Tutorial 4

Analysis and Design of Algorithms

Divide and Conquer II

Divide & Conquer Recap

- Divide & conquer is an **algorithmic paradigm** in which the problem is solved by **dividing** it into smaller parts, **conquering** them and possibly **combining** the sub problems together.

- In order to find an upper bound on the time needed to run a D&C algorithm, we write a **recurrence** and we solve it.

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ aT(b) + D(n) + C(n) & \text{otherwise} \end{cases}$$

where a is the number of subproblems, b is the size of each subproblem in terms of n, $D(n)$ is the divide time, $C(n)$ is the combine time.

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To solve the recurrence:

1. Recursion Tree (last tutorial)

2. Master Theorem (this tutorial)

Master Theorem

We use the master theorem to solve recurrences of the form T(n) = aT(n/b) + f(n) , where a ≥ 1 and b > 1 and f(n) is asymptotically positive

The Master Theorem

Let $a \geq 1$ and $b > 1$ be constants, $f(n)$ be an asymptotically positive function, and $T(n) = aT(\frac{n}{b}) + f(n)$.
$T(n)$ can be asymptotically bounded in 3 cases:

1. If $f(n) = O(n^{log_b(a)-\varepsilon})$ for some $\varepsilon > 0$, then $T(n) = \Theta(n^{log_b(a)})$.
2. If $f(n) = \Theta(n^{log_b(a)})$, then $T(n) = \Theta(n^{log_b(a)} log(n))$.
3. If $f(n) = \Omega(n^{log_b(a)+\varepsilon})$ for some $\varepsilon > 0$ and $af(\frac{n}{b}) \leq cf(n)$ for $0 < c < 1$, then $T(n) = \Theta(f(n))$.

Exercise 4-1

Use the master theorem to get the asymptotic notation of the following recurrences. If you fail to get the case, state the gap in which the recurrence falls.

i. $T(n) = 2T(n/2) + n^3$

ii. $T(n) = 16T(n/4) + n^2$

iii. $T(n) = 8T(n/2) + n^2$

iv. $T(n) = 5T(n/5) + \frac{n}{\lg(n)}$

v. $T(n) = 5T(n/5) + n\lg(n)$

Exercise 4-3 From CLRS (©MIT Press 2001)

The recurrence $T(n) = 7T(n/2)+n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A.

All done!