Homework 2

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Question 1

Functional equation:

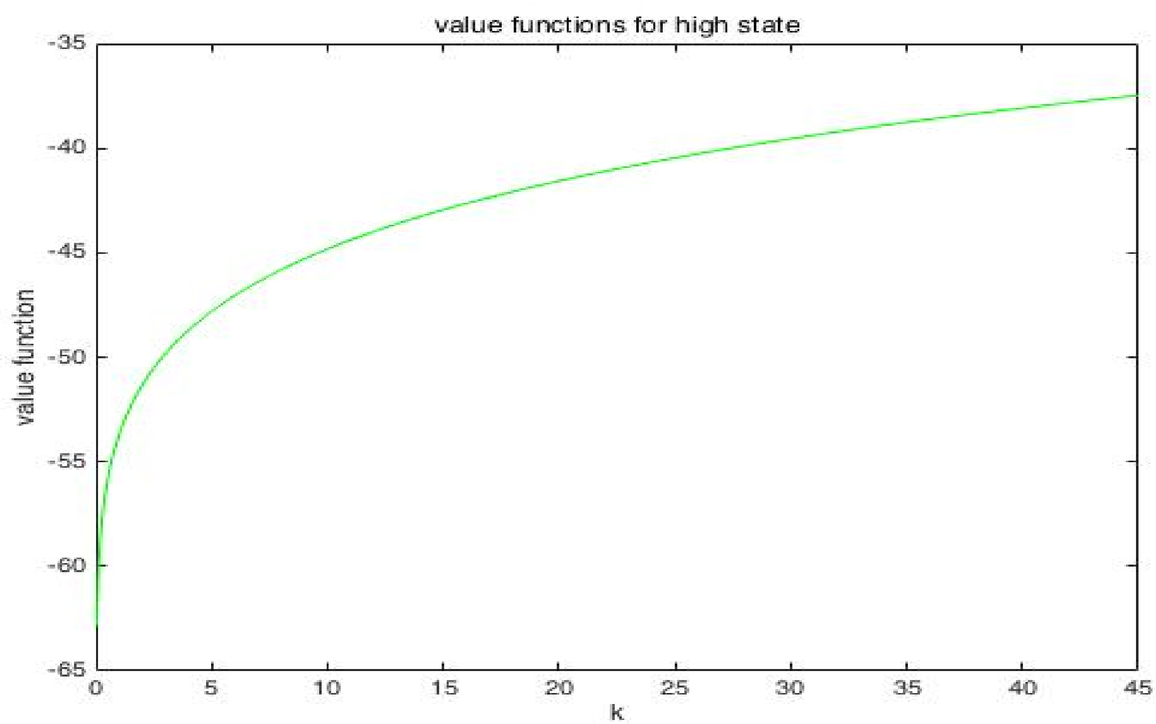
$(A_t(S^t)K_t^\alpha + (1-\delta)K_t - K_{t+1})^{-\sigma} = \beta \sum_{S^{t+1} \in S^{t+1\prime}} \pi(S^{t+1}|S^t)(A_{t+1}(S^{t+1})K_{t+1}^\alpha + (1-\delta)K_{t+1} - K_{t+2})^{-\sigma} \alpha A_{t+1}(S_{t+1})K_{t+1}^{\alpha-1}$

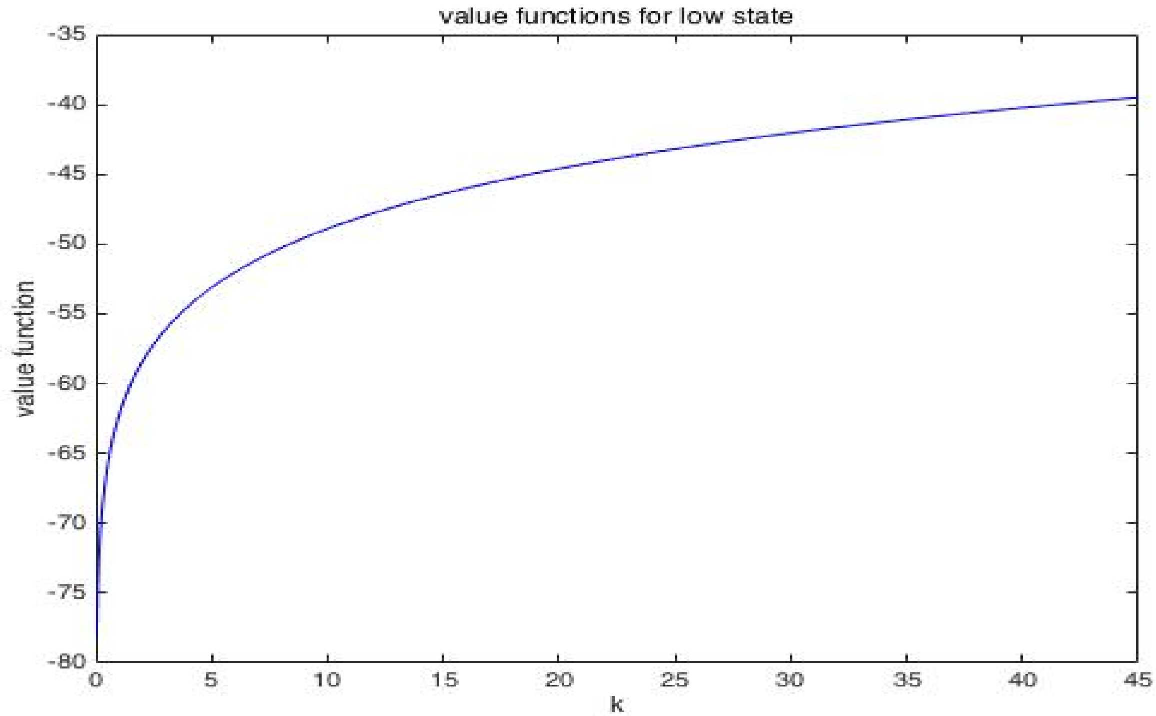
State Variables: K_t

Control Variables: K_{t+1}, C_t. Or we can use K_{t+1} to represent C_t, such that the only control variable is K_{t+1}.

Question 2

I use Matlab to solve this question. I try to solve it by writing down two functions, one for high productivity state and one for low productivity state. However, the figures look very weird, yaxis is from 10^{16} to 1.4×10^{16}. Then I find out the problem, it's because I forgot to use dot product symbol when calculating return. 😂
Below are the revised outcomes:

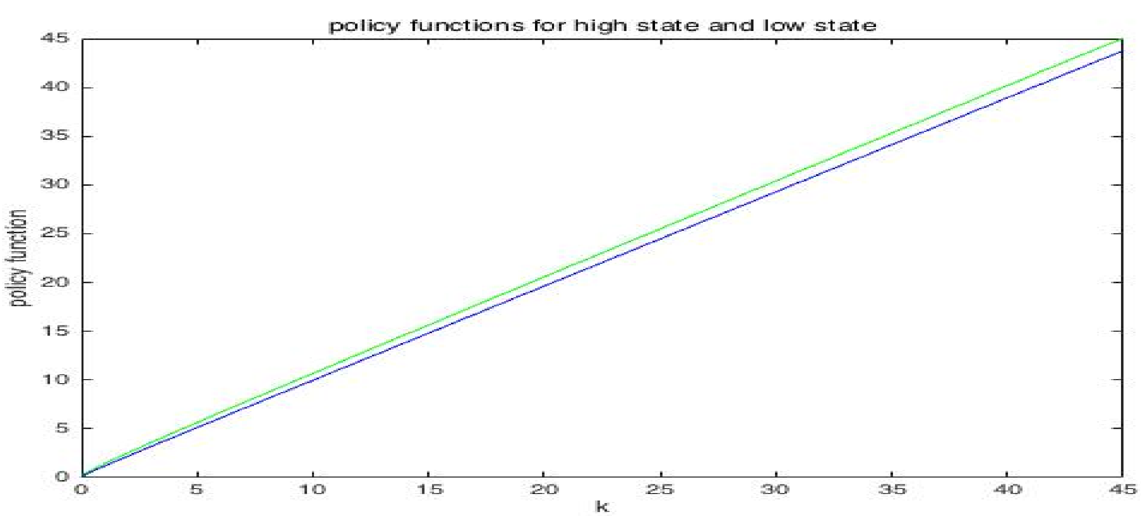




As we can see from the figure, value functions are increasing and concave. Also, value of value function in high state is bigger, which means value functions are increasing on $\mathcal{A}$ too.

Question 3

Below is the figure for policy functions, where the green line is for high productivity state and the blue line is for the low productivity state:



As we can see from the figure above, the policy functions are increasing on both $\mathcal{K}$ and $\mathcal{A}$.

Question 4

I write down the transition matrix and then calculate the invariant distribution using Matlab. Then I get the probabilities for different states in each period, by using Markov chain.

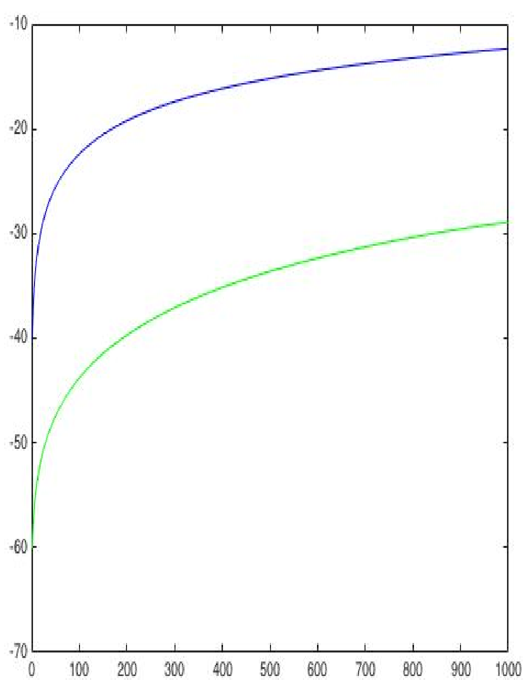
After that, I generate a random sequence r, and use it to compare with probabolities in each period. If the probability of high state in this period is greater than r, then I set $\mathcal{A}$ equals to $\mathcal{A}_h$, otherwise I set $\mathcal{A}$ equals to $\mathcal{A}_l$.

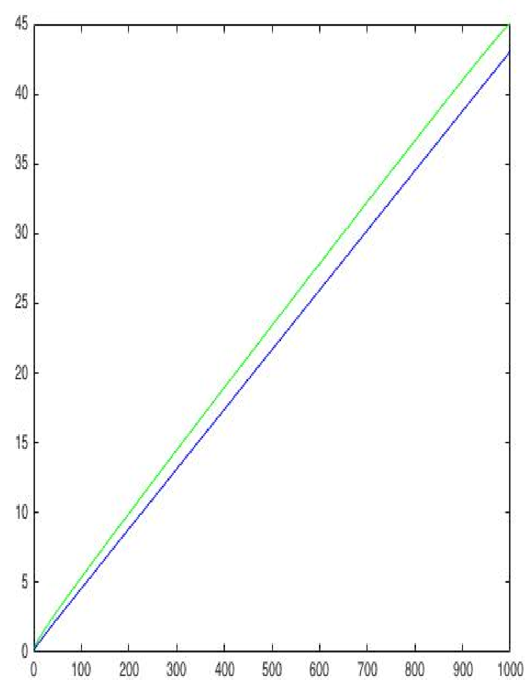
Using the above technology shocks to calculate output, and get the standard deviation of output for $\mathcal{A}_h = 1.1$ is 0.9126, which is way too bigger than our desired value 0.018. I try to set different values for $\mathcal{A}_h$, and the minimum for std is 0.076 I can get. Somehing is not right...

Please see appendix for Matlab code.

Question 5

I use one equation instead of two to represent the two-state value function. For this quesion, it takes 8 seconds for Matlab to finish, while for question 2 it takes 38 seconds. Get results as below, where green line is for high state and blue line is for low state in both graphs:





Appendix

Steps for VFI

1. Deciede on a grid, $\mathscr{K}$, the minimun value I set is 0.2, and the maximun value for k I set is 5. Also there are 20 numbers in k grid $\mathscr{K}$.
2. Technology shocks are A_h and A_l, together with transition matrix $\prod = \pi_{ij}$.
3. For each $k_i \in \mathscr{K}$ and ℓ equals l and h, compute the value funcitons.
4. Use a loop, which stops when absolute value between V_{i+1} and V_i is less than ϵ.

Matlab code

1. Code for Q2 & Q3

```
close all
clear all
%%%%%%%%parameters%%%%%%%%%
delta=0.025;                 %depreciation rate
alpha=0.35;                  %capital share of production
beta=0.99;                   %discount rate
sigma=2;                     %utility function parameter
pi=[0.977 0.023;0.074 0.926];%transition matrix

%%%%%%%%k grid%%%%%%%%%%%%%
ah = 1.1;                    %high productivity
al=0.678;                    %low productivity
knum=1000;                   %set number of k equals 1000
kmin=0 ;                     %set k min equals 0
%?*syms kmax
%?eqn=kmax==ah*kmax^alpha+(1-delta)*kmax
%?kmax=solve(eqn,kmax)        %calculate the k max[tooooo big
%kmax = 1.1*(alpha*ah/(1/beta-1+delta))^(alpha/(1-alpha))...
%+(1-delta)*(alpha*1.1/(1/beta-1+delta))^(1/(1-alpha));
kmax=45;
k=linspace(kmin,kmax,knum);  %decide on k grid
kmat=repmat(k',[1 knum]);    %this is a matrix which will be useful later

%%%%%%%%variables%%%%%%%%%%
conh=ah*kmat.^alpha+(1-delta)*kmat-kmat';
conl=al*kmat.^alpha+(1-delta)*kmat-kmat';
reth=conh.^(1-sigma)/(1-sigma);
retl=conl.^(1-sigma)/(1-sigma);
```

```
reth(conh < 0)=-Inf;
retl(conl < 0)=-Inf;


%%%%%%%%Iteration%%%%%%%%%%%
dis = 1; tol = 1e-06;        % tolerance for stopping
v_guess = zeros(2, knum);   %?????
while dis > tol

vh_mat = reth + beta *(pi(1,1)* repmat(v_guess(1,:), [knum 1])...
+pi(1,2)*repmat(v_guess(2,:), [knum 1]));
vl_mat = retl + beta *pi(2,2)* repmat(v_guess(2,:), [knum 1])+...
beta *pi(2,1)* repmat(v_guess(1,:), [knum 1]);

[vfnh, ph_indxh] = max(vh_mat, [], 2);
vfnh = vfnh';
[vfnl, pl_indxl] = max(vl_mat, [], 2);
vfnl = vfnl';


dis =[max(abs(vfnl-v_guess(2,:)));max(abs(vfnh - v_guess(1,:)))]    %?????
v_guess =[vfnh;vfnl];
%?????

end

gh = k(ph_indxh);
gl = k(pl_indxl);

figure
plot(k,vfnh,'g');
xlabel('k');
ylabel('value function');
title('value functions for high state');

figure
plot (k,vfnl,'b');
xlabel('k');
ylabel('value function');
title('value functions for low state');


figure
plot (k, gh,'g');
xlabel('k');
```

```
ylabel('policy function');
title('policy functions for high state and low state') ;
hold on;
plot(k , gl,'b');
hold off;
```

2. Code for Q4

```
close all;
clear all;

%%%%%%%%parameters%%%%%%%%%%
delta=0.025;                  %depreciation rate
alpha=0.35;                   %capital share of production
beta=0.99;                    %discount rate
sigma=2;                      %utility function parameter
pi=[0.977 0.023;0.074 0.926];%transition matrix
pi_s=pi^1000;                 %invariant distribution
ah=1e-10+1;                   %set a value for high ptoductivity shock
al=(1-ah*pi_s(1))/pi_s(3);    %calculate low prod shock

%%%%%%%%simulation%%%%%%%%%%
nStates = 1000;
initialProbabilityState=[1 0];
states = zeros(nStates,2);
states(1,:) = initialProbabilityState;
for ns = 2:nStates
states(ns,:) =states(ns-1,:)*pi ;
end                           %using markov chain to generate a prob seq

for ns=1:nStates
r=rand();
if states(ns,1)>r;
a(ns)=ah;
else a(ns)=al;
end
end                          %get a sequence of a, length of 1000


%%%%%%%set k grid%%%%%%%%%%
knum=1000;                    %set number of k equals 1000
kmin=0 ;
kmax=45;
k=linspace(kmin,kmax,knum);   %decide on k grid
kmat=repmat(k',[1 knum]);
```

```
%%%%%%%std of output%%%%%%
for ns=1:1000
y(ns)=a(ns)*k(ns).^alpha;
y_std=std(y);
end
```

3. Code for Q5

```
close all
delta=0.025;                %depreciation rate
alpha=0.35;                 %capital share of income
beta=0.99;                  %discount rate
sigma=2;                    %utility parameter


nbk=1000;                   %number of data points int he grid
nba=2;                      %number of value for the shocks
crit=1;                     %convergence criterion
epsi=1e-1;                  %convergence parameter

phh=0.977;                 %this is pi hh
pll=0.926;                 %p ll
PI=[phh 1-phh;1-pll,pll]; %transition matrix
ah=1.1;
al=0.678;
A=[ah al];                 %stochastic matrix


kmin=0.1;
kmax=45;
k=linspace(kmin,kmax,nbk)';% k grid
c=zeros(nbk,nba);
util=zeros(nbk,nba);
v=zeros(nbk,nba);
Tv=zeros(nbk,nba);


%%%%%%%%%%%%%%%%%%set up%%%%%%%%%%%%%%%%%%%%

while crit>epsi
for i=1:nbk
for j=1:nba
c=A(j)*k(i)^alpha+(1-delta)*k(i)-k;
neg=find(c<0);
```

```
c(neg)=NaN;
util(:,j)=(c.^(1-sigma))/(1-sigma);
util(neg,j)=-1e12;
end
[Tv(i,:),pl(i,:)]=max(util+beta*(v*PI));
end
crit=max(abs(Tv-v));
v=Tv;
i=i+1;
end

gh=k(pl(:,1));
gl=k(pl(:,2));

figure
plot(v(:,1),'g-*');
hold on ;
xlabel=('k');
ylabel=('value function');
title=('value functions');
plot(v(:,2),'b-o');
hold off;

figure
plot(gh,'g-*');
xlabel=('k');
ylabel=('policy function');
title=('policy functions');
hold on;
plot(gl,'b-o');
hold off;
```