HOMEWORK 3 HUGGETT'S MODEL

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Q1. Recursive problem

The recursive problem can be stated as

$$v(s,a) \quad = \quad \max_{a' \in \Gamma(s,a)} \left\{ \frac{(y(s)+a-qa')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{s'|s}\left[v(s',a')\right] \right\}$$

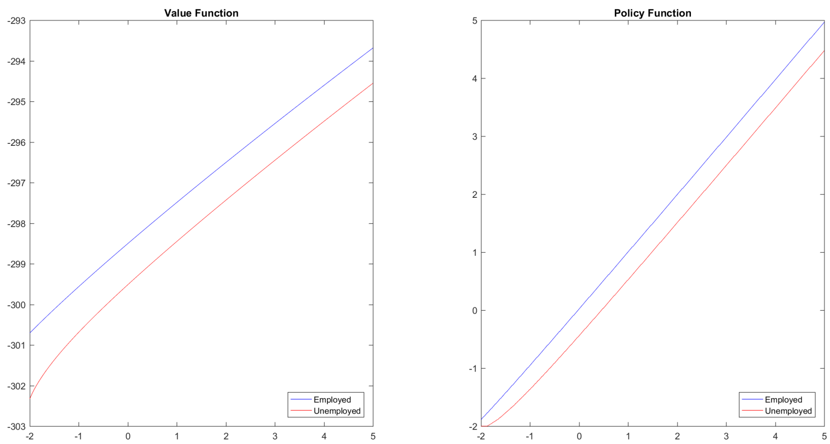
by given budget constraint $c = y(s) + a - qa'$ and asssets bound $a_{t+1} \geq \underline{a}$. The state variables are $s = \{e, u\}$ and $a = \{\underline{a}, \bar{a}\}$ i.e. $\underline{a} = -2$ and $\bar{a} = 5$ in homework, the control variable is a' from $\Gamma(s,a) = \left\{a' \colon \underline{a} \leq a' \leq \frac{y(s)+a}{q}\right\}$.

Q2. Risk-free interest rate in steady-state.

Since the assets will be repaid in next period, q is the risk-free asset price. Following the step, the steady-state risk-free interest rate is $r^* = \frac{1}{q^*} - 1 = \frac{1}{0.9943} - 1 = 0.5733\%$.

Given q^*, the value function and policy function is shown in the Figure 1.

Figure 1. Value Function and Policy Function, $q^* = 0.9943$



The approximate stationary distribution μ^* is plotted in following Figure 2.

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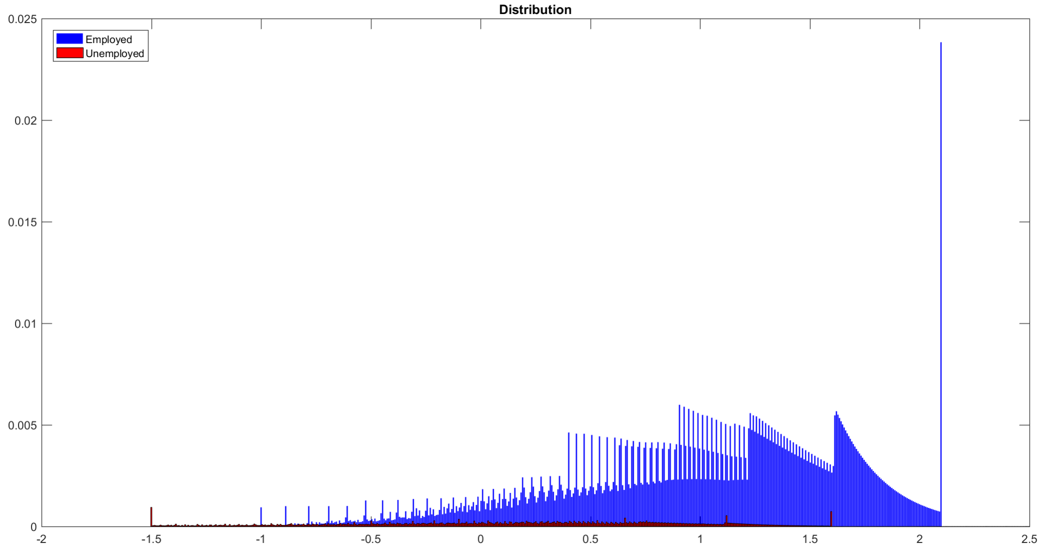
Figure 2. Approximate Stationary Distribution μ^*

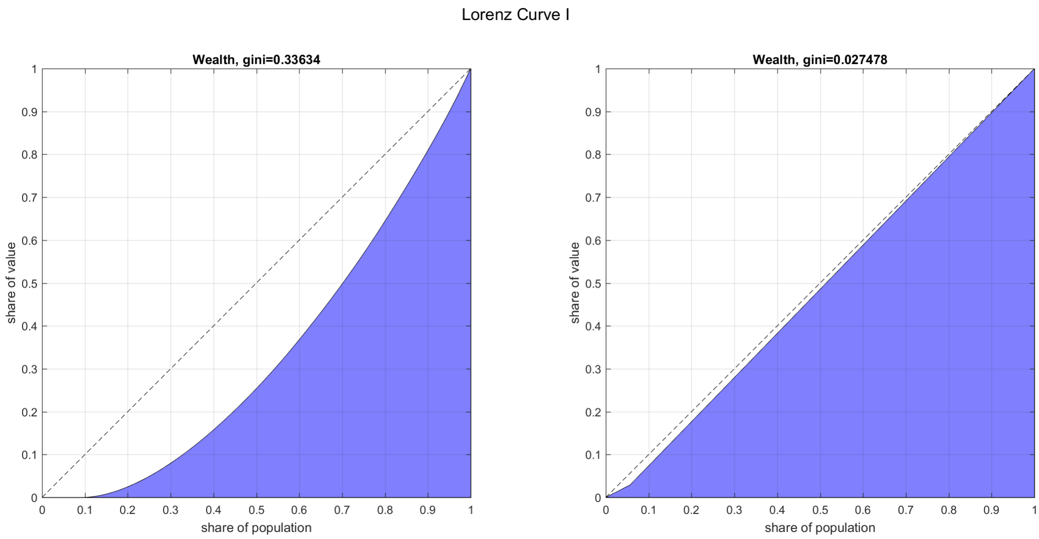


Q3. Lorenz Curve and Gini Coeffient

I produce two groups of Lorenz curve and Gini coefficient. By comparing those two groups, the results are quite similar to each other.

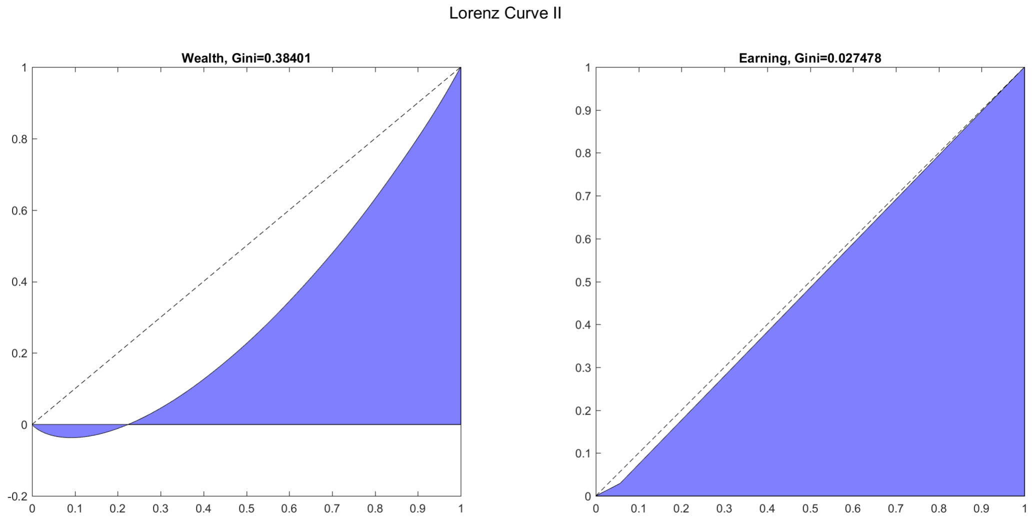
The first group is produced by online function package gini.m which requires non-negative value of wealth and earning. The gini coefficient of wealth is 0.33634 while gini coefficient of earnings is 0.027478. Type I Lorenz curve is plotted in Figure 3.

Figure 3. Lorenz Curve of Type I



The second group is produced by self-coding program which does not require non-negativity. The gini coefficient of wealth is 0.38401 while gini coefficient of earnings is 0.027478. Type II Lorenz curve is plotted in Figure 4.

FIGURE 4. Lorenz Curve of Type II



Q4. Extra Credit: Welfare Effects

In order to find the first-best utility, I solve the household problem with state-contingent bonds. Households problem is set as:

$$\max_{c_t,a_t} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t(s^t) \frac{c_t(s^t)^{1-\sigma}}{1-\sigma}$$
$$\text{st. } c_t(s^t) + \sum_{s_{t+1}\in S} q_{t+1}(s_{t+1}, s^t) a_{t+1}(s_{t+1}, s^t) = y(s_t) + a(s^t)$$

where under CRRA utiltiy, the ratio of $c_t^i(s^t)/c_t^j(s^t)$ $\forall i \neq j$ is a constant, therefore $c_t(s^t) = c$ $\forall s_t, t$. Under market clear, $\sum_i c_t^i(s^t) = \pi_e \times y(s_0 = e) + \pi_u \times y(s_0 = u)$ where π_e and π_u are long-run stationary distribution given by Π^{1000}, i.e. $\pi_e = 0.9434$ and $\pi_u = 0.0566$. $c_t(s^t) = c = \pi_e \times y(s_0 = e) + \pi_u \times y(s_0 = u) = 0.9717$.

$$\begin{aligned} W^{FB} &= \sum_{t=0}^{\infty} \beta^t U(c) \\ &= \frac{c^{1-\sigma}}{1-\sigma} \times \frac{1}{1-\beta} \\ &= -298.3702 \end{aligned}$$

53.76% of the household will be better of and the welfare gain will be 0.13%. Equivalent welfare is plotted in Figure 5.

FIGURE 5. Consumption Equivalent, λ with a'