# $\boxed{\text{Power of an oscillator}}$

$$P = \frac{1}{2} k A_o^2$$

$$\Rightarrow P \propto A_o^2$$

---

here $A_o \equiv \rho$

$$\Rightarrow P \propto \rho^2 \Rightarrow \text{where } \boxed{r = \frac{\omega}{\omega_o}}$$

$$\Downarrow$$

$$\rho^2 = \frac{1}{k^2 [(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2] \cdot \frac{1}{\omega_o^4}}$$

$$= \frac{1}{k^2 [(1 - r^2)^2 + (\frac{r}{Q})^2]}$$

**BUT**

$$\rho^2 \approx \frac{1}{4k^2 [(1 - r)^2 + (\frac{\gamma}{2})^2]}$$

$$\rho^2 = \frac{1}{\underbrace{4m^2\omega_0^2}_{a}\left[(\omega_0-\omega)^2 + \underbrace{\frac{\gamma^2}{4}}_{b}\right]}$$

P

$$\rho^2 = \frac{1}{a\left[(\omega_0-\omega)^2 + b\right]}$$

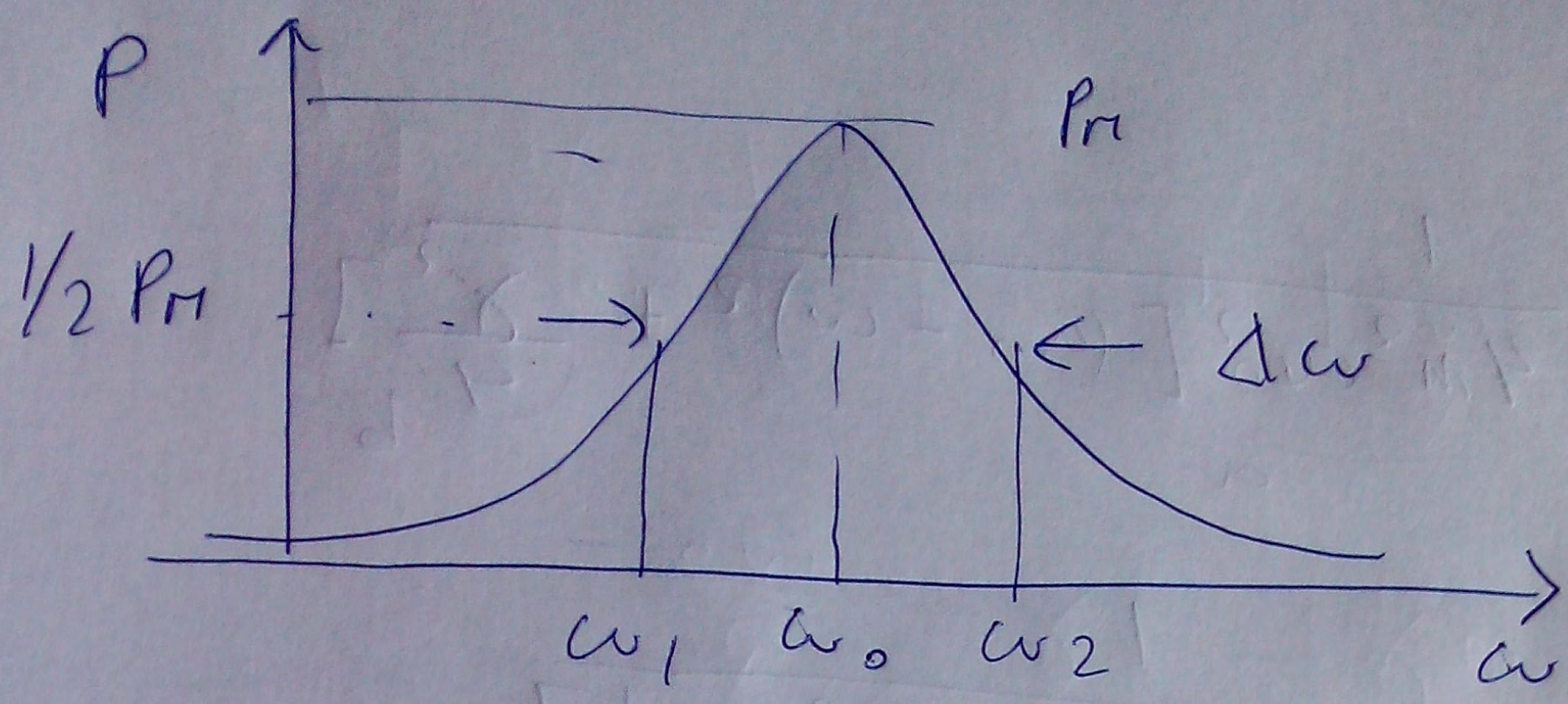
$$\frac{dP}{dw} = \left[\frac{1}{a}\left([\omega_0-\omega]^2+b\right)^{-1}\right]'$$

$$= +\frac{1}{a}\left[[\omega_0-\omega)^2+b\right]^{-2} 2(\omega_0-\omega)$$

$$\frac{1}{a}\left[(\omega_0-\omega)^2+b\right]^{-2} 2(\omega_0-\omega)$$

$$= \boxed{\frac{2(\omega_0-\omega)}{a\left[(\omega_0-\omega)^2+b\right]^2}} = \frac{dP}{dw}$$

$$\frac{dP}{dw} = 0 \Rightarrow \boxed{\omega = \omega_0}$$ resonance when $\gamma$ is small




$$P_M \propto \left. \frac{1}{4m^2\omega_0^2\left[(\omega_0-\omega)^2+\frac{\gamma^2}{4}\right.} \right|_{\omega=\omega_0}$$

$$P_M \equiv \left. \frac{1}{\cancel{4}m^2\omega_0^2\,\frac{\gamma^2}{\cancel{4}}} \right|_{m=\frac{k}{\omega_0^2}} = \frac{1}{\frac{k^2\gamma^2}{\omega_0^2}}$$

$$\boxed{P_M = \left(\frac{\omega_0}{k\gamma}\right)^2}$$

$$\frac{1}{2} P_M = \frac{1}{4 \frac{k^2}{\omega_0^4} \omega_0^2 \left[ (\omega_0 - \omega)^2 + \frac{\gamma^2}{4} \right]}$$

$$\boxed{\frac{1}{2} P_M = \frac{\omega_0^2}{4 k^2 \left[ (\omega_0 - \omega)^2 + \frac{\gamma^2}{4} \right]}}$$

$$\Rightarrow \frac{\cancel{\omega_0^2}}{\cancel{k^2} \gamma^2} \cdot \frac{1}{2} = \frac{\cancel{\omega_0^2}}{4 \cancel{k^2} \left[ (\omega_0 - \omega)^2 + \frac{\gamma^2}{4} \right]}$$

$$\frac{1}{\gamma^2} = \frac{1}{2 \left[ (\omega_0 - \omega)^2 + \frac{\gamma^2}{4} \right]}$$

$$\left[ (\omega_0 - \omega)^2 + \frac{\gamma^2}{4} \right] = \frac{\gamma^2}{2}$$

$$(\omega_0 - \omega)^2 = \left( \frac{\gamma}{2} \right)^2$$

$$\Rightarrow (\omega_0 - \omega) = \pm \frac{\gamma}{2}$$

$$\boxed{\omega = \omega_0 \pm \frac{\gamma}{2}}$$

$$\omega_1 = \omega_0 - \frac{\gamma}{2}$$

$$\omega_2 = \omega_0 + \frac{\gamma}{2}$$

$$\boxed{\omega_2 - \omega_1 = \gamma \equiv \Delta\omega}$$

Correct result:

$\gamma \approx \Delta\omega$ for small $\gamma$ and $\omega_r \approx \omega_0$

$$\gamma = \frac{\omega_0}{Q} \Rightarrow \Delta\omega = \frac{\omega_0}{Q}$$

$$\therefore \boxed{\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}}$$

when plotting "power" versus frequency!