HOMEWORK 2 STOCHASTIC VFI REPORT

HUAYAN GENG (HGENG1@BINGHAMTON.EDU)

Q1. Functional Equation

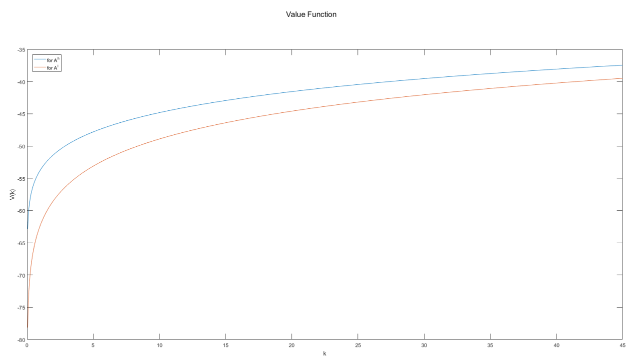
The value function of dynamic programming problem is

$$v(K,\, A) \quad = \quad \max_{0 \le K' \le AK^{\alpha} + (1-\delta)K} \left\{ \frac{(AK^{\alpha} + (1-\delta)K - K')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}[v(K',\, A')|A] \right\}$$

where K_t and A_t are state variables while $K_{t+1} \in [0,\, A_t K_t^{\alpha} + (1-\delta)K_t]$ is control variable.

Q2. Value Function

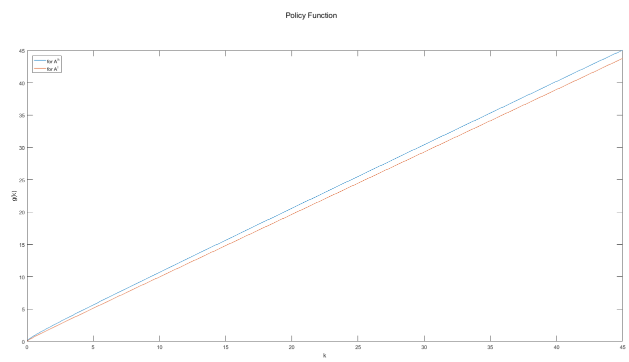
When I solve the model, I generate value function for A^h and A^l separately. Therefore, I got two vectors of value function.



The value function is an increasing and concave function over K for each state of A.

Q3. Policy Function

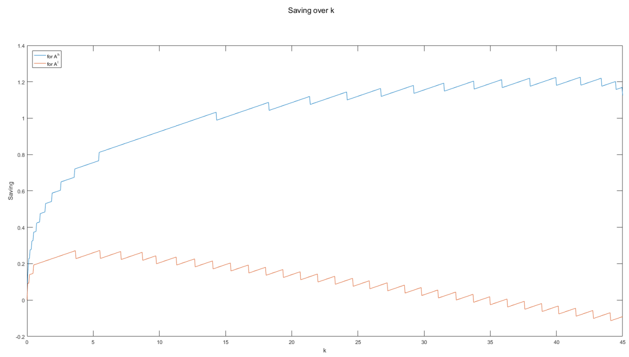
Similarly, in my code, I get two vectors of policy function.



The policy function is increasing in K clearly indication by increasing curve. Also, it is increasing in A because by holding K, policy function for A^h is always higher than that for A^l.

Date: 09/22/2017.

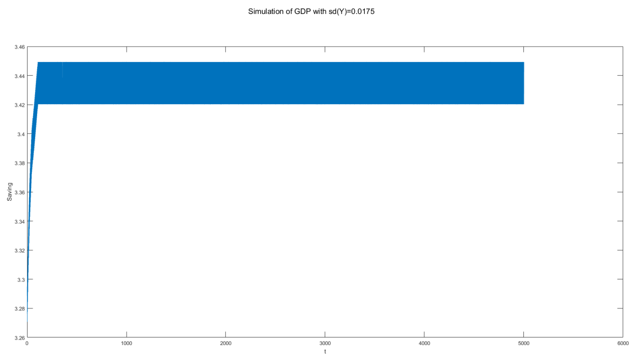
Finally, savings over K for each A are increasing in both K and A.



Q4. Simulation of A

Based on Markov probability matrix Π, the long run probability for A^h and A^l can be generated by $\bar{\Pi} = \Pi^{1000}$, i.e. $\bar{\Pi} = \begin{bmatrix} 0.7629 & 0.2371 \\ 0.7629 & 0.2371 \end{bmatrix}$. Therefore $\begin{bmatrix} \bar{\pi}^h & \bar{\pi}^l \end{bmatrix} = \begin{bmatrix} 0.7629 & 0.2371 \end{bmatrix}$. By adjusting A^h, I can get pair of $A = \begin{bmatrix} A^h & A^l \end{bmatrix}$ satisfying $A^h\bar{\pi}^h + A^l\bar{\pi}^l = 1$ to do VFI to find policy function $K_{t+1} = g(A_t, K_t)$ given A. Then, I randomly drawing the sequence of random numbers from uniform distribution and use it to construct a 5000 periods sequence of $\{A_t\}_0^{5000}$ where $A_0 = A^h$ (it could be A^las well) according to the transition probabilities. At the same time I generate sequence of $\{K_{t+1}\}_0^{5000}$ where $K_0 = 30$ (as well as $\{K_t\}$) from policy function above. As defined, $Y_t = A_t(s_t)^\alpha K_t$. Finally, I adjust value of A^h to appropriate value satisfying $\sigma(Y) < 1.8\%$.

My result is $A^h = 1.0020$ and $A^l = 0.9936$ and corresponding $\sigma(Y) = 17.5\%$.



Extra Credit: Time Efficiency of Vectorization

I choose to solve same problem by using loop. Here, I apply the same idea but change element product into loop.

	Vectorization	Loop
Time	23.6378s	135.3539

Vectorization is much more efficient than loop.