ECON 634 Problem Set 2 Answer

Ruohao Zhang

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1. The functional equation is:

$$V(K_t, A_t) = \max_{K_{t+1}} \frac{(A_t K_t^{\alpha} + (1-\delta) K_t - K_{t+1})^{1-\sigma}}{1-\sigma} + \beta E[V(K_{t+1}, A_{t+1})] \tag{1}$$

State variables are K_t and A_t, control variable is $K_{t+1} \in [0, A_t K_t^{\alpha} + (1-\delta)k_t]$.

2. From the value function, can get Euler equation:

$$U^{'}(C_t) = \beta E \left\{ U^{'}(C_{t+1})[\alpha A_{t+1} K_{t+1}^{\alpha-1} + (1-\delta)] \right\} \tag{2}$$

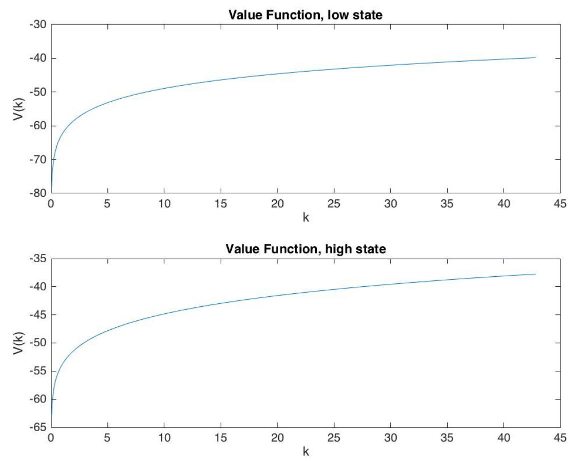
The possible maximum K is using the "steady state K" if all states are A^H and assume no consumption. That "steady state K" is:

$$K^* = (\frac{\alpha A^H}{\frac{1}{\beta} + \delta - 1})^{\frac{1}{1-\alpha}} \tag{3}$$

Then the possible maximum K, denoted by $\bar{K}$ is:

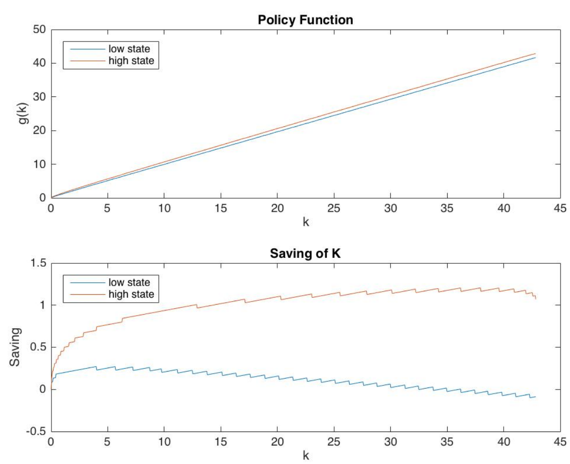
$$\bar{K} = A^H (\frac{\alpha A^H}{\frac{1}{\beta} + \delta - 1})^{\frac{\alpha}{1-\alpha}} + (1-\delta)(\frac{\alpha A^H}{\frac{1}{\beta} + \delta - 1})^{\frac{1}{1-\alpha}} \tag{4}$$

Using MATLAB to solve the value function, can get the plot of value function for both states:



The plot shows that for both states, value function is always increasing and concave.

3. Plot the policy function over K for each state of A and savings over K for each A:



The graph shows that policy function is increasing in both K and A, saving is increasing in A, but not in K.

4. After simulation, I find that $A^l = 0.995$, $A^h = 1.0016$ will result a standard deviation less than 1.8%. I generate two simulated A_t series, started with low states and high states respectively. The started capital level is the steady state capital calculated from the policy functions of two states. The numerical result shows that these two steady state capital level is very close to each other ($K^{ss,l} = 33.1$, $K^{ss,h} = 34.2$). Based on the transition matrix, the state is very likely to keep. Therefore, I assume that the economy is very easy to stay on the steady state. The result shows that for both series, given

$A^l = 0.995$, $A^h = 1.0016$, the standard deviation is around 1%.

5. The code using loops is included in the MATLAB code “loops.m”, and the vectorization code is included in the MATLAB code “VFIdeterministic.m”. By running and timing the program, the result is following (the timing of loop is on the top, and the timing of vectorization on the bottom).

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
loops	1	1117.228 s	1115.588 s	

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
VFIdeterministic	1	26.766 s	25.706 s	

It is obvious that vectorization is much faster than loops in calculation.