Homework 3

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October 6, 2017

Contents

Question 1

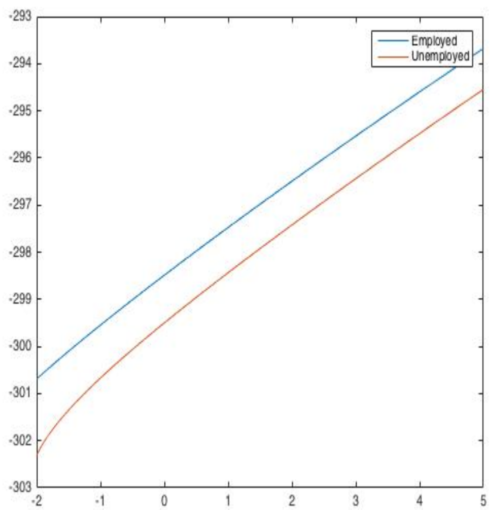
The recursive problem is:

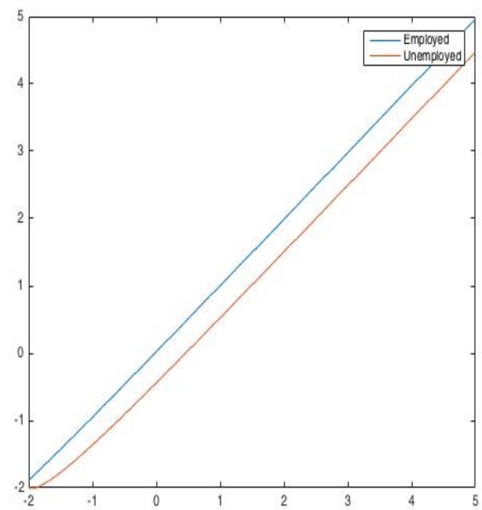
$$v(s,a) = \max_{a' \in \tau(s,a)} \frac{(y(s) + a - q(a'))^{1-\sigma}}{1-\sigma} + \beta \sum_{s' \in \{u,e\}} \pi(s'|s) v(s', a')$$

States variables: a, s;
Control variables: a', s';
State space: $a \in [-2, 5]$, $s \in \{u, e\}$;
Constraint Correspondence: $\tau(s,a) = [-2, 5] \cap [-2, \frac{y(s)+a}{q}]$.

Question 2

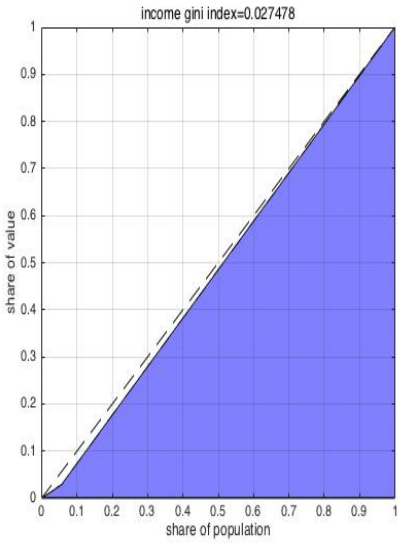
To answer this question, we have to do the value function iteration and the distribution iteration using Matlab. The result for asset price is $q = 0.9943$, hence the risk free interest rate is $r = \frac{1}{q} = 1.0057$. Below are the value funcions and policy functions respectively:

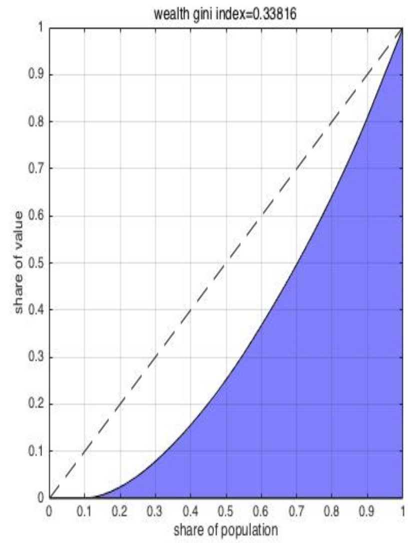




Question 3

Below are the Lorentz curve for income and wealth. Since the $'ginipackage'$ in Matlab can not deal with negative numbers, I use $'0'$ to subsitute all the negative numbers in wealth.As we can see from the Lorentz curve for income, it is very close to the 45 degree line. It is because the the unemployment rate is relatively low in the stationary distribution. While for the wealth part, an agent accumulates wealth in good times and runs it down in bad times.





Question 4

To know the welfare impact of incomplete market, we have to do the following steps:

1 Calculate the expected utility if there was perfect insurance, here is:

$$W^{FB} = \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} \tag{4.1}$$

Since everyone starts out with zero assets and is perfect insured, we can use the steady state of $\mu(s,a)$ and the endowment when employed and unemployed to calculate the consumption, where C_t should be equal to percentage of employed times y_high plus percentage of unemployed times y_low. By using matlab, $C_t = 0.9717$. Then we can get $W^{FB} = -298.37$ using equation 4.1.

2 To calculate the implement λ, we should let the expected utility under incomplete market with implement euqals to W^{FB}.

$$\lambda(s,a) = (\frac{W^{FB}}{v(s,a)})^{1/(s-\sigma)} - 1 \tag{4.2}$$

Using Matlab, λ is a 2 by m matrix, where m is the number of different assets.

3 Calculate the economy wide gain,using λ and steady state of $Mu(s,a)$, the result I get is 0.0013.

Appendix

Matlab codes:

```
% PROGRAM NAME: ps4huggett.m
clear ;
close all;

% PARAMETERS
beta = .9932; %discount factor
sigma = 1.5; % coefficient of risk aversion
b = 0.5; % replacement ratio (unemployment benefits)
y_s = [1, b]; % endowment in employment states
PI = [.97 .03; .5 .5]; % transition matrix


% ASSET VECTOR
a_lo = -2; %lower bound of grid points
a_hi = 5;%upper bound of grid points
num_a = 600;

a = linspace(a_lo, a_hi, num_a); % asset (row) vector

% INITIAL GUESS FOR q
q_min = 0.98;
q_max = 1;

% ITERATE OVER ASSET PRICES
aggsav = 1 ;
while abs(aggsav) >= 0.01 ;
q_guess = (q_min + q_max) / 2;

% CURRENT RETURN (UTILITY) FUNCTION
cons = bsxfun(@minus, a', q_guess * a);
cons = bsxfun(@plus, cons, permute(y_s, [1 3 2]));
ret = (cons .^ (1-sigma)) ./ (1 - sigma); % current period utility
ret(cons<0)=-Inf;
% INITIAL VALUE FUNCTION GUESS
v_guess = zeros(2, num_a);

% VALUE FUNCTION ITERATION
v_tol = 1;
while v_tol >.0001;
% CONSTRUCT RETURN + EXPECTED CONTINUATION VALUE

vf=bsxfun(@plus,ret,permute(beta*PI*v_guess,[3,2,1]));

% CHOOSE HIGHEST VALUE (ASSOCIATED WITH a' CHOICE)
[vfn,pol_indx]=max(vf,[],2);
```

```
v_tol=[max(abs(vfn(:,:,1)' - v_guess(1,:))) ; max(abs(vfn(:,:,2)' - v_guess(2,:)))];
v_guess=[vfn(:,:,1)';vfn(:,:,2)'];
end;

% KEEP DECSISION RULE
pol_indx=permute(pol_indx, [3 1 2]);
pol_fn=a(pol_indx);

% SET UP INITITAL DISTRIBUTION
Mu=ones(2,num_a)/(2*num_a);

% ITERATE OVER DISTRIBUTIONS
m_tol=1;
while m_tol>0.0001
[emp_ind, a_ind, mass] = find(Mu > 0); % find non-zero indices

MuNew = zeros(size(Mu));


for ii = 1:length(emp_ind)
apr_ind = pol_indx(emp_ind(ii), a_ind(ii)); % which a prime does the policy fn prescribe
MuNew(:, apr_ind) = MuNew(:, apr_ind) +[PI(emp_ind(ii),1)*Mu(emp_ind(ii),a_ind(ii));PI(e
% which mass of households goes to which exogenous state?
end
m_tol=max(max(abs(MuNew-Mu)));
Mu=MuNew;
end

aggsav=Mu(1,:)*a'+Mu(2,:)*a';
if aggsav>0;
q_min=q_guess;
else q_max=q_guess;
end
q_min;
q_max;
end

figure;
plot(a,vfn(:,:,1),a,vfn(:,:,2)),legend('Employed','Unemployed');

figure;
plot(a, pol_fn(1,:),a,pol_fn(2,:)),legend('Employed','Unemployed');


%%%%%for lorenz curve and gini index
```

```
p=[Mu(2,:);Mu(1,:)];
income=[repmat(y_s(2),1,num_a);repmat(y_s(1),1,num_a)];
g=gini(p,income,true);
title(['income gini index=',num2str(g)]);
wealth=[bsxfun(@plus,a,y_s(2));bsxfun(@plus,a,y_s(1))];
wealth(wealth<0)=0;
wg=gini(p,wealth,true);
title(['wealth gini index=',num2str(wg)]);
%%%%%%%%%%%extra credits%%%%%%%%%%
pe=sum(Mu(1,:));
pu=sum(Mu(2,:));
c_fb=pe*y_s(1)+pu*y_s(2);
W_fb=c_fb^(1-sigma)/((1-sigma)*(1-beta));
lambda=(W_fb./v_guess).^(1/(1-sigma))-1;

%econmy wide gain
gain_e=lambda(1,:)*Mu(1,:)';
gain_u=lambda(2,:)*Mu(2,:)';
gain_tot=gain_e+gain_u;
```