# 619 HW 3

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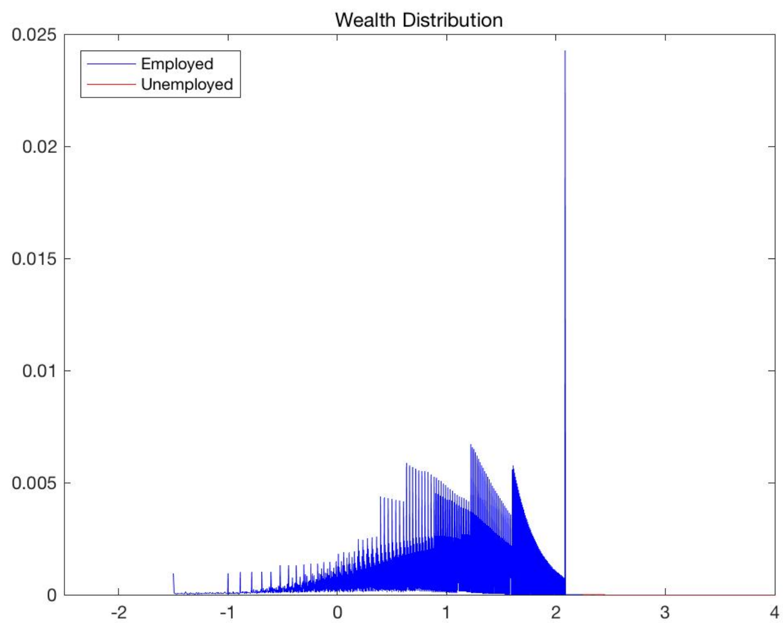
**1. Recursive Problem** The households' problem can be expressed recursively as:

$$
\begin{aligned}
v(s,a) &= \max_{a' \in \Gamma(s,a)} \frac{c^{1-\sigma}}{1-\sigma} + \beta \sum_{s' \in Z} \pi(s'|s) v(s',a') \\
s.t \quad & c = y(s) + a - qa' \\
& a' \leq \bar{a}
\end{aligned}
\tag{1}
$$

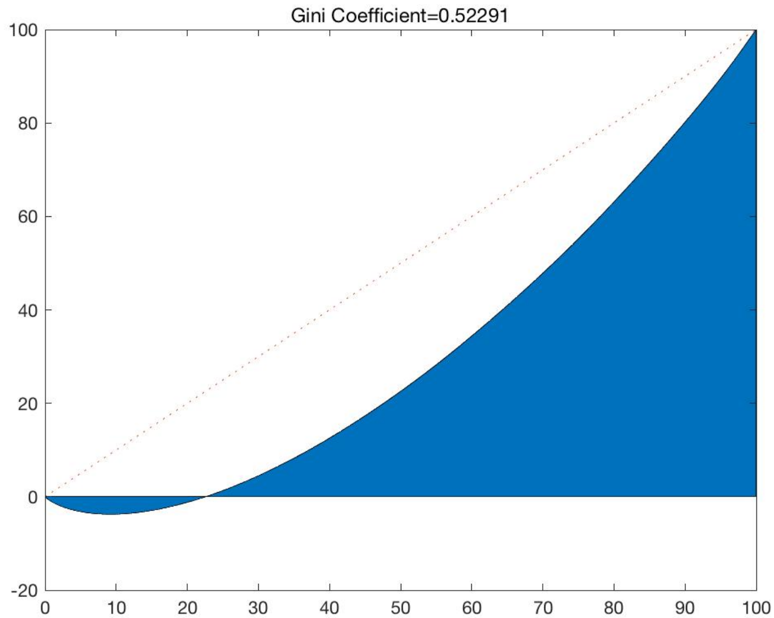
**2. Market Clearing Bond Price** Using value function iteration method in MATLAB, the answer for a steady state bond price $q = 0.9943$. At steady state, the wealth distribution is plotted below:

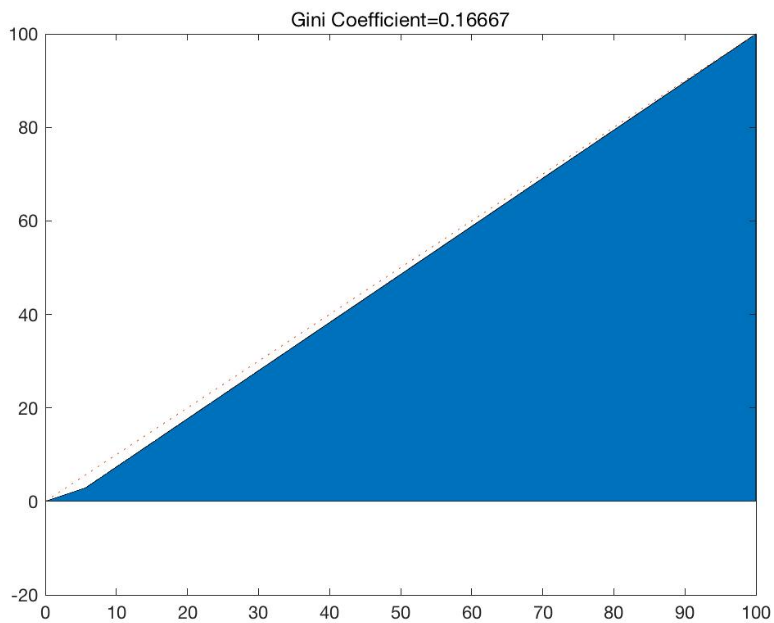



**3. Lorenz Curve and Gini Index** From our calculation, there are some negative wealth in the distribution. To solve this, we use alternative definition:

$$
G = \frac{\sum_{i=1} \sum_{j=1} |x_i - x_j|}{\sum_{i=1} \sum_{j=1} x_j}
\tag{2}
$$

The results are displayed below. The Gini index is 0.5229 for wealth, and 0.1667 for income.


Gini Coefficient=0.52291
100
80
60
40
20
0
-20
0
10
20
30
40
50
60
70
80
90
100


Gini Coefficient=0.16667
100
80
60
40
20
0
-20
0
10
20
30
40
50
60
70
80
90
100

**4.Extra credit: welfare implication** With zero assets and perfect insurance, households will have a perfectly smooth consumption allocation through out all periods. And with market clearing condition we have:

$$\begin{aligned}\bar{c} &= \pi_u \cdot y(u) + \pi_e \cdot y(e) \\ &= 0.9434 \times 1 + 0.0566 \times 0.5 \\ &= 0.9717\end{aligned} \tag{3}$$

Hence the "first-best" welfare $W^{FB}$ is:

$$\begin{aligned}W^{FB} &= \frac{\bar{c}}{1-\beta} \\ &= \frac{0.8717}{1-0.9932} \\ &= -298.3702\end{aligned} \tag{4}$$

Comparing with $v(s,a)$, there are 37.15% households who will benefit from this plan. Using $\lambda(s,a) = \left(\frac{W^{FB}}{v(s,a)}\right)^{\frac{1}{1-\sigma}} - 1$, we can compute the consumption equivalence, and the economy-wide welfare, which is 0.0014.