Course: Econ 634, Fall 2017
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Homework No 2.

1. **Question 1.**

   Suggested Answer:
   The dynamic programming problem is,

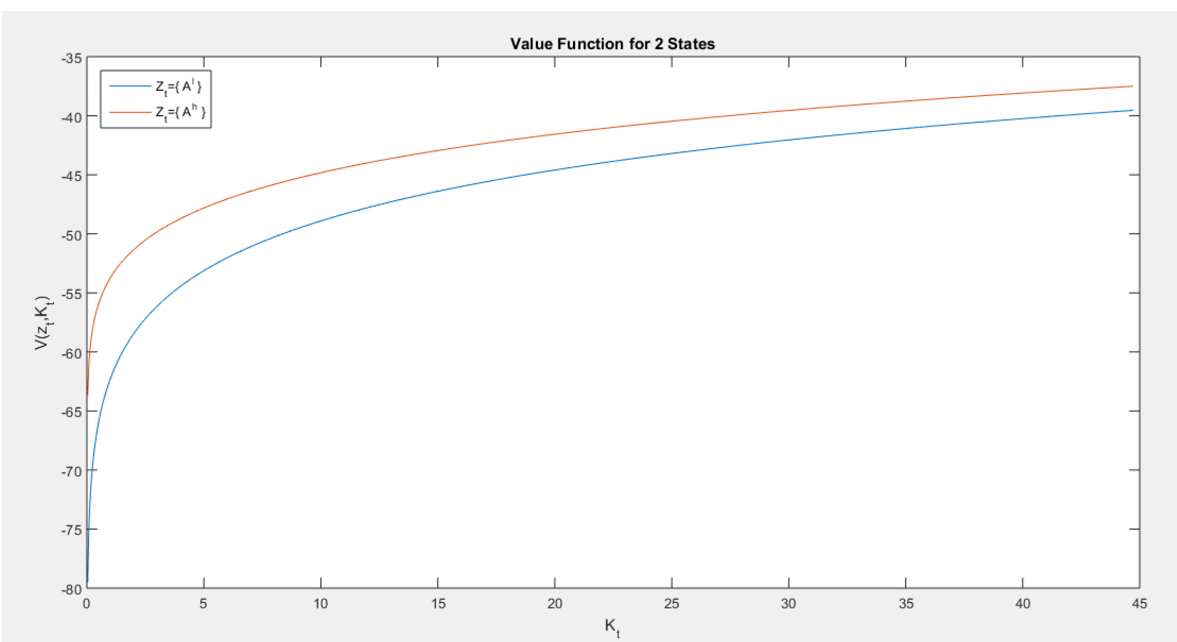
$$v(K, A) = \max_{0 \leq K' \leq AK^\alpha + (1-\delta)K} \left\{ \frac{(AK^\alpha + (1-\delta)K - K')^{1-\sigma}}{1-\sigma} + \beta \, \mathbb{E}\left[v(K', A')|A\right] \right\} \tag{1}$$

   The state variables are (K, A) and the control variable is K'. Additionally, for the case of a Markov process (with 2 states for A), we can express $\mathbb{E}\left[v(K', A')|A\right]$ as,

$$\mathbb{E}\begin{pmatrix} \left[v(K', A')|A^h\right] \\ \\ \left[v(K', A')|A^l\right] \end{pmatrix} = \begin{pmatrix} \pi_{hh} * v(K', A^h) + \pi_{hl} * v(K', A^l) \\ \\ \pi_{lh} * v(K', A^h) + \pi_{ll} * v(K', A^l) \end{pmatrix}$$

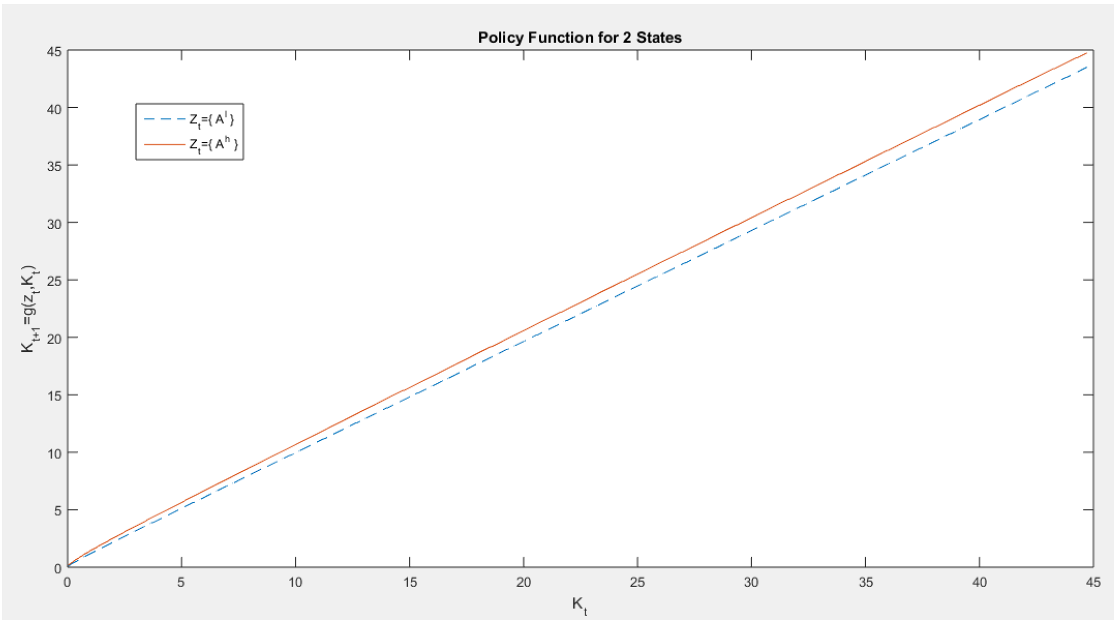
2. **Question 2.**

   Suggested Answer:
   As we can observe in the following figure, the value function for each state of A is an increasing and concave function of K.

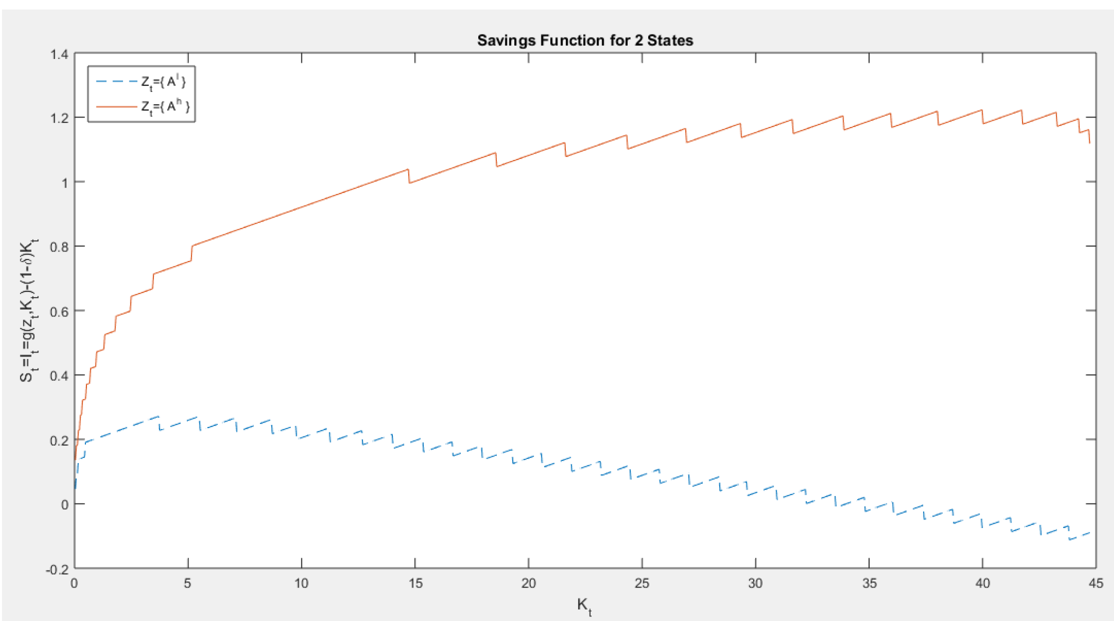


3. **Question 3.**

   Suggested Answer:
   As we can observe in the following figure, the policy function is increasing in K and A.

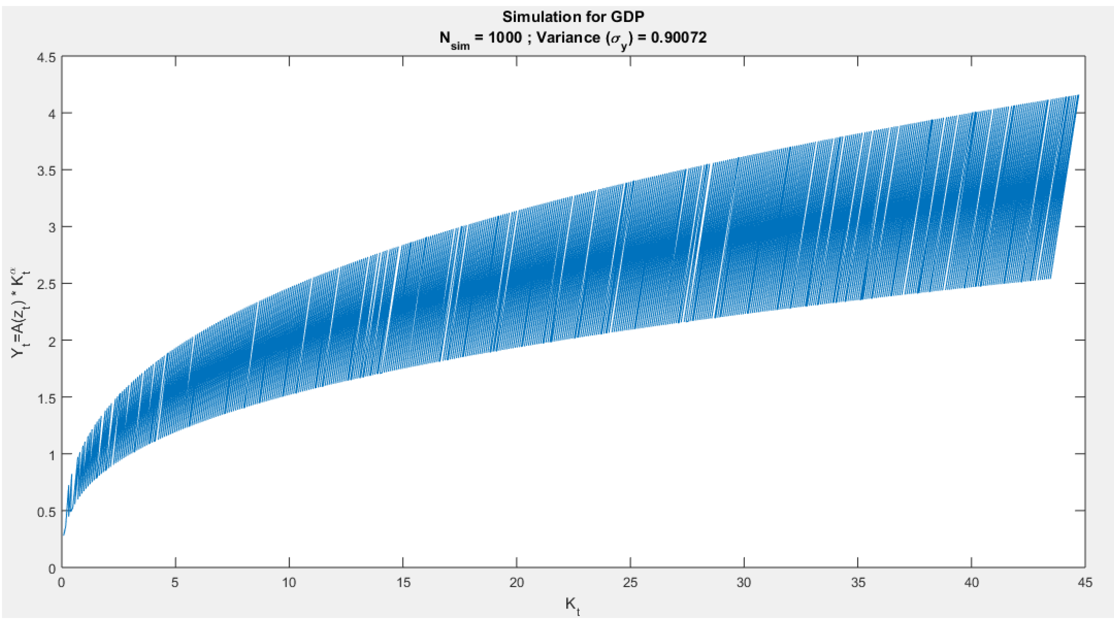


On the other hand, we may argue that for savings the function is increasing in A, but concave in K.



4. **Question 4.**

Suggested Answer:
For this question I randomly simulated productivity shocks (A) using the probabilities given by the Markov process. Specifically, given an initial state (i.e. A^l; however, it doesn't matter if we use A^h), and a sequence of random numbers (specifically, I used $x_t \overset{i.i.d.}{\sim} U[0,1]$), it is possible to compare the probability of continuing in the same state (i.e. π_{ll} for A^l) with the random probability of changing (or switching) to a different state (i.e. comparing π_{ll} with x). Thus, we can draw one particular realization for the random sequence of productivity shocks, $\left\{A_t^j\right\}_{j\in\{l,h\};t=1}^{T}$. Finally, for each particular state or realization of A in period $(t-1)$, I used the policy function to find the optimal level of capital K_t. Hence, having values for A and K, it is possible to construct the time series of GDP ($\{y_t\}$) and calculate the standard deviation.



Thus, the standard deviation we obtained from the simulation is $\sigma = 0.9$, which is less than the standard deviation for the time series of quarterly output in the U.S.

5. **Question 5. Extra-credit.**

Suggested Answer:
In this part I replaced the vectorization process by $for-loops$ in two parts of the Matlab code: (i) for the construction of the consumption series and the utility function, and (ii) for the construction of the value function during the value function iteration process. Thus, the following table presents the comparison of the times in Matlab when we use vectorization vs $for-loops$. As we can notice, the time needed for the $for-loops$ process is more than 5 times the time with vectorization.

$time\,using\,vectorization$	$time\ using\ for-loops$
75.4988	421.1558

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Annex. Matlab Codes

- **The main file PS2_Ch.m solves the stochastic problem using a value function iteration approach:**

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Binghamton University                       %
% PhD in Economics                            %
% ECON634 Advanced Macroeconomics             %
% P.S. 2                                      %
% Fall 2017                                   %
% Luis Chancí (lchanci1@binghamton.edu)       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all; a  = 0.35; b = 0.99; d = 0.025; s = 2;
z   = [.678 1.1]';                          % A^l and A^h
p   = [.926 (1-.977)]';                     % pi(ll) and pi(hl)
kss= (a*inv(inv(b)-(1-d)))^inv(1-a);        % I use the k(steady state)
k   = linspace(0.001*kss, 1.3*kss, 1000); %  between 0.1% and 130% of kss
km = repmat(k',[1 length(k)]);
c   = cat(3,z(1)*km.^a+(1-d)*km-km',z(2)*km.^a+(1-d)*km-km');
u   = c.^(1-s)/(1-s); u(c<0)=-Inf;
v0 = zeros(length(z),length(k));
e   = 1; t0 = tic;
while e>1e-06
    for m=1:length(z)
        v(:,:,m) = u(:,:,m)+b*(p(m)*repmat(v0(1,:), [length(k) 1])+...
                    (1-p(m))*repmat(v0(2,:), [length(k) 1]));
        [vfn(m,:),idx(m,:)] = max(v(:,:,m),[],2);
    end
    e = max(max(abs(vfn - v0)));
    v0 = vfn;
end; t = toc(t0); g = k(idx);    % Now we can plot

% Simulation.
S = bsxfun(@gt,[.926;.977],rand(1,length(k)));
y = [];  j = 1;
for i = 1:length(k)
    y = [y z(j)*g(j,i)^a];
    if S(j,i) == 1
        if j == 1; j = 2; elseif j == 2; j = 1; end
    end
end                         % Now we can get var(y)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```


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- **The file PS2_Ch_Extra.m is the code for question 5 (Extra-credit).**

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Binghamton University                      %
% PhD in Economics                           %
% ECON634 Advanced Macroeconomics            %
% P.S. 2  (Q5. Extra-credit)                 %
% Fall 2017                                  %
% Luis Chancí (lchanci1@binghamton.edu)      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all; a  = 0.35; b = 0.99; d = 0.025; s = 2;
z   = [.678 1.1]';
p   = [.926 (1-.977)]';
kss= (a*inv(inv(b)-(1-d)))^inv(1-a);
k   = linspace(0.001*kss, 1.3*kss, 1000);

t0 = tic;
u = cat(3,zeros(length(k)),zeros(length(k)));
for m = 1:length(z)
    for i = 1:length(k)
        for j = 1:length(k)
            c = z(m)*k(i)^a+(1-d)*k(i)-k(j);
            if  c>0; u(i,j,m)= c^(1-s)/(1-s);
            else     u(i,j,m)= -Inf; end
end; end; end

v   = cat(3,zeros(length(k)),zeros(length(k)));
v0  = zeros(length(z),length(k));
e   = 1;
while e>1e-06
    for m=1:length(z)
        for i = 1:length(k)
            for j = 1:length(k)
                v(i,j,m) = u(i,j,m)+b*(p(m)*v0(1,j)+...
                                   (1-p(m))*v0(2,j));
        end; end
        [vfn(m,:),idx(m,:)] = max(v(:,:,m),[],2);
    end
    e = max(max(abs(vfn - v0)));
    v0 = vfn;
end; t = toc(t0);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```