German University in Cairo - GUC
Faculty of Media Engineering and Technology - MET
Department of Digital Media Engineering and Technology
Assoc. Prof. Dr.- Rimon Elias

Monday, December 23rd, 2020

DMET 502 Computer Graphics

Winter Semester 2020/2021

Midterm Exam (Version II)

Barcode

Major: Pick one

☐ DMET
☐ CSEN

Instructions: **Read Carefully Before Proceeding.**

1- Non-programmable calculators are allowed.
2- This is a **closed book exam**.
3- Write your solutions in the space provided.
4- The exam consists of **(4) questions.**
5- This exam booklet contains **(9) pages** including this page. The last two pages are a formula sheet. **Keep them attached**.
6- Total time allowed for this exam is **(120) minutes.**
7- When you are told that time is up, stop working on the test.

Good Luck!

Don't write anything below ;-)

Question	1	2	3	4	∑
Possible Marks	30	31	9	17	87
Final Marks					

Question 1 [2D Graphics]:

a) [14 marks] In Cohen-Sutherland Algorithm (listed in the formula sheet) for line clipping, if the clip polygon used is a pentagon instead of a rectangle, determine the minimum length of each outcode (i.e., the number of binary digits). Determine all the outcodes in this case.

Answers to Question 1 a):

In the original algorithm, the clip polygon was a rectangle; thus, the minimal space required corresponds to the number of borders (i.e., 4 bits).

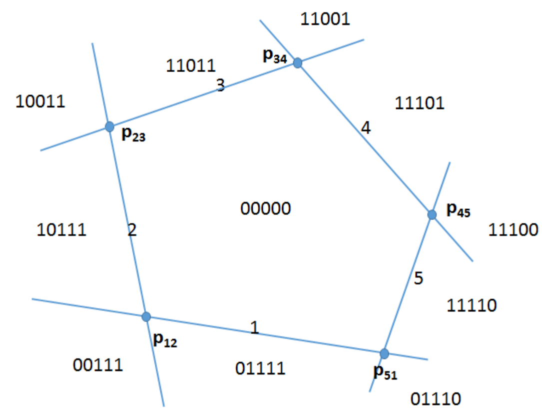
Notice that the number of regions = 4 (number of borders) * 2 (sides per border) + 1 (the clip rectangle) = 9 regions.

Applying the same idea to the pentagon, the minimal space required corresponds to the number of borders (i.e., 5 bits). [3 marks]

The number of regions = 5 (number of borders) * 2 (sides per border) + 1 (the clip region) = 11 regions.

Consider the figure and the outcodes. The borders of the pentagon are numbered. An outcode consists of 5 binary digits.

From left to right, the binary digits corresponds to border 1, border 2, …, border 5. Bit = 1 in front of the corresponding border and 0 otherwise. [1 mark each outcode = 11 marks]



b) [9 marks] Propose an algorithm to generate these outcodes.

Answers to Question 1 b):

1. Determine the linear equations for each of the sides.
2. Determine the vertices by intersecting lines; $\mathbf{p}_{12}$, $\mathbf{p}_{23}$, $\mathbf{p}_{34}$, $\mathbf{p}_{45}$, $\mathbf{p}_{51}$
3. Outcode = 00000

4. Apply vertex $\mathbf{p}_{23}$ to adjacent border 1. For each region on the same side of border 1 as $\mathbf{p}_{23}$. Outcode OR 10000
5. Apply vertex $\mathbf{p}_{34}$ to adjacent border 2. For each region on the same side of border 2 as $\mathbf{p}_{34}$. Outcode OR 01000
6. Apply vertex $\mathbf{p}_{45}$ to adjacent border 3. For each region on the same side of border 3 as $\mathbf{p}_{45}$. Outcode OR 00100
7. Apply vertex $\mathbf{p}_{51}$ to adjacent border 4. For each region on the same side of border 4 as $\mathbf{p}_{51}$. Outcode OR 00010
8. Apply vertex $\mathbf{p}_{12}$ to adjacent border 5. For each region on the same side of border 5 as $\mathbf{p}_{12}$. Outcode OR 00001

9. Return outcode

[1 mark each step = 9 marks]

c) [7 marks] Modify the Cohen-Sutherland Algorithm to work with a clip pentagon.

Answers to Question 1 c):

1. Determine outcode for each endpoint. [1 mark]
2. Dealing with the two outcodes of a border: [1 mark]
    a. Bitwise-OR the bits. If this results in 00000, trivially accept. [1 mark]
    b. Otherwise, if both outcodes are equal, trivially reject. . [2 marks]
    c. Otherwise, segment the line. The outpoint is replaced by the intersection point. Go to Step 2. [1 mark]
3. If trivially accepted, draw the line. [1 mark]

Any other logical alternative can be considered

Question 2 [2D Transformations]:

a) [17 marks] A triangle is to be reflected about an axis inclined at an angle θ with respect to the x-axis and intersecting the y-axis at $[0, y_0]^T$. Prove that the homogeneous transformation matrix is:

$$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) & -y_0\sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) & y_0(\cos(2\theta)+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Answers to Question 2 a):

Steps: [10 marks]

1. Translate the triangle such that the point of intersection between the axis of reflection and the y-axis is moved to the origin
2. Rotate the triangle through an angle $-\theta$ such that the axis of reflection coincides with the x-axis.
3. Reflect the triangle about the x-axis.
4. Rotate back through an angle θ.
5. Translate back using the same translation vector of Step 1 but along the opposite

$$\mathtt{M}_1 = \mathtt{T}([0, -y_0]^T) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathtt{M}_2 = \mathtt{R}(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathtt{M}_4 = \mathtt{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathtt{M}_3 = \mathtt{Ref}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathtt{M}_5 = \mathtt{T}([0, y_0]^T) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Matrices [5 marks]

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta).$$
Refer to Slide 8 (2D Transformations). (In the Formula Sheet)

$$\mathtt{M} = \mathtt{M}_5\mathtt{M}_4\mathtt{M}_3\mathtt{M}_2\mathtt{M}_1$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ldots$$

$$\ldots \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2\theta) & \sin(2\theta) & -y_0\sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) & y_0(\cos(2\theta)+1) \\ 0 & 0 & 1 \end{bmatrix}.$$

Order of multiplication [1 mark]
Final matrix [1 mark]

b) [14 marks] In the previous question, if the same axis is expressed using its slope m rather than its inclination angle θ, re-express the transformation matrix above in terms of m instead of θ.

Answers to Question 2 b):

We know that

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$ Refer to Slide 8 (2D Transformations). (In the Formula Sheet)

Hence, the transformation matrix can be expressed as [4 marks]

$$\mathtt{M} = \begin{bmatrix} \cos^2(\theta) - \sin^2(\theta) & 2\sin(\theta)\cos(\theta) & -2y_0\sin(\theta)\cos(\theta) \\ 2\sin(\theta)\cos(\theta) & -\cos^2(\theta) + \sin^2(\theta) & y_0(\cos^2(\theta) - \sin^2(\theta) + 1) \\ 0 & 0 & 1 \end{bmatrix}$$

Since the slope $m = \tan(\theta)$; [2 marks]

hence, $\sin(\theta) = m/\text{sqrt}(m^2+1)$ [3 marks]

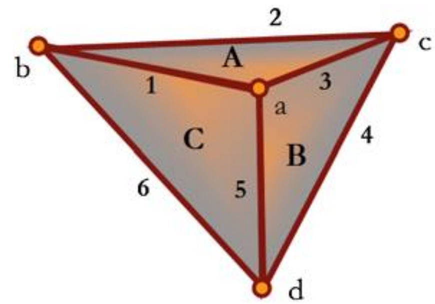
and $\cos(\theta) = 1/\text{sqrt}(m^2+1)$ [3 marks]

Substituting the values of $\sin(\theta)$ and $\cos(\theta)$ in the previous matrix, we get

$$\mathtt{M} = \begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} & \frac{-2y_0 m}{m^2+1} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & \frac{2y_0}{m^2+1} \\ 0 & 0 & 1 \end{bmatrix}$$ [2 marks]

Question 3 [Solids]:

[9 marks] Consider the tetrahedron shown where the vertices are indicated by lowercase letters (i.e., "a," "b," "c," . . .), the faces by uppercase letters (i.e., "A," "B," "C," . . .) and the edges by digits (i.e., "1," "2," "3," . . .). Considering polygonal modeling, determine the adjacency lists for face "A," edge "1" and vertex "a."



Answers to Question 3:

[3 marks * 3]

Example 4.11

Entity	Faces	Edges	Vertices
Face A	B, C, D	3, 1, 2	a, b, c
Edge 1	A, C	2, 3, 5, 6	b, a
Vertex a	B, C, A	3, 5, 1	c, d, b

Question 4 [3D Transformations]:

[17 marks] We want to rotate a point through an angle of 34.87° about a line passing through the origin and the point $[1, 1, \sqrt{3}]^T$. Determine transformation matrix required. You **must start** with rotation about the *z*-axis. Also, you **must avoid** rotating about the *y*-axis.

Answers to Question 4:

Prob 5.4

Steps:

1. Rotate through 45° about the *z*-axis [Step 1 mark; parameter = 1 mark]
2. Rotate through $\tan^{-1}(\text{sqrt}(3)/\text{sqrt}(2))^\circ$ (90-50.768=39.23) about the *x*-axis[Step 1 mark; parameter = 2 marks]
3. Rotate through 34.87° about the *z*-axis [Step 1 mark; parameter = 1 mark]1
4. Rotate through -39.23 about the *x*-axis [Step, parameter = 1 mark]
5. Rotate through -45° about the *z*-axis [Step, parameter = 1 mark]

$$M_1 = \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(39.23) & -\sin(39.23) \\ 0 & \sin(39.23) & \cos(39.23) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.7746 & -0.6324 \\ 0 & 0.6324 & 0.7746 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} \cos(34.87) & -\sin(34.87) & 0 \\ \sin(34.87) & \cos(34.87) & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.8205 & -0.5717 & 0 \\ 0.5717 & 0.8205 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(39.23) & \sin(39.23) \\ 0 & -\sin(39.23) & \cos(39.23) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.7746 & 0.6324 \\ 0 & -0.6324 & 0.7746 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} \cos(45) & \sin(45) & 0 \\ -\sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrices [5 marks]

Order [2 marks] $M_5 M_4 M_3 M_2 M_1$

Final Matrix [1 mark]

$$M = \begin{bmatrix} 0.8564 & -0.4069 & 0.3179 \\ 0.4788 & 0.8564 & -0.1935 \\ -0.1935 & 0.3179 & 0.9282 \end{bmatrix}$$

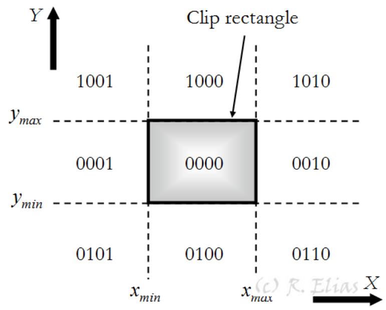
Formula Sheet

Input: x_0, y_0, x_1, y_1

```
steep = |y1 − y0| > |x1 − x0|
if (steep = TRUE) then
    swap (x0, y0)
    swap (x1, y1)
end if

if (x0 > x1) then
    swap (x0, x1)
    swap (y0, y1)
end if

if (y0 > y1) then
    δy = −1
else
    δy = 1
end if

Δx = x1 − x0
Δy = |y1 − y0|
y = y0
error = 0

for (x = x0 to x1) do
    if (steep = TRUE) then
        Plot [y, x]^T
    else
        Plot [x, y]^T
    end if
    error = error + Δy
    if (2 × error ≥ Δx) then
        y = y + δy
        error = error − Δx
    end if
end for
```

end

1. Determine outcode for each endpoint.
2. Dealing with the two outcodes:
   a. Bitwise-OR the bits. If this results in 0000, trivially accept.
   b. Otherwise, bitwise-AND the bits. If this results in a value other than 0000, trivially reject.
   c. Otherwise, segment the line. The outpoint is replaced by the intersection point. Go to Step 2.
3. If trivially accepted, draw the line.



$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1} + \underbrace{\begin{bmatrix} t_x \\ t_y \end{bmatrix}}_{\mathrm{t}}.$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}}_{\mathrm{P}_2} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathrm{T}([t_x,t_y]^T)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}}_{\mathrm{P}_1}$$

$$\begin{aligned} \sin(\alpha+\theta) &= \sin(\alpha)\cos(\theta) + \cos(\alpha)\sin(\theta) \\ \cos(\alpha+\theta) &= \cos(\alpha)\cos(\theta) - \sin(\alpha)\sin(\theta) \end{aligned}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\dot{\mathrm{R}}(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}}_{\dot{\mathrm{S}}(s_x,s_y)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\dot{\mathrm{Ref}}_x} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{\dot{\mathrm{Ref}}_y} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix}}_{\dot{\mathrm{Sh}}_x(sh_x)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathrm{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix}}_{\dot{\mathrm{Sh}}_y(sh_y)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathrm{P}}_1}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -t_x \\ -t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{s_x} & 0 \\ 0 & \frac{1}{s_y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

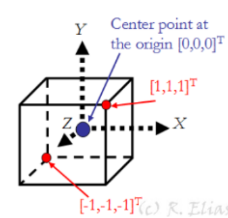
Vertex #	x	y	z

Edge #	Start vertex	End vertex

Edge	Vertices		Faces		Left traverse		Right traverse	
Name	Start	End	Left	Right	Pred	Succ	Pred	Succ

Vertex	edge

Face	edge

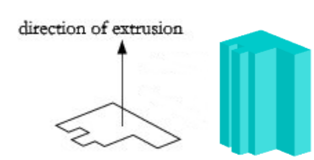


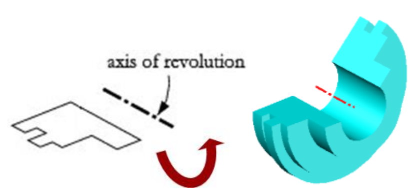
translate(scale(Block, < 1, 1.5, 1.5 >), < 1, 2, 3 >)

Face	Vertices
A	$[x_1, y_1, z_1]^T$, $[x_2, y_2, z_2]^T$, $[x_3, y_3, z_3]^T$
B	$[x_2, y_2, z_2]^T$, $[x_4, y_4, z_4]^T$, $[x_3, y_3, z_3]^T$
⋮	⋮

Vertex	Coordinates
1	$[x_1, y_1, z_1]^T$
2	$[x_2, y_2, z_2]^T$
⋮	⋮

Face	Vertices
A	1, 2, 3
B	2, 4, 3
⋮	⋮





$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1} + \underbrace{\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}}_{\mathbf{t}}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}}_{\mathbf{P}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathtt{T}([t_x, t_y, t_z]^T)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}}_{\mathbf{P}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\dot{\mathtt{R}}_x(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}}_{\dot{\mathtt{R}}_y(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{R}}_z(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}}_{\dot{\mathtt{S}}(s_x, s_y, s_z)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_{xy}} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_{yz}} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_{zx}} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ sh_{xy} & 1 & 0 \\ sh_{xz} & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_x(sh_{xy}, sh_{xz})} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & sh_{yx} & 0 \\ 0 & 1 & 0 \\ 0 & sh_{yz} & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_y(sh_{yx}, sh_{yz})} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & sh_{zx} \\ 0 & 1 & sh_{zy} \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_z(sh_{zx}, sh_{zy})} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$