ECON 634 Homework 3

Ruohao Zhang

October 3, 2017

1. The maximization question can be described by:

$$\underset{c_t, a_t}{Max}\, E_0 \left(\sum_{t=0}^{\infty} \beta^t U(c_t) \right)$$
$$st.\, c_t + q_t a_{t+1} = y(s_t) + a_t \quad \forall t,\, s_t.$$

The value function will become:

$$V(a_t,\, s_t) = \underset{a_{t+1} \in \Gamma(a_t,\, s_t)}{Max} U(y(s_t) + a_t - q_t a_{t+1}) + \beta E\left[V(a_{t+1},\, s_{t+1})\right]$$

Where state variables are a_t and s_t, control variable is a_{t+1}. State space is $a_t \in [-2,\, 5]$, $s_t \in \{e,\, u\}$. Constraint correspondence is $a_{t+1} \in \left[-2,\, min\left(5,\, \frac{y(s_t)+a_t}{q_t}\right)\right]$.

2. $q = 0.9943$ satisfies the market clear condition. Risk free interest rate is 5.7%. Given that interest rate, the value function and policy function is shown in the following graph.

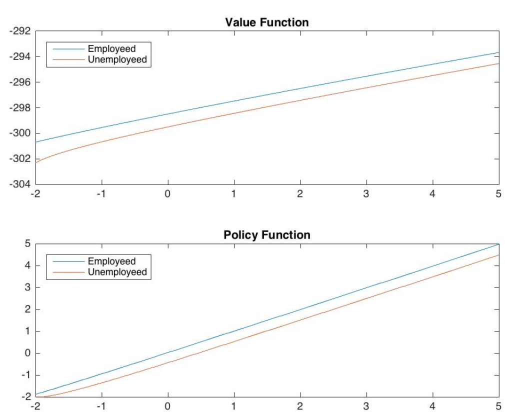

Figure 1: Value function and Policy function

3. The wealth Gini coefficient is 0.3359 and the earning Gini coefficient is 0.0275.Figure 2 gives Lorenz curve for both wealth and earning.

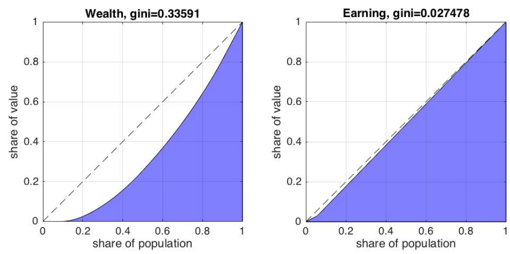

Figure 2: Lorenz curve

4.

(1). In that case, assume all consumers started at $t=0$ without any state, and will randomly get employment state in $t=1$. Households want to solve the following problem:

$$\underset{c_t, a_t}{Max} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t(s^t) \frac{c_t(s^t)^{1-\sigma}}{1-\sigma}$$

$$st.\ c_t(s^t) + \sum_{s_{t+1} \in S} q_{t+1}(s_{t+1},\, s^t) a_{t+1}(s_{t+1},\, s^t) = y(s_t) + a_t(s^t) \quad \forall t,\, s^t,\, i.$$

By solving this problem, we get

$$q_{t+1}(s_{t+1},\, s^t|\, s_0) = \beta^{t+1} \pi(s_{t+1},\, s^t|\, s_0) \left[\frac{c_0(s_0)}{c_{t+1}(s_{t+1},\, s^t,\, s_0)} \right]^{\sigma} \quad \forall t,\, s^t,\, i.$$

Under market clear condition, $c_t^1(s^t) + c_t^2(s^t) = Pr(s=e) \times y(s=e) + Pr(s=u) \times y(s=u)$.

The households will choose the consumption to make marginal utility same for each period. Indeed, the allocation will be $c_t(s^t) = \hat{c}$ for all periods and states. The stationary distribution of employment keeps on $\begin{pmatrix} 0.9434 & 0.0566 \end{pmatrix}$, so the total endowment in the society is $Y = \pi_e \cdot y(e) + \pi_u \cdot y(u) = 0.9717$. Due to market clear condition, $c_t(s^t) = \hat{c} = Y = 0.9717$. The expected utility will be

$$W^{FB} = \sum_{t=0}^{\infty} \beta^t U(\hat{c}) = \frac{\hat{c}^{1-\sigma}}{1-\sigma} \cdot \frac{1}{1-\beta} = -298.3702$$

(2). Since we have $W^{FB} = -298.3702$, By comparing W^{FB} with value function, the portion of population who are better off is 0.5190.

(3). The consumption equivalent plot w.r.t asset is shown in the following graph.

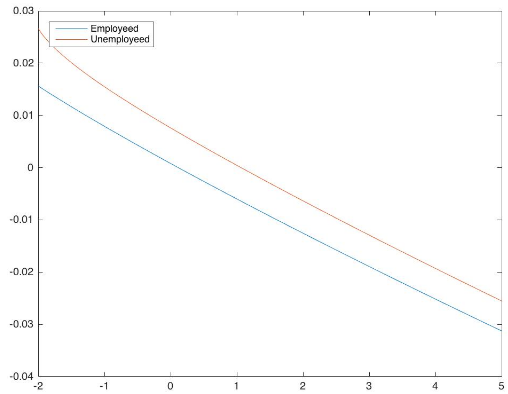

Figure 3: Consumption equivalent

and the economy-wide welfare gain is 0.12%.