

ECON 634 Problem Set 3

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1. The recursive problem is

$$\begin{aligned} \max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \\ \text{s.t. } c_t + q_t a_{t+1} = y(s_t) + a_t \\ a_{t+1} \in \Gamma(s_t, a_t) \\ a_{t+1} \geq \underline{a}, \forall t, \text{ and } a_0 \text{ given.} \end{aligned}$$

The functional equation of the dynamic programming problem is

$$V(s, a) = \max_{a' \in \Gamma(s, a)} \left\{ \frac{[y(s) + a - qa']^{(1-\sigma)}}{1-\sigma} + \beta \mathbb{E}_{s'|s} V(s', a') \right\}$$

where s and a are the state variables and s' and a' are the control variables.

The state space is the combination of the possible states of asset

$$\mathcal{A} = \{a_1, a_2, \dots, a_n\}$$

and the possible states of employment

$$\mathcal{S} = \{e, u\}.$$

The constraint correspondence is

$$\Gamma(a, s) = \left\{ a' : a' \geq 0, a' \leq \frac{y(s) + a}{q} \right\}.$$

2. The risk-free interest rate in the economy in the steady state is
3. Yes, according to the plot of the policy function over K , it is concave and increasing in K .