German University in Cairo
Media Engineering and Technology
DMET 502 Computer Graphics, Winter 2023
Assoc. Prof. Dr. Rimon Elias

Practice assignment 3 solution

2D Transformations

Q 1: Consider a square whose vertices are $\mathbf{a} = [2,1]^T, \mathbf{b} = [6,1]^T, \mathbf{c} = [6,5]^T$, and $\mathbf{d} = [2,5]^T$. It is required to rotate this square about its center through an angle of $35°$; this is followed by a reflection about a line passing through the origin with an inclination of $35°$ relative to the x-axis. Derive a single transformation matrix to perform all the calculations.

Solution:

$$M1 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M2 = \begin{bmatrix} cos(35°) & -sin(35°) & 0 \\ sin(35°) & cos(35°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M3 = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

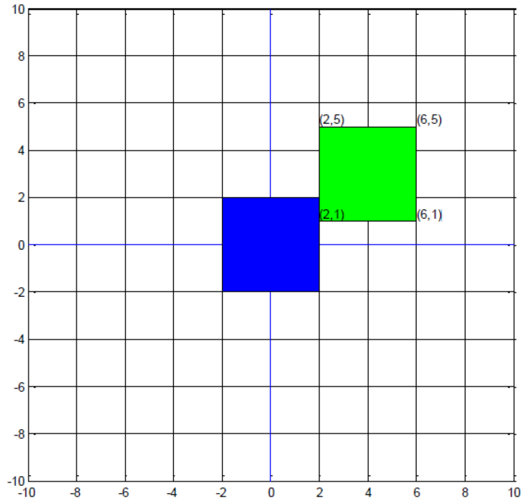
$$M4 = \begin{bmatrix} cos(-35°) & -sin(-35°) & 0 \\ sin(-35°) & cos(-35°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

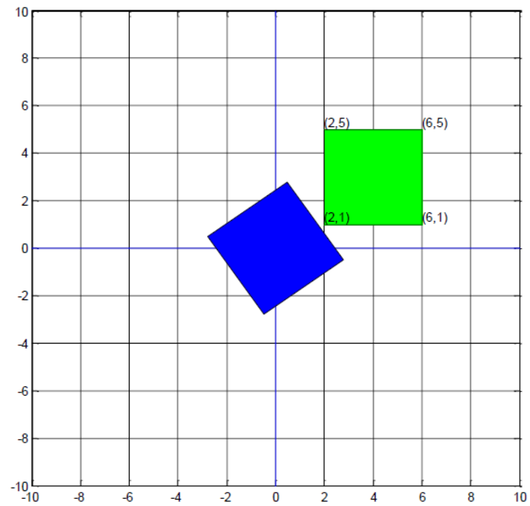
$$M6 = \begin{bmatrix} cos(35°) & -sin(35°) & 0 \\ sin(35°) & cos(35°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M6 * M5 * M4 * M3 * M2 * M1$$

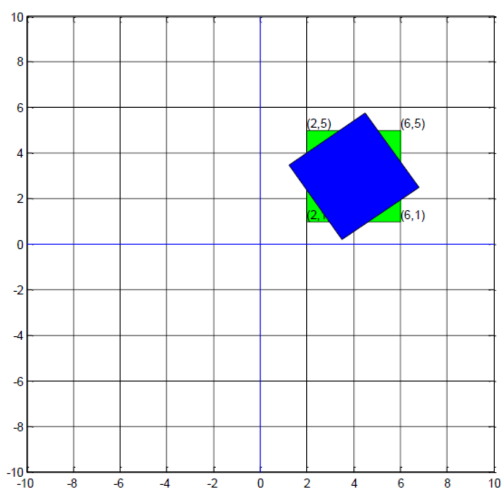
$$M = \begin{bmatrix} 0.8192 & 0.5736 & -0.8102 \\ 0.5736 & -0.8192 & 2.8959 \\ 0 & 0 & 1 \end{bmatrix}$$



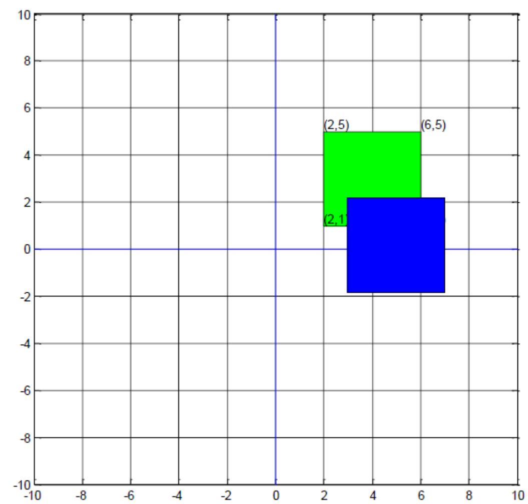
(a)



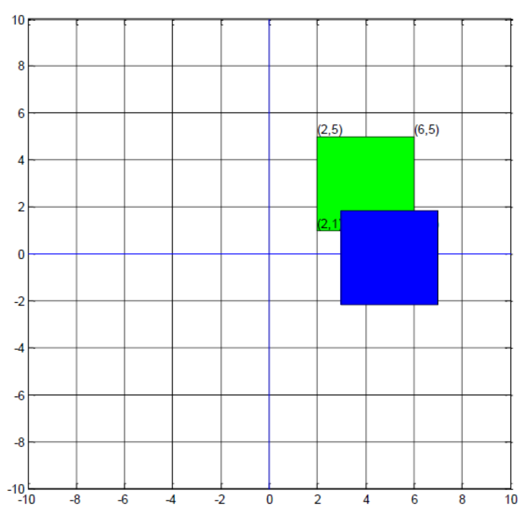
(b)



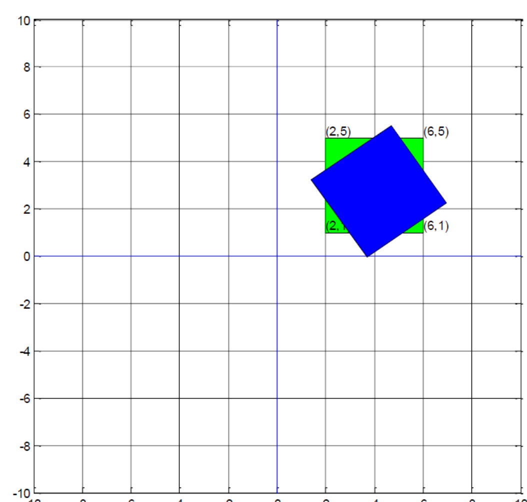
(c)



(d)



(e)



(f)

Q 2: Given a 2D polygon specified by the vertices $[1,1]^T$, $[3,1]^T$, $[5,3]^T$, and $[2,4]^T$, develop a single transformation matrix that:

- reflects it about the x-axis,
- rotates it about its center through an angle of $25°$.

Determine the coordinates of the transformed polygon.

Solution:

$$M1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

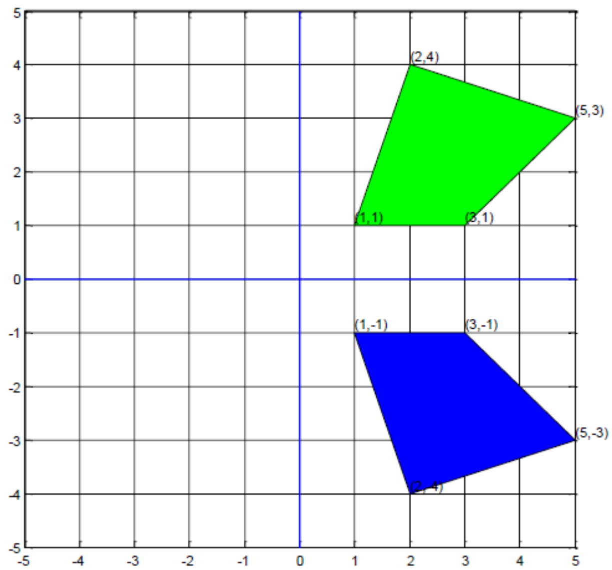
$$M2 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M3 = \begin{bmatrix} cos(25°) & -sin(25°) & 0 \\ sin(25°) & cos(25°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

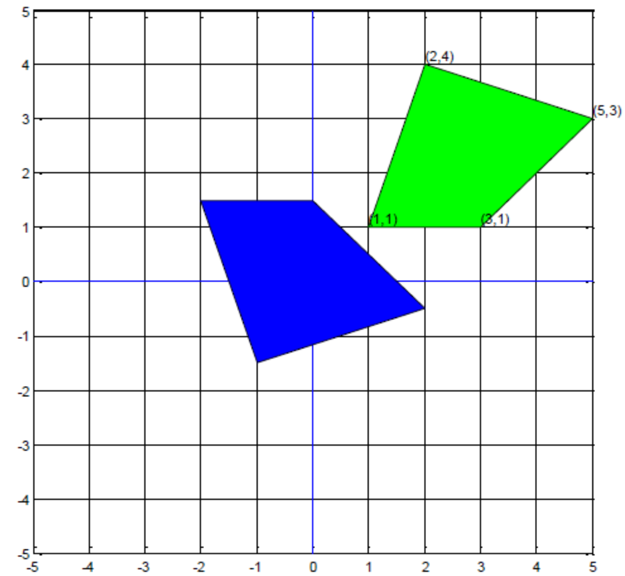
$$M4 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M4 * M3 * M2 * M1$$

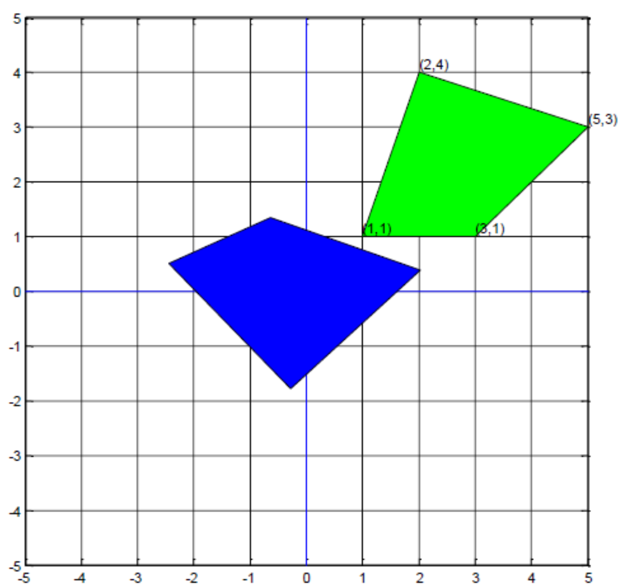
$$M = \begin{bmatrix} 0.9063 & 0.4226 & -0.7755 \\ 0.4226 & -0.9063 & -1.5021 \\ 0 & 0 & 1 \end{bmatrix}$$



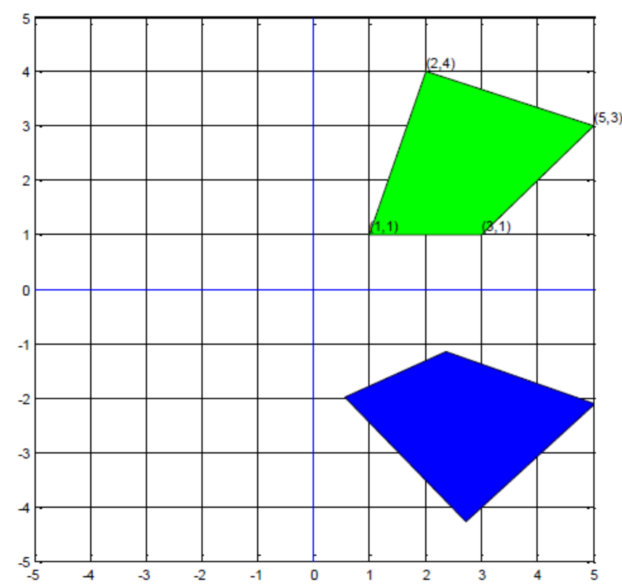
(a)



(b)



(c)



(d)

Q 3: Given the previous polygon, derive the transformation matrix that performs the following operations:

- reflects the polygon about the line $Y + X = 1$,
- translates it by -1 and 2 in the x and y directions respectively,
- rotates it about the point $[2, 2]^T$ through an angle of $180°$.

Plot the original and transformed polygons.

Solution:

$$M1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, M2 = \begin{bmatrix} cos(45°) & -sin(45°) & 0 \\ sin(45°) & cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, M4 = \begin{bmatrix} cos(-45°) & -sin(-45°) & 0 \\ sin(-45°) & cos(-45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

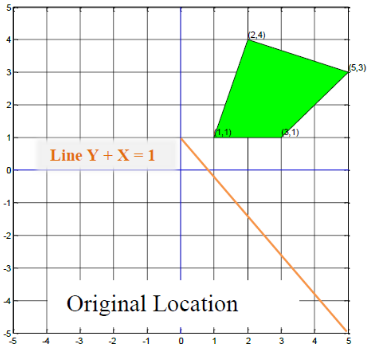
$$M5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, M6 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M7 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}, M8 = \begin{bmatrix} cos(180°) & -sin(180°) & 0 \\ sin(180°) & cos(180°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

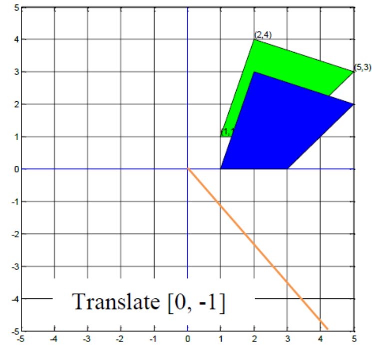
$$M9 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M9 * M8 * M7 * M6 * M5 * M4 * M3 * M2 * M1$$

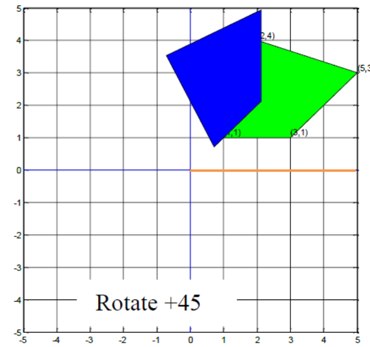
$$M = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$


Line Y + X = 1
Original Location
(2,4)
(5,3)
(1,1)
(3,1)

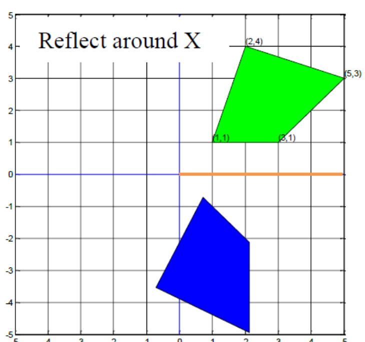
(a)


Translate [0, -1]
(2,4)
(5,3)

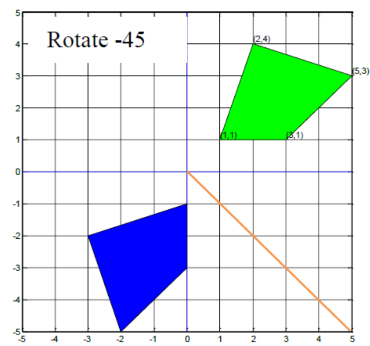
(b)


Rotate +45
(5,3)

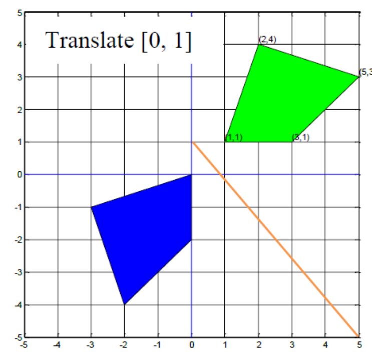
(c)


Reflect around X
(2,4)
(5,3)
(1,1)
(3,1)

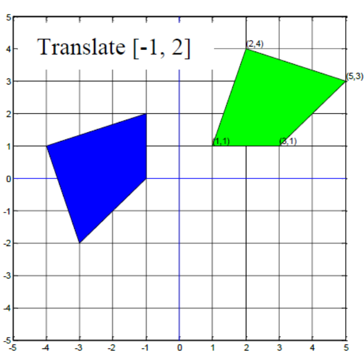
(d)


Rotate -45
(2,4)
(5,3)
(1,1)
(3,1)

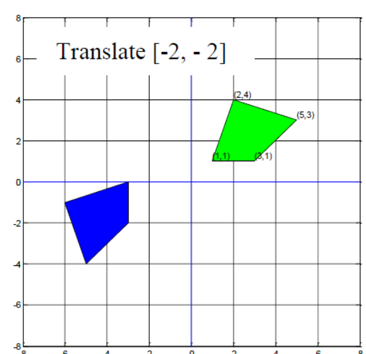
(e)


Translate [0, 1]
(2,4)
(5,3)
(1,1)
(3,1)

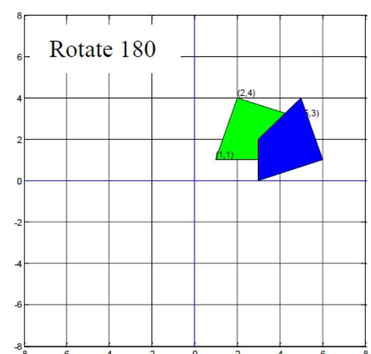
(f)


Translate [-1, 2]
(2,4)
(5,3)
(1,1)
(3,1)

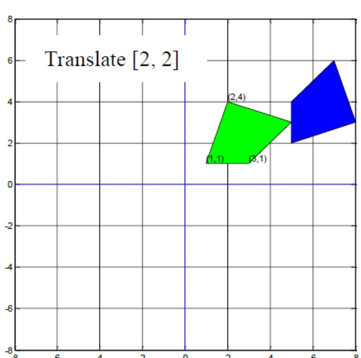
(g)


Translate [-2, - 2]
(2,4)
(5,3)
(1,1)
(3,1)

(h)


Rotate 180
(2,4)
(1,1)

(i)


Translate [2, 2]
(2,4)
(1,1)
(3,1)

(j)

Q 4: Consider a triangle with vertices located at $[1, 0]^T, [0, 1]^T$, and $[-1, 0]^T$. If this triangle is sheared by a factor of 3 in the x-direction and then rotated by $35°$ about the origin, determine the coordinates of the transformed triangle.

Solution:

$$M1 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M2 = \begin{bmatrix} cos(35°) & -sin(35°) & 0 \\ sin(35°) & cos(35°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

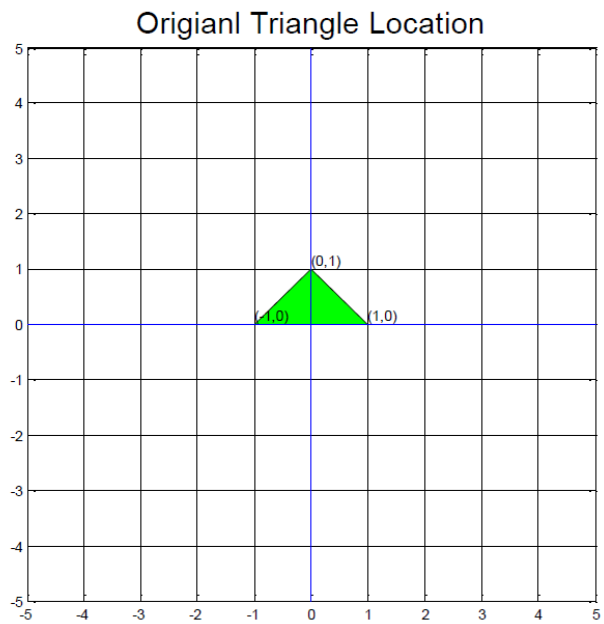
$M = M2 * M1$

$$M = \begin{bmatrix} 0.81 & 1.88 & 0 \\ 0.57 & 2.53 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

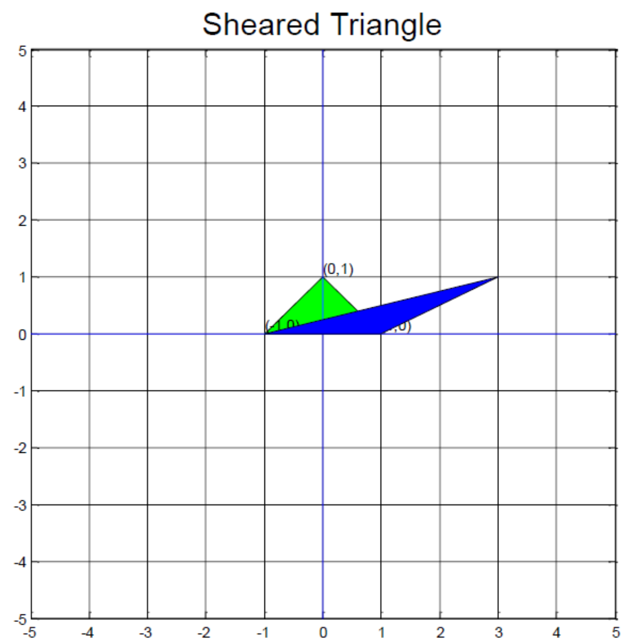
$P_1 = (1, 0)$
$P_2 = (0, 1)$
$P_3 = (-1, 0)$

$P_1' = M * P_1$
$P_2' = M * P_2$
$P_3' = M * P_3$

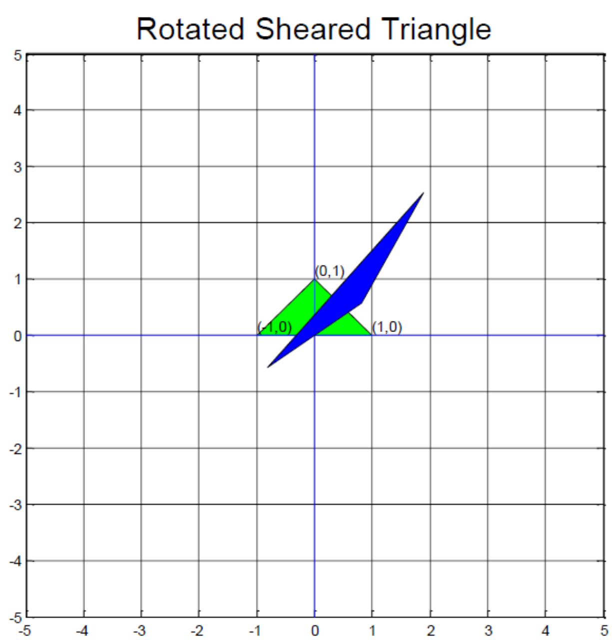
$P_1' = (0.81, 0.57)$
$P_2' = (1.88, 2.53)$
$P_3' = (-0.81, -0.57)$


Origianl Triangle Location
(0,1)
(-1,0)
(1,0)

(a)


Sheared Triangle
(0,1)

(b)


Rotated Sheared Triangle
(0,1)
(-1,0)
(1,0)

(c)

Q 5: Consider a square with vertices located at $[1,0]^T, [0,-1]^T, [-1,0]^T$, and $[0,1]^T$. If this square is scaled up by a factor of 2 in the x-direction, and then rotated by $35°$ about the origin, determine the coordinates of the transformed square.

Solution:

$$M1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M2 = \begin{bmatrix} cos(35°) & -sin(35°) & 0 \\ sin(35°) & cos(35°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

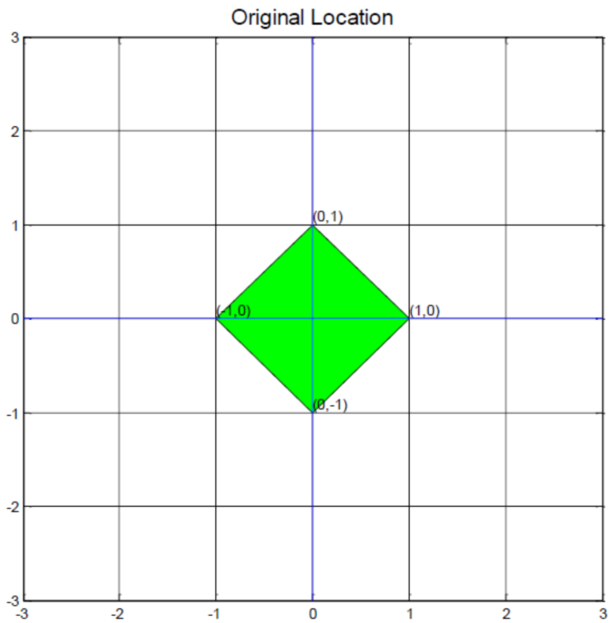
$M = M2 * M1$

$$M = \begin{bmatrix} 1.63 & -0.57 & 0 \\ 1.14 & 0.81 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

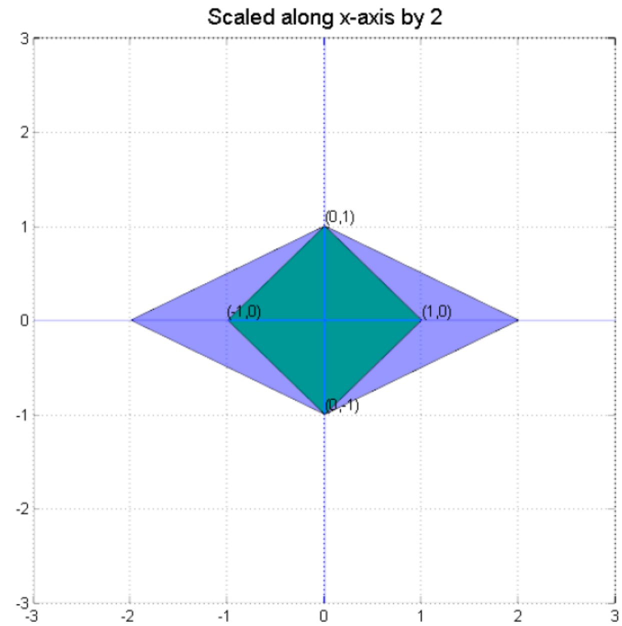
$P_1 = (1,0)$
$P_2 = (0,-1)$
$P_3 = (-1,0)$
$P_4 = (0,1)$

$P_1' = M * P_1$
$P_2' = M * P_2$
$P_3' = M * P_3$
$P_4' = M * P_4$

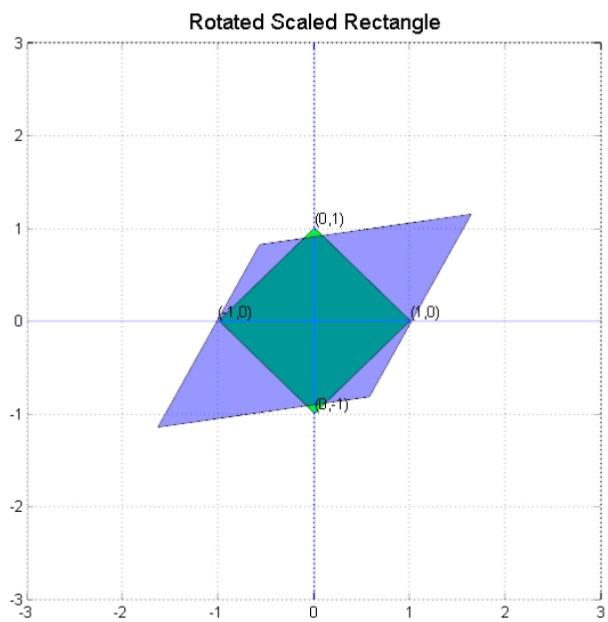
$P_1' = (1.63, 1.14)$
$P_2' = (0.57, -0.81)$
$P_3' = (-1.63, -1.14)$
$P_4' = (-0.57, 0.81)$


Original Location
(0,1)
(-1,0)
(1,0)
(0,-1)
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

(a)


Scaled along x-axis by 2
(0,1)
(-1,0)
(1,0)
(0,-1)
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

(b)


Rotated Scaled Rectangle
(0,1)
(-1,0)
(1,0)
(0,-1)
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

(c)

Q 6: Consider a unit square centered at the point [5,5]T with sides parallel to the two major axes. Find the 2D transformation matrix that transforms this square into another square with vertices $[1, 0]^T$, $[0, 1]^T$, $[-1, 0]^T$, and $[0, -1]^T$.

Solution:

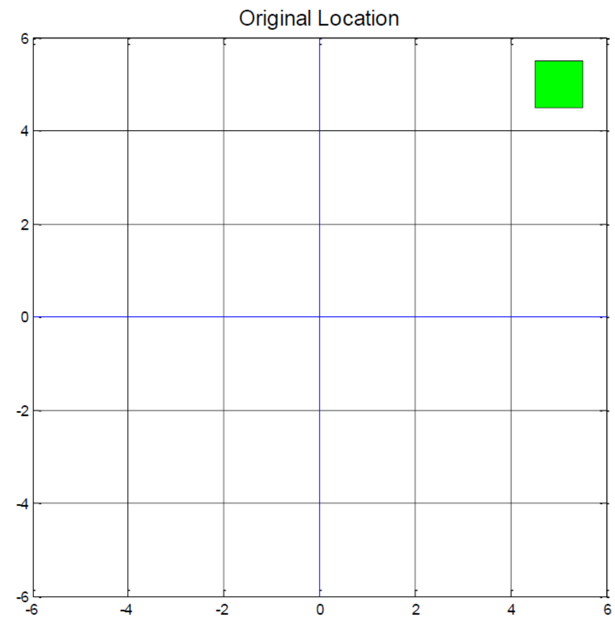
$$M1 = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M2 = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

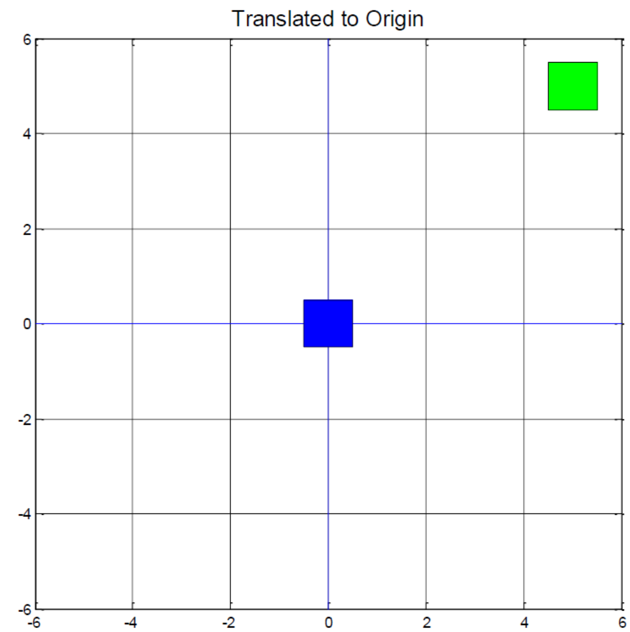
$$M3 = \begin{bmatrix} cos(45°) & -sin(45°) & 0 \\ sin(45°) & cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M3 * M2 * M1$$

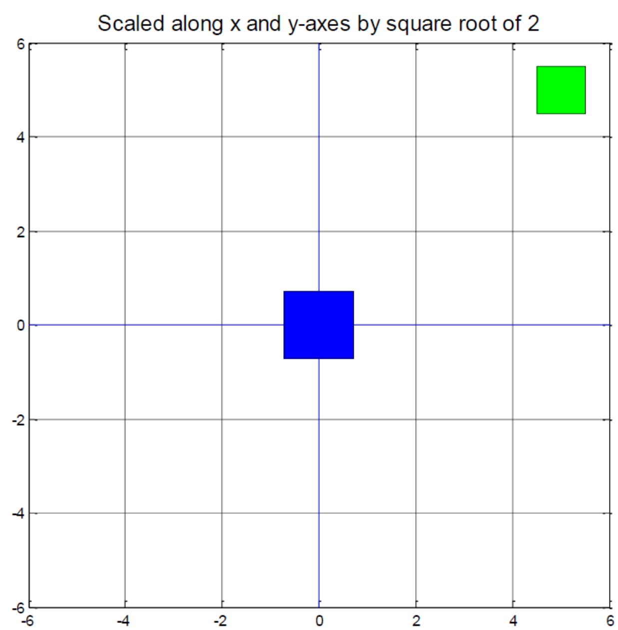
$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$


Original Location
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

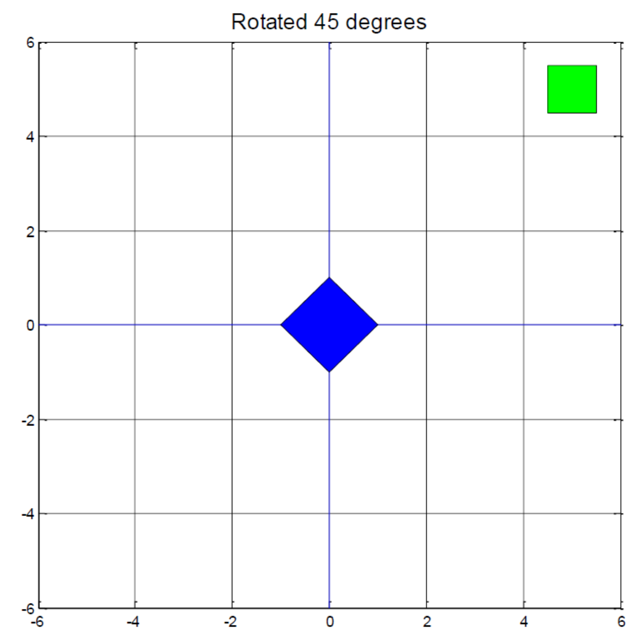
(a)


Translated to Origin
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

(b)


Scaled along x and y-axes by square root of 2
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

(c)


Rotated 45 degrees
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

(d)

Q 7: A triangle whose vertices are **a** $= [1,0]^T$, **b** $= [0,1]^T$, and **c** $= [-1,0]^T$, is translated by the vector $[3,2]^T$, and then rotated by $45°$ about the point $[1,-1]^T$. Determine the coordinates and the area of the transformed triangle.

Solution:

$$M1 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M3 = \begin{bmatrix} cos(45°) & -sin(45°) & 0 \\ sin(45°) & cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

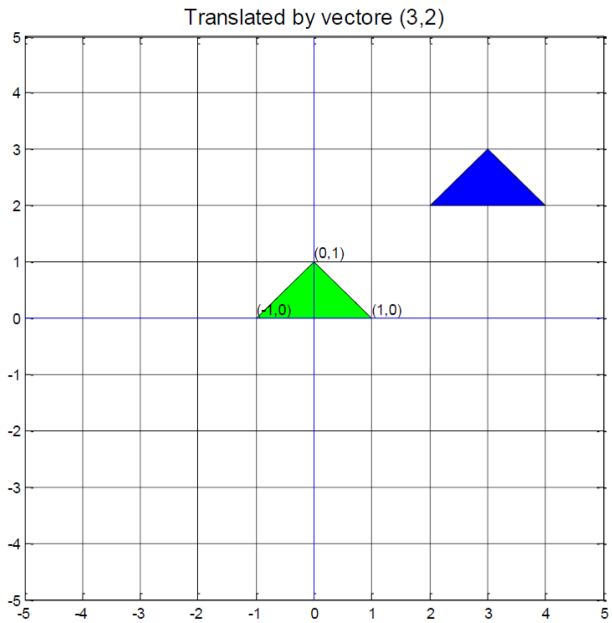
$$M = M4 * M3 * M2 * M1$$

$$M = \begin{bmatrix} 0.707 & -0.707 & 0.292 \\ 0.707 & 0.707 & 2.535 \\ 0 & 0 & 1 \end{bmatrix}$$

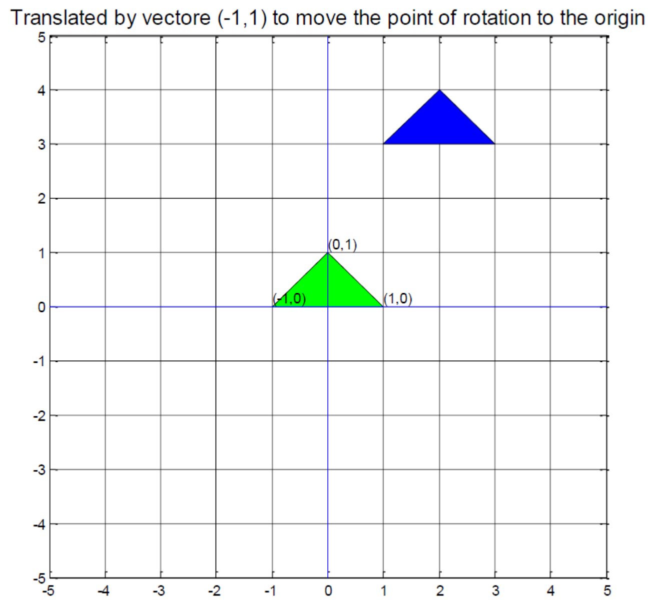
$P_1' = (1, 3.242)$
$P_2' = (-0.414, 3.242)$
$P_3' = (-0.414, 1.828)$

Translation and rotation operations do not affect the triangle dimensions, hence the area of the triangle is

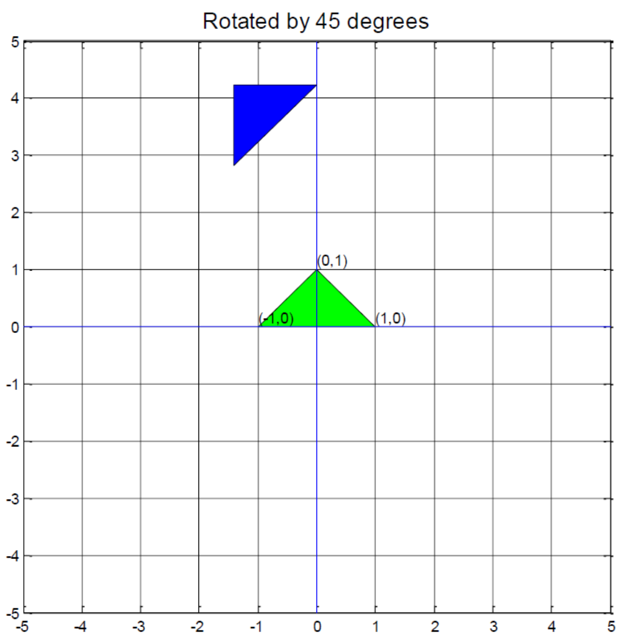
$A = \frac{1}{2} \times base \times height$
$A = \frac{1}{2} \times 2 \times 1 = 1$


Translated by vectore (3,2)
(0,1)
(-1,0)
(1,0)

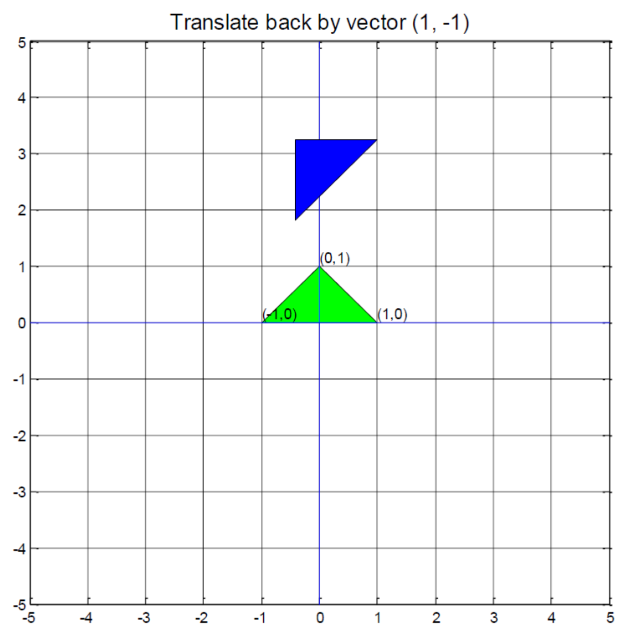
(a)


Translated by vectore (-1,1) to move the point of rotation to the origin
(0,1)
(-1,0)
(1,0)

(b)


Rotated by 45 degrees
(0,1)
(-1,0)
(1,0)

(c)


Translate back by vector (1, -1)
(0,1)
(-1,0)
(1,0)

(d)

Q 8: A unit square with sides parallel to the x and y axes, and centered at $[5,5]^T$, is to be transformed to a rectangle whose vertices are $[1,1]^T$, $[-1,-1]^T$, $[-4,2]^T$, and $[-2,4]^T$. Find the transformation matrix.

Solution:

$$M1 = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

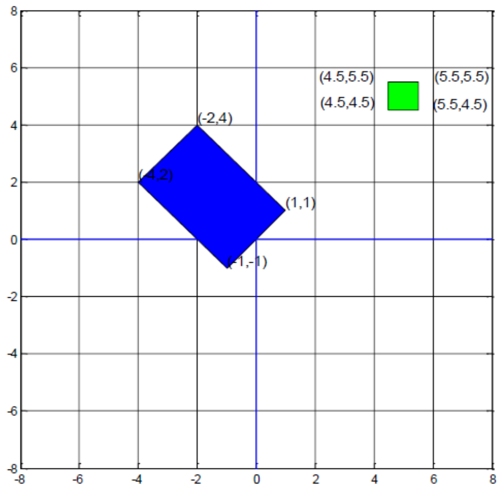
$$M2 = \begin{bmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & 3\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{3\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

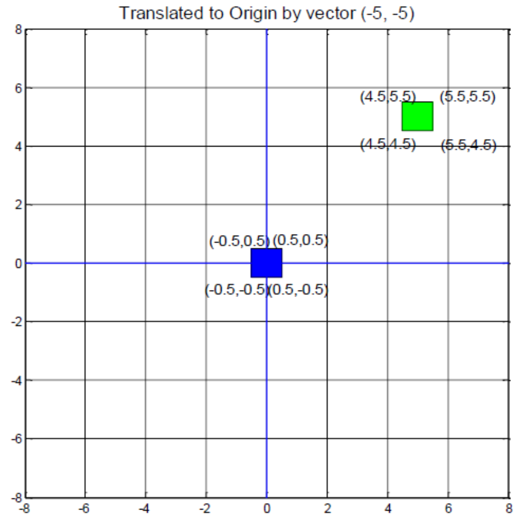
$$M4 = \begin{bmatrix} cos(45^\circ) & -sin(45^\circ) & 0 \\ sin(45^\circ) & cos(45^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M4 * M3 * M2 * M1$$

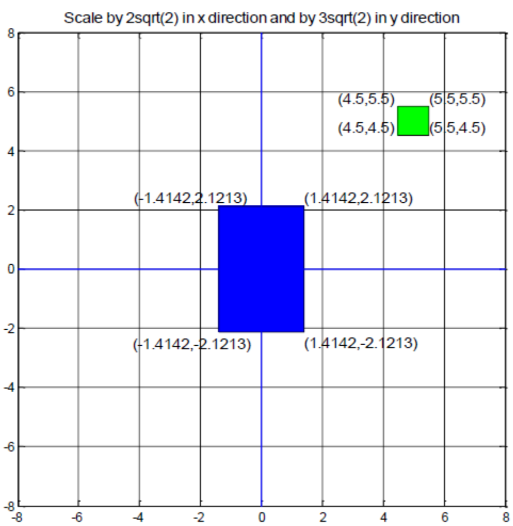
$$M = \begin{bmatrix} 2 & -3 & 3.5 \\ 2 & 3 & -23.5 \\ 0 & 0 & 1 \end{bmatrix}$$


(4.5,5.5)
(5.5,5.5)
(4.5,4.5)
(5.5,4.5)
(-2,4)
(-4,2)
(1,1)
(-1,-1)

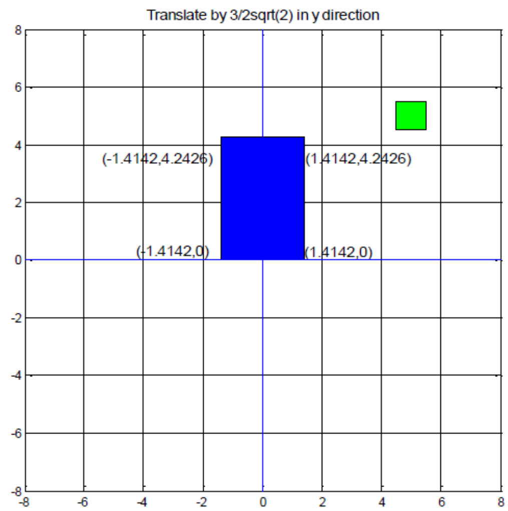
(a)


Translated to Origin by vector (-5, -5)
(4.5,5.5)
(5.5,5.5)
(4.5,4.5)
(5.5,4.5)
(-0.5,0.5)
(0.5,0.5)
(-0.5,-0.5)
(0.5,-0.5)

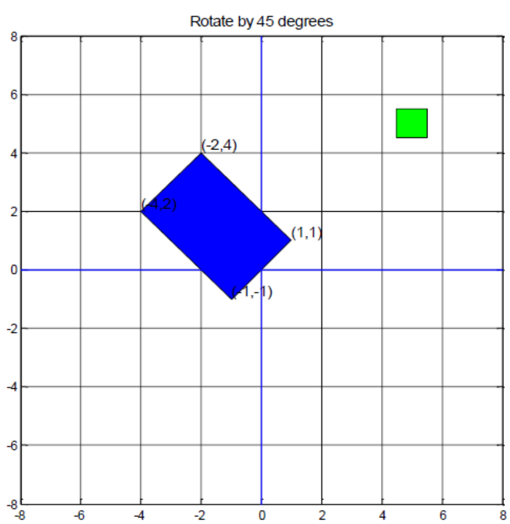
(b)


Scale by 2sqrt(2) in x direction and by 3sqrt(2) in y direction
(4.5,5.5)
(5.5,5.5)
(4.5,4.5)
(5.5,4.5)
(-1.4142,2.1213)
(1.4142,2.1213)
(-1.4142,-2.1213)
(1.4142,-2.1213)

(c)


Translate by 3/2sqrt(2) in y direction
(-1.4142,4.2426)
(1.4142,4.2426)
(-1.4142,0)
(1.4142,0)

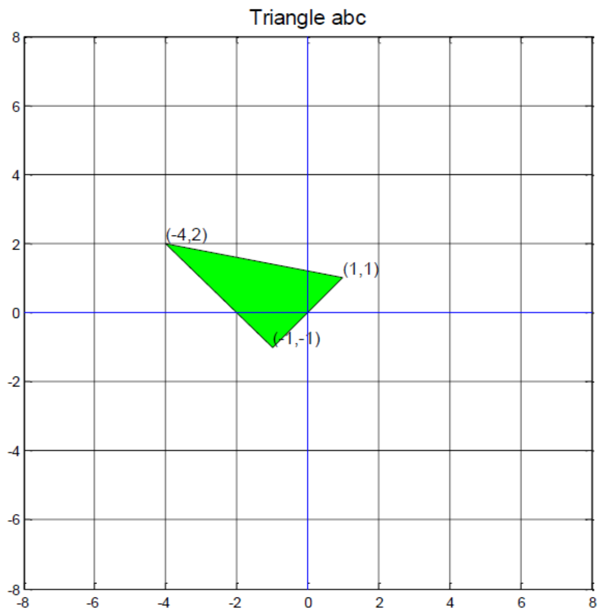
(d)

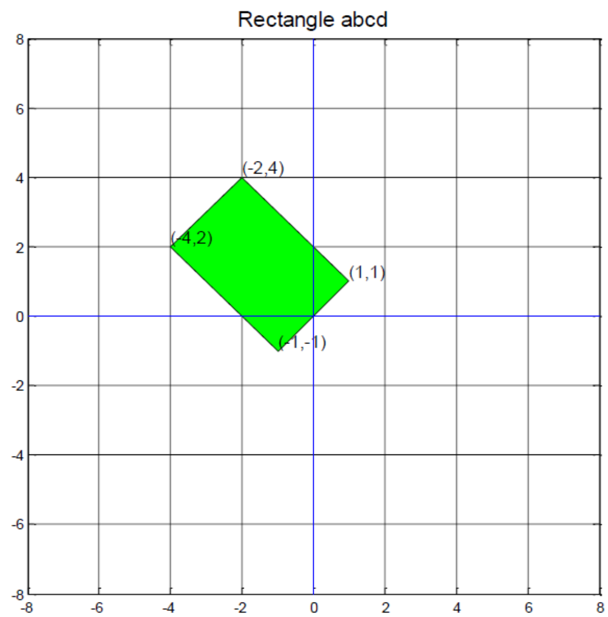

Rotate by 45 degrees
(-2,4)
(-4,2)
(1,1)
(-1,-1)

(e)

Q 9: The points **a** $= [1, 1]^T$, **b** $= [-1, -1]^T$, and **c** $= [-4, 2]^T$, form a right-angled triangle. Find the point d, such that **abcd** is a rectangle. The rectangle **abcd** is rotated $45°$ about its center, and then scaled to double its original size. Find the required transformation matrix, and sketch the rectangle after the transformation.

Solution:
Since abc is a right-angled triangle, then the vector from a to b is the same as the vector c to d. So if $d = (x, y)$ then $(-2, -2) = (-4 - x, 2 - y)$. So $x = -2$ and $y = 4$, then $d = (-2, 4)$.





$$M1 = \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & -1.5 \\ 0 & 0 & 1 \end{bmatrix}$$

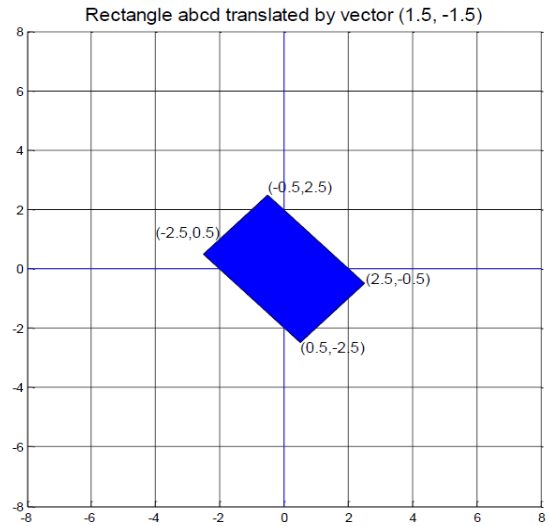
$$M2 = \begin{bmatrix} cos(45°) & -sin(45°) & 0 \\ sin(45°) & cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

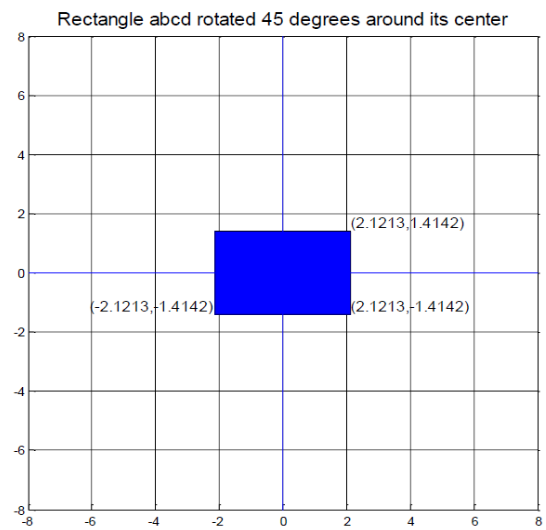
$$M3 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

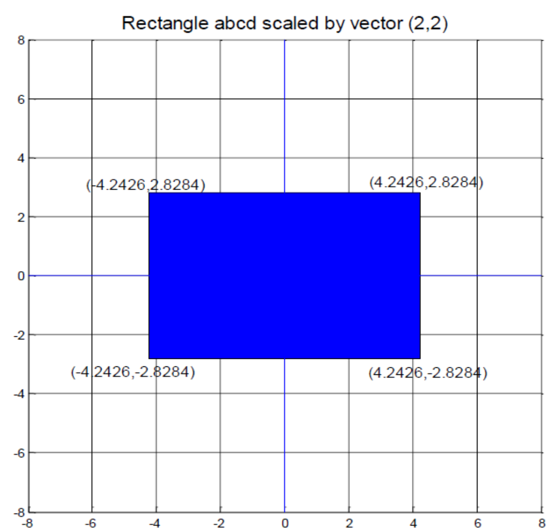
$$M4 = \begin{bmatrix} 1 & 0 & -1.5 \\ 0 & 1 & 1.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M4 * M3 * M2 * M1$$

$$M = \begin{bmatrix} 1.4142 & -1.4142 & 2.7426 \\ 1.4142 & 1.4142 & 1.5 \\ 0 & 0 & 1 \end{bmatrix}$$







Q 10: A triangle whose vertices are **a** $= [1, 0]^T$, **b** $= [0, 1]^T$, and **c** $= [-1, 0]^T$, is sheared by a factor of 3 in the x-direction and then rotated by $45°$ about the point $[1, -1]^T$. Determine the coordinates and area of the transformed triangle.

Solution:

1. $M1$, shearing by 3 along the x-axis
2. $M2$, translate the point $[1, -1]^T$ to the origin, which means the translation vector is $[-1, 1]^T$
3. $M3$, rotate by $45°$
4. $M4$, translate back by $[1, -1]^T$

$M1 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$M2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

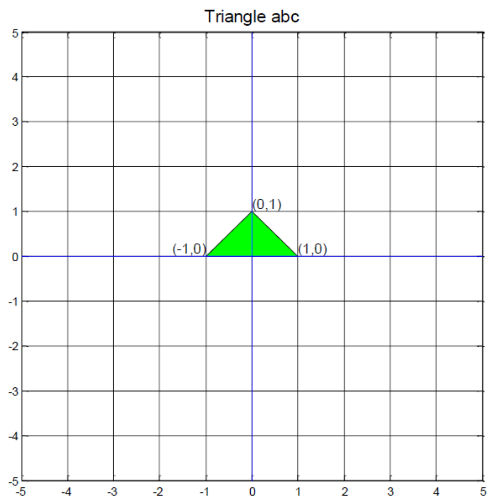
$M3 = \begin{bmatrix} cos(45°) & -sin(45°) & 0 \\ sin(45°) & cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$M4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

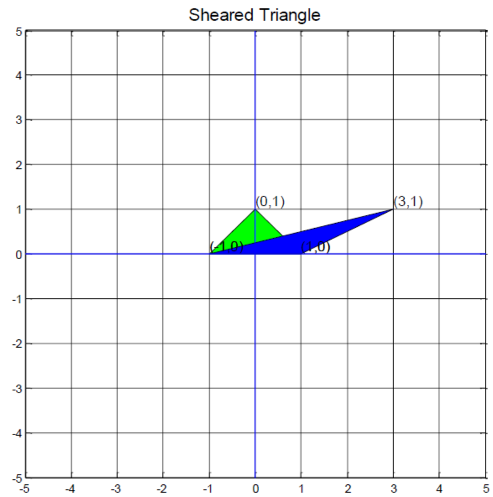
$M = M4 * M3 * M2 * M1$

$M = \begin{bmatrix} 0.707 & 1.414 & -0.414 \\ 0.707 & 2.828 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

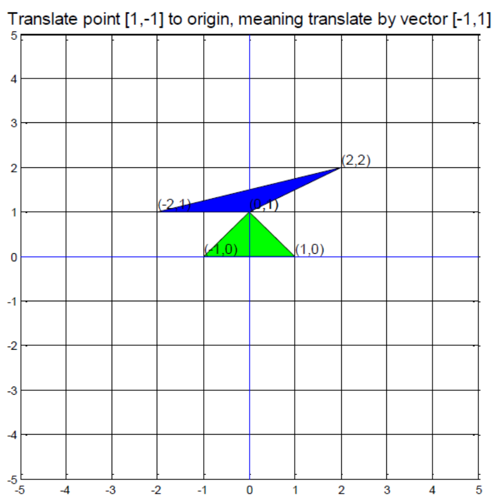
Area of Triangle:
Before the transformations the base of the triangle is equal to 2 (distance between -1 and 1 along the x-axis), while the height is equal to 1 (distance from 0 to 1 along the y-axis).
By shearing along the x-axis, the height of the triangle is maintained since no values along the y-axis are changed. The base of the triangle is also unchanged, since the values of the points lying on the x-axis remain as they were after the shearing. In addition, the translation and rotation operations do not affect the dimensions of the triangle. Therefore, the area of the triangle is calculated as follows:
$A = \frac{1}{2} \times base \times height$
$A = \frac{1}{2} \times 2 \times 1 = 1$


Triangle abc
(0,1)
(-1,0)
(1,0)

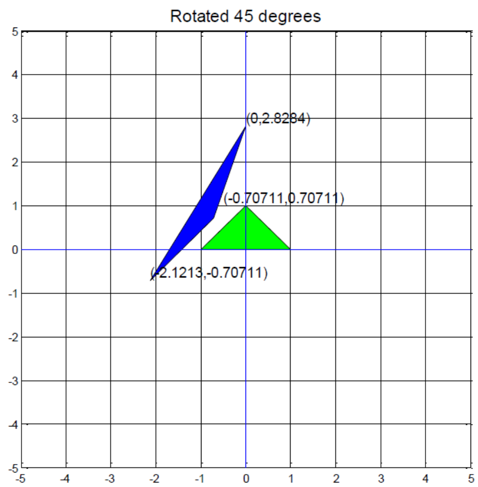
(a)


Sheared Triangle
(0,1)
(3,1)

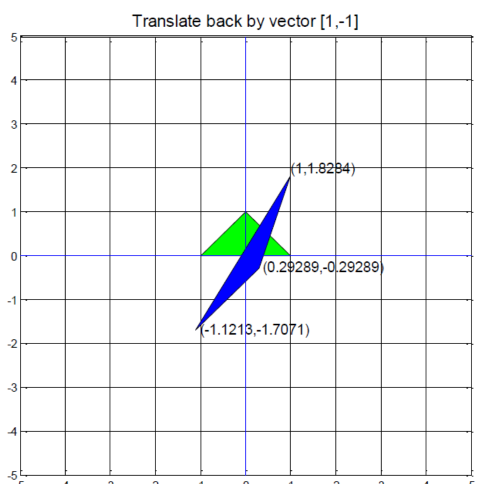
(b)


Translate point [1,-1] to origin, meaning translate by vector [-1,1]
(2,2)
(1,0)

(c)


Rotated 45 degrees
(0,2.8284)
(-0.70711,0.70711)

(d)


Translate back by vector [1,-1]
(1,1.8284)
(0.29289,-0.29289)
(-1.1213,-1.7071)

(e)

Q 11: Construct one possible sequence of primitive transformation matrices in homogeneous coordinates that maps the following shape $ABCD$ into the new shape $A'B'C'D'$. Compute a single composite transformation matrix that combines the sequence you found.

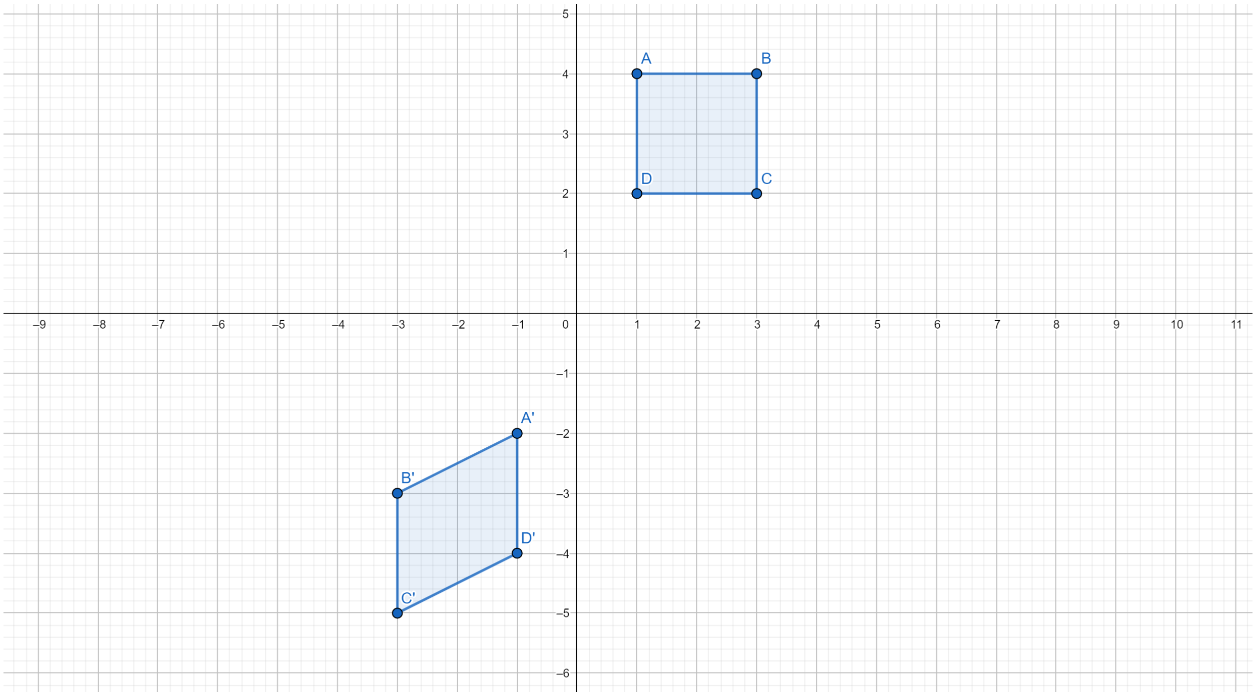


Figure 1: Transformation

Solution: To calculate the shearing factor, use a point before shearing to be $[x_1, y_1]$ and the corresponding point after shearing to be $[x_2, y_2]$. An edge from the object must coincide with one of the axes.

$[x_1, y_1] = [2, 4]$ (Point B before translating the shape, AD coincides with the y-axis)

$[x_2, y_2] = [2, 3]$ (Point B after shearing the shape, $A'D'$ coincides with the y-axis)

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

$y_2 = sh_y * x_1 + y_1$

$3 = sh_y * 2 + 4$

$sh_y = -0.5$

First Solution:

Shearing before reflection: Shearing factor $= -0.5$

1. Translate by $[-1, 0]^T$
   $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Shear in y by -0.5
   $\begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3. Reflect about the y-axis
   $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4. Translate by $[-1, -6]^T$
   $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

$M = M4 * M3 * M2 * M1$

$$M = \begin{bmatrix} -1 & 0 & 0 \\ -0.5 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Second Solution:

1. Translate by $[-1, -2]^T$
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$
2. Shear in y by -0.5
$$\begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
3. Reflect about the y-axis
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
4. Translate by $[-1, -4]^T$
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$M = M4 * M3 * M2 * M1$

$$M = \begin{bmatrix} -1 & 0 & 0 \\ -0.5 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Third Solution:

Reflection before shearing: Shearing factor $= +0.5$

1. Translate by $[-1, 0]^T$
   $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Reflect about the y-axis
   $$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Shear in y by $+0.5$
   $$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Translate by $[-1, -6]^T$
   $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$M = M4 * M3 * M2 * M1$

$$M = \begin{bmatrix} -1 & 0 & 0 \\ -0.5 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Fourth Solution:

1. Translate by $[-1,-6]^T$
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
2. Reflect about the y-axis
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
3. Shear in y by $+0.5$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
4. Translate by $[-1,0]^T$
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$M = M4 * M3 * M2 * M1$

$$M = \begin{bmatrix} -1 & 0 & 0 \\ -0.5 & 1 & -5.5 \\ 0 & 0 & 1 \end{bmatrix}$$