Course: Econ 634, Fall 2017
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Homework No 3. Huggett's Model.

1. **Question 1. The Recursive Problem.**
   Suggested Answer:
   The households' constraints are $a_{t+1} \geq \underline{a}$ and $c_t = y(s_t) + a_t - q_t a_{t+1}$, thus, the recursive problem can be expressed as

$$v(s,a) = \max_{a' \in \Psi(s,a)} \left\{ \frac{(y(s) + a - q * a')^{1-\sigma}}{1-\sigma} + \beta * \mathbb{E}_{s'|s}\left[v(s',a')\right] \right\} \tag{1}$$

   with,

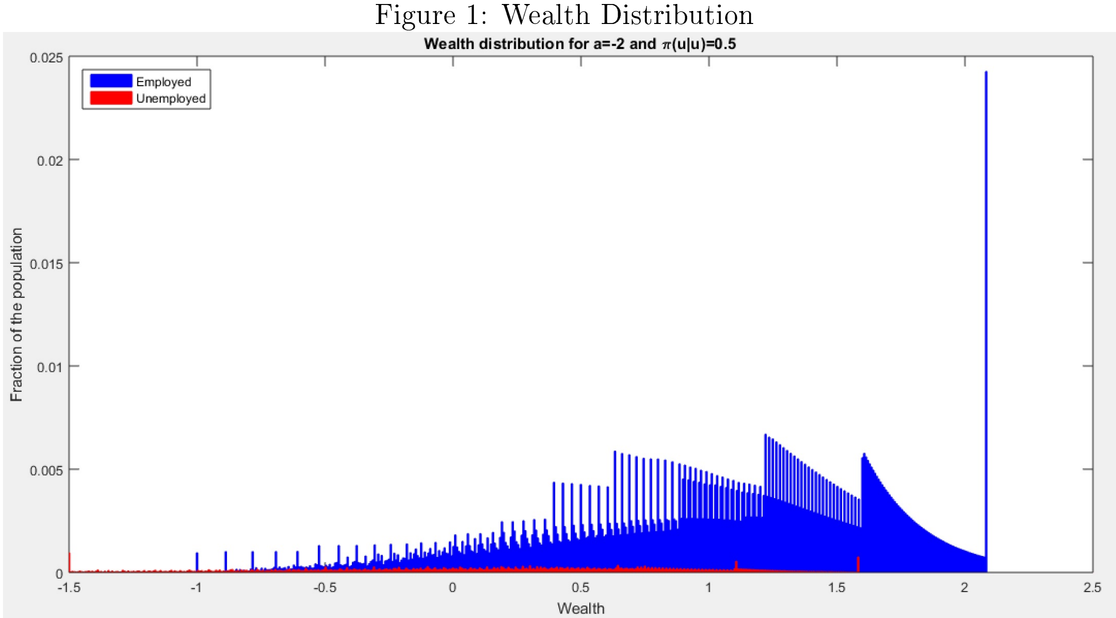
$$\Psi(s,a) = \left\{ a' : \ \underline{a} \leq a' \leq \frac{y(s)+a}{q} \right\}$$

2. **Question 2. Risk-free interest rate in the economy in steady-state.**

   Suggested Answer:
   After finding the fixed points in $T\mu^* = \mu^*$ and $Tv^* = u + \beta * \mathbb{E}\{v'^*\}$, in steady-state the price is $q^* = 0.9943$, thus, the interest rate is $r^* =$.

   In addition to the result in steady-state for q, we can plot the steady-state distribution of wealth (figure 1).

Figure 1: Wealth Distribution



3. **Question 3. The Gini coefficient and the Lorenz curve for earnings and wealth.**

   Suggested Answer:
   The following two figures show the Lorenz Curve for wealth (figure 2) and income (figure 3). Additionally, the Gini coefficients are 0.4 for wealth and 0.055 for income.

Figure 2: Lorenz Curve for Wealth

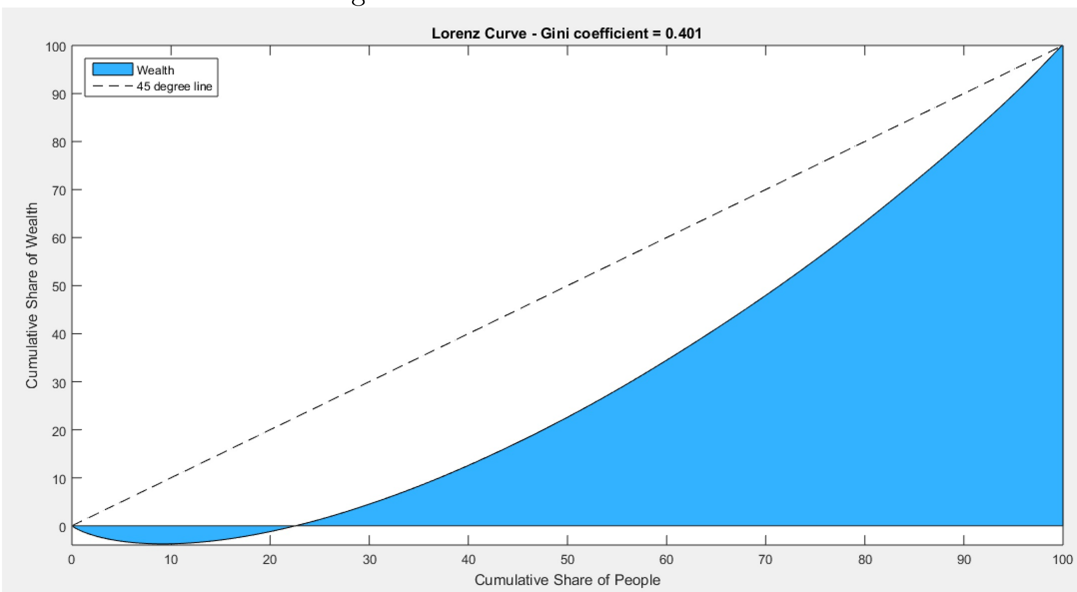


Figure 3: Lorenz Curve for Income

4. **Question 4. (Extra-credit) - Welfare.**

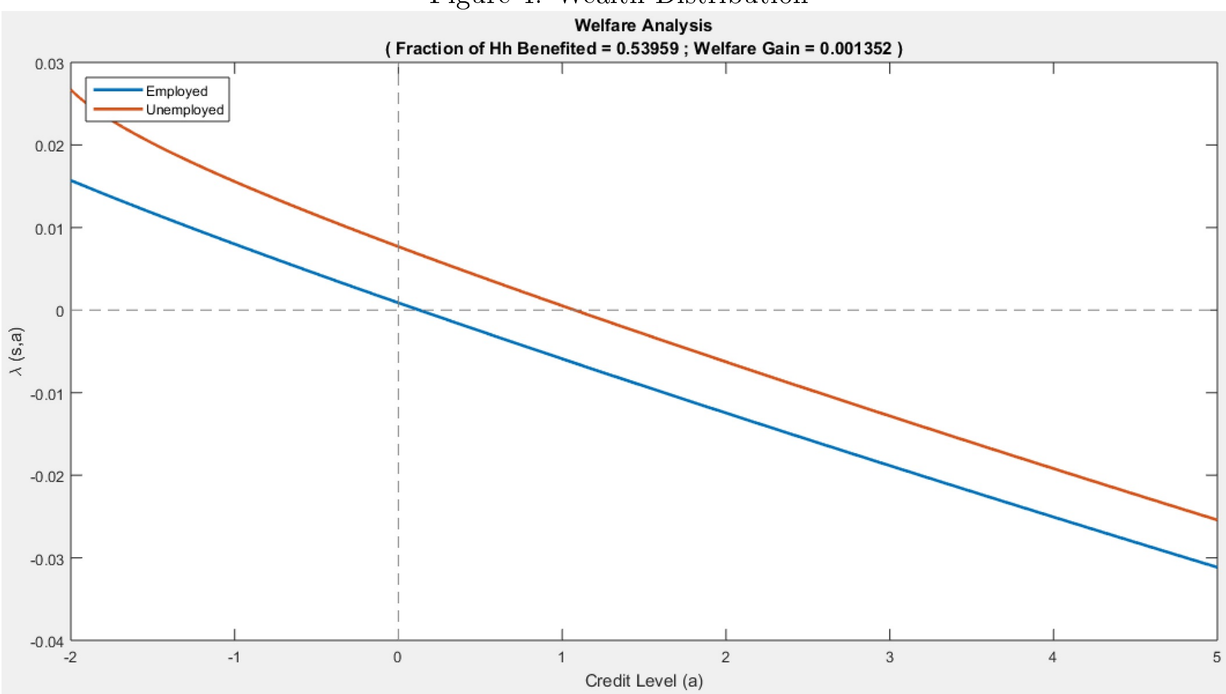
Suggested Answer:
Firstly, we need to find the stationary or invariant probability distribution for the first-order Markov process, $p' = p'\pi$.

$$\left[\begin{pmatrix} 0.97 & 0.5 \\ 0.03 & 0.5 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Using this information and the fact that $p_1 + p_2 = 1$, we can find $p_1 = 0.9434$ and $p_2 = 0.0566$.

Figure (4) shows the results for consumption equivalent. The fraction of households benefited with the plan is 0.54 (or in percentage, 54%), while the economy-wide welfare gain is 0.00135.

Figure 4: Wealth Distribution



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Annex. Matlab Codes

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Binghamton University                            %
% PhD in Economics                                 %
% ECON634 Advanced Macroeconomics                  %
% Fall 2017                                        %
% Luis Chancí (lchanci1@binghamton.edu)            %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all; close all; clc;

% PARAMETERS
beta  = 0.9932; sigma = 1.5; b    = 0.5;
y_s   = [1, b]; PI    = [.97 .03; .5 .5];

% ASSET VECTOR
num_a = 1000;  a = linspace(-2, 5, num_a);

% INITIAL GUESS FOR q
q_min = 0.98; q_max = 1.1;

aggsav = 1;
while (abs(aggsav) >= 0.01)
    q_guess = (q_min + q_max) / 2;
    c  = bsxfun(@plus, bsxfun(@minus,a',q_guess*a),permute(y_s,[1 3 2]));
    u  = (c.^(1-sigma))./(1-sigma); u(c<0)=-Inf;
    v0 = zeros(2, num_a);
    e1 = 1;
    while e1 >1e-06
          v = u + beta * repmat(permute((PI*v0),[3 2 1]),[num_a 1 1]);
          [vfn,idx] = max(v,[],2);
          e1 = max(max(abs(permute(vfn,[3 1 2]) - v0)));
          v0 = permute(vfn,[3 1 2]);
    end
    pol_indx = permute(idx,[3 1 2]);
    g  = a(pol_indx);
    Mu = ones(size(g))/numel(g);
    e2 = 1;
    while e2>=1e-06;
        [emp_ind, a_ind, mass] = find(Mu);
        MuNew  = zeros(size(Mu));
        for ii = 1:length(emp_ind)
            apr_ind = pol_indx(emp_ind(ii), a_ind(ii));
            MuNew (:, apr_ind) = MuNew (:, apr_ind) + ...
                  (PI(emp_ind(ii), :) * mass(ii))';
        end
        e2 = max(max(abs(MuNew  - Mu)));
        Mu = MuNew;
    end
    aggsav = sum(sum(Mu.*g));
    if aggsav>0
       q_min = q_guess; else; q_max = q_guess;
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Lorenz Curve and Gini Coeff.
f  = reshape(Mu',[numel(Mu) 1]);
y  = [reshape(bsxfun(@plus,repmat(a, [2,1]),y_s')',[2*num_a 1]),...% Wealth
          reshape(repmat(y_s',[1 num_a])',[2*num_a 1])];                 % Income
L    = []; G = [];
for i=1:2;
    s = cumsum(sortrows([f,f.*y(:,i),y(:,i)],3));
    L = [L,bsxfun(@rdivide,s,s(end,:))*100];
    G = [G,1 - sum((s(1:end-1,2)+s(2:end,2)).*diff(s(:,1)))];
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Welfare
c_bar   = 1*0.9434+0.5*.0566;                          % Using invariant Pr.
W_FB    = ((c_bar^(1-sigma))/(1-sigma))*inv(1-beta);% Welfare First Best
lambda  = ((v0.^(-1)).*W_FB).^(1/(1-sigma))-1;         % Consumption equivalent
Frac    = sum(sum((lambda>0).*Mu));                    % Fraction W_FB > v(s,a)
WG      = sum(sum(lambda.*Mu));
%%%%%%%%%%%%%%     Now we can Plot  %%%%%%%%%%%%%%
```