] + E[\varepsilon^2] + E[2(f(x) - \hat{f}(x))\varepsilon] \\
 &= E[(f(x) - \hat{f}(x))^2] + E[\varepsilon^2]
 \end{aligned}$$

$$\text{So, } E[(\hat{Y} - Y)^2] = E[(\hat{f}(x) - f(x))^2] + E[\varepsilon^2] = E[(\hat{f}(x) - f(x))^2] + 0$$

Adding and subtracting  $E[\hat{f}(x)]$ , we get:-

$$\begin{aligned}
 E[(\hat{f}(x) - f(x))^2] &= E[(\hat{f}(x) - f(x) + E[\hat{f}(x)] - E[\hat{f}(x)])^2] \\
 &= E\left[\left((\hat{f}(x) - E[\hat{f}(x)]) + (E[\hat{f}(x)] - f(x))\right)^2\right] \\
 &= E\left[(\hat{f}(x) - E[\hat{f}(x)])^2 + (E[\hat{f}(x)] - f(x))^2 + 2(\hat{f}(x) - E[\hat{f}(x)])(E[\hat{f}(x)] - f(x))\right] \\
 &= E[(\hat{f}(x) - E[\hat{f}(x)])^2] + (E[\hat{f}(x)] - f(x))^2 + 2\{\hat{f}(x) \cdot (E[\hat{f}(x)] - f(x)) - E[\hat{f}(x)] \cdot (E[\hat{f}(x)] - f(x))\}
 \end{aligned}$$

$$\begin{aligned}
 &= E[(\hat{f}(x) - E[\hat{f}(x)])^2] + (E[\hat{f}(x)] - f(x))^2 + 2\{\hat{f}(x) \cdot E[\hat{f}(x)] - \hat{f}(x) \cdot f(x) \\
 &\quad - (E[\hat{f}(x)])^2 + E[\hat{f}(x)] \cdot f(x)\}
 \end{aligned}$$

We know:  $E[X + Y] = E[X] + E[Y]$  so,

$$\begin{aligned}
 &= E[(\hat{f}(x) - E[\hat{f}(x)])^2] + E[(E[\hat{f}(x)] - f(x))^2] \\
 &\quad + 2 E[\hat{f}(x) \cdot E[\hat{f}(x)] - \hat{f}(x) \cdot f(x) + E[\hat{f}(x)] \cdot f(x)]
 \end{aligned}$$