$$m\ddot{z} + b\dot{z} + kz = F_0 \cos(\omega t) \quad 1//$$

- solution is the real part of:

$$z = \hat{z} e^{i\omega t} \quad \text{where} \quad \hat{z} = z_0 e^{i\phi} \quad 2//$$

assume:

$$\begin{cases} b \equiv m\gamma \ (\text{definition}) \\ \text{and} \\ F \equiv \hat{F} e^{i\omega t} \quad \text{where} \quad \hat{F} = F_0 e^{i(0)} \end{cases} \quad 3//$$

$$\therefore \boxed{\hat{z} = \frac{\hat{F}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}} \quad 4//$$

TRANSFER FUNCTION

then

$$\begin{cases} \hat{z} = \rho e^{i\phi} F_0 \\ z_0 = \rho F_0 \cos(\omega t + \phi) \end{cases} \quad 5//$$

$$\hat{z} = F_0 R \qquad 6//$$

$$R = \frac{1}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$\left\{ \begin{aligned} R &\equiv \rho e^{i\phi} \\ \rho^2 &\equiv R R^* \end{aligned} \right. \qquad 7//$$

$$R = \frac{1}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$R^* = \frac{1}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

$$\therefore \rho^2 = \frac{1}{m^2} \left| \frac{1}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]} \right._{i^2 = -1} \qquad 8//$$

$\therefore \; |z_0|^2 = e^2 F_0^2$ from 5//

modulus of z_0 squared.

$$z_0^2 = \frac{F_0^2}{m^2\left((\omega_0^2-\omega^2)^2+(\gamma\omega)^2\right)}$$

$$\therefore \frac{z_0}{F_0/k} \equiv \frac{1}{\left[\frac{(\omega_0^2-\omega^2)^2}{\omega_0^4}+\frac{(\gamma\omega)^2}{\omega_0^4}\right]^{1/2}}$$

9//

$m = \frac{k}{\omega_0^2}$

$$\boxed{\gamma \equiv \frac{\omega_0}{Q}}$$ from the definition of Q

i.e. $$\boxed{Q = 2\pi \frac{\frac{1}{2}k z_0^2}{b z_0^2 \pi \omega_0}}$$

10//

STANDARD TRANSFER FUNCTION

$$\therefore \boxed{\frac{z_0}{F_0/K} \equiv \frac{1}{\left[(1-r^2)^2 + \left(\frac{r}{Q}\right)^2\right]^{1/2}}}$$

11

$$r \equiv \frac{\omega}{\omega_0}$$

<u>However:</u>

from 4

$$\hat{z} = \frac{F_0}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

IF $\underline{\underline{\omega \approx \omega_0}}$

$$\hat{z} \approx \frac{F_0}{m(2\omega_0(\omega_0 - \omega) + i\gamma\omega_0}$$

12

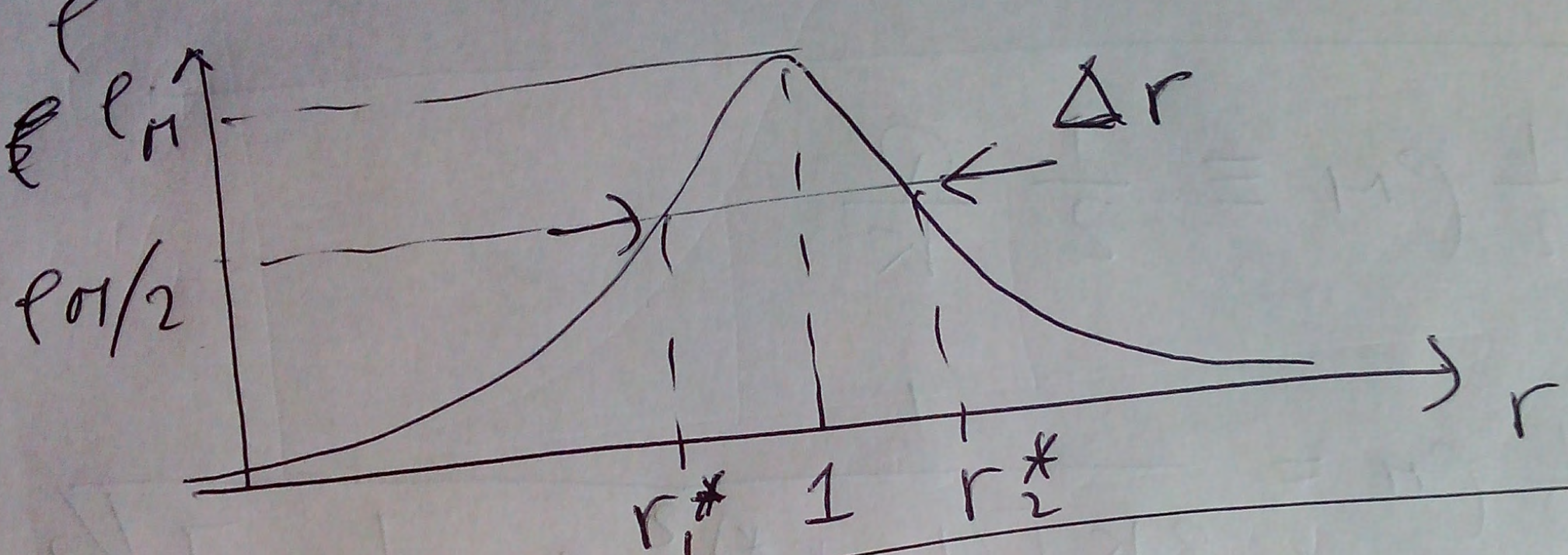
$$\boxed{z_0^2 \cong \rho^2 F_0^2}$$

$$\rho^2 \simeq \frac{1}{m^2 (2\omega_0)^2 \left[(\omega_0 - \omega) + i\frac{\gamma}{2}\right]\left[(\omega_0 - \omega) - i\frac{\gamma}{2}\right]}$$

$$\boxed{\rho^2 \cong \frac{1}{m^2 4\omega_0^2 \left((\omega_0 - \omega)^2 + \frac{\gamma^2}{4}\right)}}$$

13//

$$\rho \cong \frac{1}{2\frac{k}{\omega_0}\left[(\omega_0-\omega)^2+\frac{\gamma^2}{4}\right]^{1/2}}$$ 14



$$\boxed{\rho \simeq \frac{1}{2k\left[(1-r)^2+\frac{1}{4Q^2}\right]^{1/2}}}$$ 15

$$r \equiv \frac{\omega}{\omega_0}$$

$$\boxed{z_0 \simeq \rho F_0}$$ 16

$$r = 1 \Rightarrow \rho_M \simeq \frac{1}{2K\left[\frac{1}{4Q^2}\right]^{1/2}}$$

$$\rho_M \simeq \frac{\cancel{2}\,Q}{2K\,[\,1\,]} = \boxed{\frac{Q}{K}} \quad 17 //$$

$$\frac{1}{2}\rho_M \equiv \frac{1}{2}\frac{Q}{K}$$

$$\frac{1}{2}\rho_M = \frac{1}{2K\left[(1-r^*)^2 + \frac{1}{4Q^2}\right]^{1/2}}$$

$$\frac{1}{2}\frac{Q}{K} = \frac{1}{2K\left[\,(1-r^*)^2 + \frac{1}{4Q^2}\right]^{1/2}}$$

$$r^*_{1/2} \simeq 1 \pm \sqrt{3/4}\;Q$$

$$r^*_2 - r^*_1 \simeq \sqrt{3}\;Q \equiv \Delta r \quad 18 //$$

Finally

$$\Delta r \equiv \frac{\Delta\omega}{\omega_0} \approx \sqrt{3}\, Q$$

$$\gamma = \frac{\omega_0}{Q}$$

$$\Rightarrow \boxed{\gamma = \frac{\omega_0}{\frac{\Delta\omega}{\omega_0} \cdot \sqrt{3}} \approx \frac{\omega_0^2}{\sqrt{3}\,\Delta\omega}}$$