German University in Cairo - GUC
Faculty of Media Engineering and Technology - MET
Department of Digital Media Engineering and Technology
Assoc. Prof. Dr.- Rimon Elias

Tuesday, November 8th, 2022

DMET 502 Computer Graphics

Winter Semester 2022/2023

Midterm Exam (Version I) Model Answers

Barcode

Major: Pick one

☐ DMET
☐ CSEN

Instructions: **Read Carefully Before Proceeding.**

1- Non-programmable calculators are allowed.
2- This is a **closed book exam**.
3- Write your solutions in the space provided.
4- The exam consists of **(3) questions.**
5- This exam booklet contains **(6) pages** including this page. The last page is a formula sheet. **Keep it attached**.
6- Total time allowed for this exam is **(120) minutes.**
7- When you are told that time is up, stop working on the test.

Good Luck!

Don't write anything below ;-)

Question	**1**	**2**	3	∑
Possible Marks	20	30	16	66
Final Marks				

Question 1 [2D Graphics]:

[20 marks] The Cohen-Sutherland Algorithm is used for line clipping if the clip polygon is a rectangle. If a circle is used instead, propose an algorithm to clip lines (i.e., to keep what is inside the circle and remove any lines or parts of lines outside the circle). You have to cover **all** possibilities that may occur.

Answers to Question 1:

1. Assume that the center point of the circle is $[x, y]^T$ and the radius is r. [1 mark]
2. Calculate the distances from each endpoints $[x_1, y_1]^T$ and $[x_2, y_2]^T$ to the center $[x, y]^T$. Let us call these d_1 and d_2 respectively. [1 mark]
3. There are 4 possibilities
   a. If $d_1 <= r$ and $d_2 <= r$, [1 mark]
      i. The whole line segment is included inside the circle and should be accepted
   b. If $d_1 > r$ and $d_2 <= r$ [1 mark]
      i. Split the line segment $<[x_1, y_1]^T[x_2, y_2]^T>$ into 2 segments $<[x_1, y_1]^T[x_3, y_3]^T$ and $<[x_3, y_3]^T[x_2, y_2]^T>$ where $[x_3, y_3]^T$ is the intersection point between and line and the circle. [1 mark]
      ii. The line segment $<[x_1, y_1]^T[x_3, y_3]^T>$ will be rejected [1 mark]
      iii. The line segment $<[x_3, y_3]^T[x_2, y_2]^T>$ will be accepted [1 mark]
   c. If $d_1 <= r$ and $d_2 > r$ [1 mark]
      i. Split the line segment $<[x_1, y_1]^T[x_2, y_2]^T>$ into 2 segments $<[x_1, y_1]^T[x_3, y_3]^T$ and $<[x_3, y_3]^T[x_2, y_2]^T>$ where $[x_3, y_3]^T$ is the intersection point between and line and the circle. [1 mark]
      ii. The line segment $<[x_3, y_3]^T[x_2, y_2]^T>$ will be rejected [1 mark]
      iii. The line segment $<[x_1, y_1]^T[x_3, y_3]^T>$ will be accepted [1 mark]
   d. If $d_1 > r$ and $d_2 > r$ 🡨 both endpoints are outside the circle. [1 mark]
      i. Determine the perpendicular distance d_3 from the center $[x, y]^T$ to the line $<[x_1, y_1]^T[x_2, y_2]^T>$ [1 mark]
      ii. If d_3 is $>$ r then the line segment $<[x_1, y_1]^T[x_2, y_2]^T>$ is outside the circle and should be rejected [1 mark]
      iii. If d_3 is $<$ r then the line segment $<[x_1, y_1]^T[x_2, y_2]^T>$ intersects the circle in 2 points $[x_3, y_3]^T$ and $[x_4, y_4]^T$ [1 mark]
      iv. The line segment $<[x_3, y_3]^T[x_4, y_4]^T>$ will be rejected [1 mark]
      v. The line segment $<[x_1, y_1]^T[x_3, y_3]^T>$ will be accepted [1 mark]
      vi. The line segment $<[x_4, y_4]^T[x_1, y_1]^T>$ will be accepted [1 mark]
      vii. If d_3 is = r then the line touches the circle in 1 point which should be accepted [1 mark]

Note that the rejected segments are mentioned before the accepted ones to make sure that the intersection points are included with the accepted segments. [1 mark]

Question 2 [2D Transformations]:

a) [9 marks] Assume that the *y*-axis is sheared using a factor -δ. Write down the appropriate axes shearing matrices for inhomogeneous and homogeneous coordinates.

If this operation is followed by another *x*-axis shearing using a factor α, determine the composite transformation matrix.

Answers to Question 2 a):

It is the same shearing matrix along the y-axis with a factor of +δ. This is as done with all axes transformation opreations.

$$\begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix}$$ [3 marks]

$$\begin{bmatrix} 1 & 0 & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ [1 mark]

$$\begin{bmatrix} 1 & -\alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} = \begin{bmatrix} 1-\alpha\delta & -\alpha \\ \delta & 1 \end{bmatrix}$$ [3 + 2 marks]

b) [4 marks] Consider a square with vertices $[1, 1]^T$, $[1, -1]^T$, $[-1, -1]^T$ and $[-1, 1]^T$. If the *y*-axis is sheared using a factor -1 followed by an *x*-axis shearing using a factor 3, determine the new coordinates of the square.

Answers to Question 2 b):

$$\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$ [1 mark]

$$\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ [1 mark]

$$\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$ [1 mark]

$$\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$ [1 mark]

c) [17 marks] Consider a square with vertices $[1, 1]^T$, $[1, -1]^T$, $[-1, -1]^T$ and $[-1, 1]^T$. If this square is sheared along the x-axis using a factor 3, determine the new coordinates of the vertices.

If this is followed by x-axis shearing using a factor 3, determine the new coordinates in this case.

Determine a single matrix defining these two consecutive operations (i.e., object shearing followed by axes shearing).

Answers to Question 2 c):

$$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$$ [3 +1 marks]

$$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}-2\\-1\end{bmatrix}$$ [1 mark]

$$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix} = \begin{bmatrix}-4\\-1\end{bmatrix}$$ [1 mark]

$$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$ [1 mark]

$$\begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$$ [3 +1 marks]

$$\begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}\begin{bmatrix}-2\\-1\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$$ [1 mark]

$$\begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}\begin{bmatrix}-4\\-1\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$$ [1 mark]

$$\begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$$ [1 mark]

We get the identity matrix

$$\begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$ [3 marks]

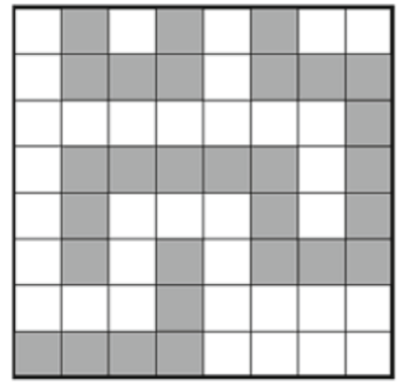
OR, in general

$$\begin{bmatrix}1 & -\beta\\0 & 1\end{bmatrix}\begin{bmatrix}1 & \alpha\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & \alpha-\beta\\0 & 1\end{bmatrix}$$

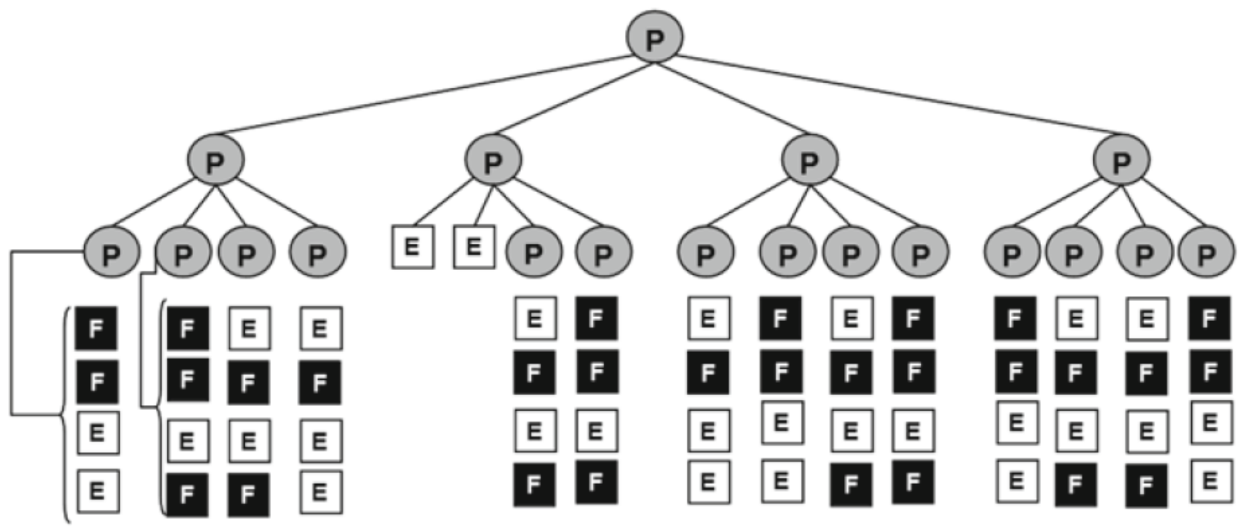
Question 3 [Solids]:

[16 marks] Construct a quadtree to partition the pattern shown. The order of constructing nodes is given by

2	3
0	1



Answers to Question 3:



Formula Sheet

Input: x_0, y_0, x_1, y_1

$steep = |y_1 - y_0| > |x_1 - x_0|$
if $(steep = TRUE)$ **then**
    swap (x_0, y_0)
    swap (x_1, y_1)
end if

if $(x_0 > x_1)$ **then**
    swap (x_0, x_1)
    swap (y_0, y_1)
end if

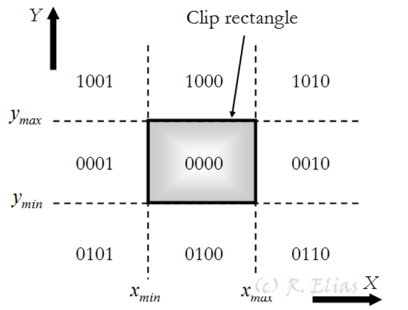
if $(y_0 > y_1)$ **then**
    $\delta y = -1$
else
    $\delta y = 1$
end if

$\Delta x = x_1 - x_0$
$\Delta y = |y_1 - y_0|$
$y = y_0$
$error = 0$

for $(x = x_0$ to $x_1)$ **do**
    if $(steep = TRUE)$ **then**
        Plot $[y, x]^T$
    else
        Plot $[x, y]^T$
    end if
    $error = error + \Delta y$
    if $(2 \times error \geq \Delta x)$ **then**
        $y = y + \delta y$
        $error = error - \Delta x$
    end if
end for

end

1. Determine outcode for each endpoint.
2. Dealing with the two outcodes:
   a. Bitwise-OR the bits. If this results in 0000, trivially accept.
   b. Otherwise, bitwise-AND the bits. If this results in a value other than 0000, trivially reject.
   c. Otherwise, segment the line. The outpoint is replaced by the intersection point. Go to Step 2.
3. If trivially accepted, draw the line.



$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1} + \underbrace{\begin{bmatrix} t_x \\ t_y \end{bmatrix}}_{\mathbf{t}}.$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}}_{\mathbf{P}_2} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathtt{T}([t_x,t_y]^T)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}}_{\mathbf{P}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\dot{\mathtt{R}}(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}}_{\dot{\mathtt{S}}(s_x,s_y)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_x} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_y} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_x(sh_x)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_y(sh_y)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -t_x \\ -t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{s_x} & 0 \\ 0 & \frac{1}{s_y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

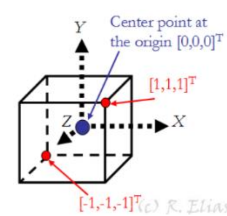
Vertex #	x	y	z

Edge #	Start vertex	End vertex

Edge	Vertices		Faces		Left traverse		Right traverse	
Name	Start	End	Left	Right	Pred	Succ	Pred	Succ

Vertex	edge

Face	edge



translate(scale(Block, < 1, 1.5, 1.5 >), < 1, 2, 3 >)

Face	Vertices
A	$[x_1, y_1, z_1]^T$, $[x_2, y_2, z_2]^T$, $[x_3, y_3, z_3]^T$
B	$[x_2, y_2, z_2]^T$, $[x_4, y_4, z_4]^T$, $[x_3, y_3, z_3]^T$
$\vdots$	$\vdots$

Vertex	Coordinates
1	$[x_1, y_1, z_1]^T$
2	$[x_2, y_2, z_2]^T$
$\vdots$	$\vdots$

Face	Vertices
A	1, 2, 3
B	2, 4, 3
$\vdots$	$\vdots$