German University in Cairo
Media Engineering and Technology
DMET 502 Computer Graphics, Winter 2023
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Practice assignment 3

2D Transformations

Q 1: Consider a square whose vertices are $\mathbf{a} = [2,1]^T, \mathbf{b} = [6,1]^T, \mathbf{c} = [6,5]^T$, and $\mathbf{d} = [2,5]^T$. It is required to rotate this square about its center through an angle of $35°$; this is followed by a reflection about a line passing through the origin with an inclination of $35°$ relative to the x-axis. Derive a single transformation matrix to perform all the calculations.

Q 2: Given a 2D polygon specified by the vertices $[1,1]^T, [3,1]^T, [5,3]^T$, and $[2,4]^T$, develop a single transformation matrix that:

- reflects it about the x-axis,
- rotates it about its center through an angle of $25°$.

Determine the coordinates of the transformed polygon.

Q 3: Given the previous polygon, derive the transformation matrix that performs the following operations:

- reflects the polygon about the line $Y + X = 1$,
- translates it by -1 and 2 in the x and y directions respectively,
- rotates it about the point $[2,2]^T$ through an angle of $180°$.

Plot the original and transformed polygons.

Q 4: Consider a triangle with vertices located at $[1,0]^T, [0,1]^T$, and $[-1,0]^T$. If this triangle is sheared by a factor of 3 in the x-direction and then rotated by $35°$ about the origin, determine the coordinates of the transformed triangle.

Q 5: Consider a square with vertices located at $[1,0]^T, [0,-1]^T, [-1,0]^T$, and $[0,1]^T$. If this square is scaled up by a factor of 2 in the x-direction, and then rotated by $35°$ about the origin, determine the coordinates of the transformed square.

Q 6: Consider a unit square centered at the point [5,5]T with sides parallel to the two major axes. Find the 2D transformation matrix that transforms this square into another square with vertices $[1,0]^T, [0,1]^T, [-1,0]^T$, and $[0,-1]^T$.

Q 7: A triangle whose vertices are $\mathbf{a} = [1,0]^T, \mathbf{b} = [0,1]^T$, and $\mathbf{c} = [-1,0]^T$, is translated by the vector $[3,2]^T$, and then rotated by $45°$ about the point $[1,-1]^T$. Determine the coordinates and the area of the transformed triangle.

Q 8: A unit square with sides parallel to the x and y axes, and centered at $[5,5]^T$, is to be transformed to a rectangle whose vertices are $[1,1]^T, [-1,-1]^T, [-4,2]^T$, and $[-2,4]^T$. Find the transformation matrix.

Q 9: The points $\mathbf{a} = [1,1]^T, \mathbf{b} = [-1,-1]^T$, and $\mathbf{c} = [-4,2]^T$, form a right-angled triangle. Find the point d, such that **abcd** is a rectangle. The rectangle **abcd** is rotated $45°$ about its center, and then scaled to double its original size. Find the required transformation matrix, and sketch the rectangle after the transformation.

Q 10: A triangle whose vertices are $\mathbf{a} = [1,0]^T, \mathbf{b} = [0,1]^T$, and $\mathbf{c} = [-1,0]^T$, is sheared by a factor of 3 in the x-direction and then rotated by $45°$ about the point $[1,-1]^T$. Determine the coordinates and area of the transformed triangle.

Q 11: Construct one possible sequence of primitive transformation matrices in homogenous coordinates that maps the following shape $ABCD$ into the new shape $A'B'C'D'$. Compute a single composite transformation matrix that combines the sequence you found.

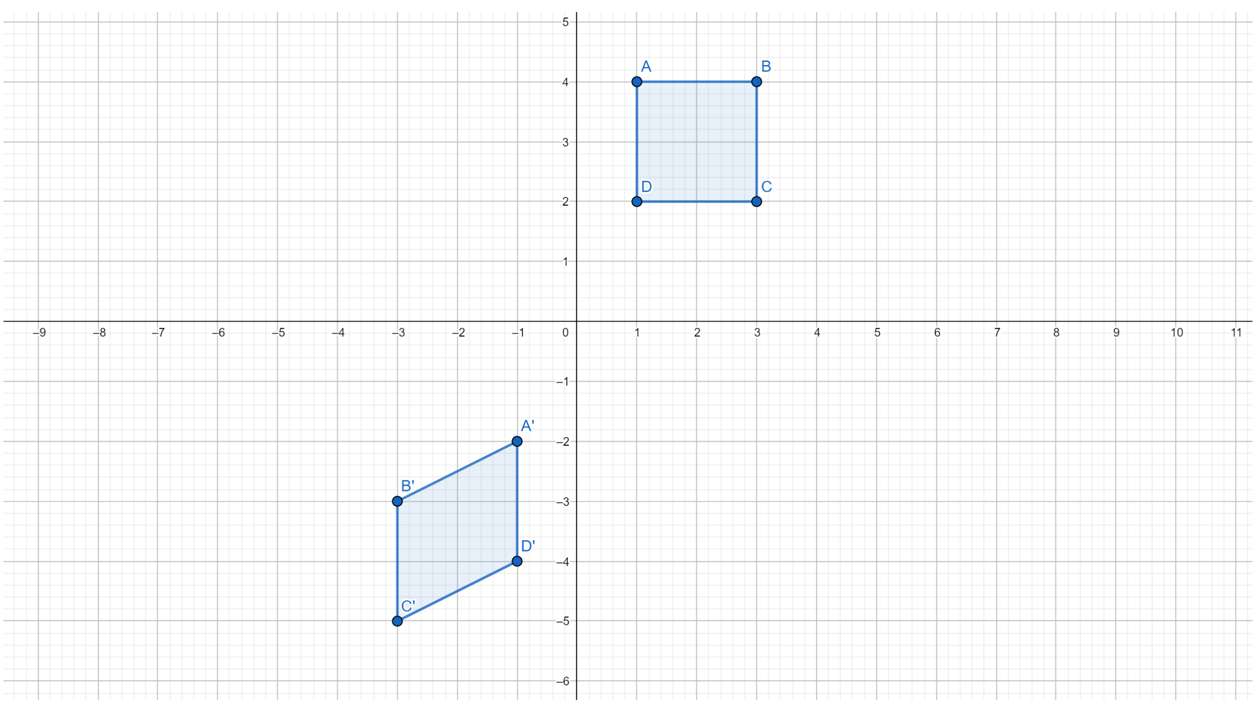

Figure 1: Transformation