Problem Set 3 – Huggett's model

In this problem set we will consider a simplified version of Huggett (JEDC, 1993). At the core is still the problem of finding the households' decision rules through value function iteration. In addition, however, we now have to find the market clearing bond price q and the distribution $\mu(s,a)$. Since these two equilibrium objects are interdependent, we have to find them simultanously.

The basic steps are

A) Guess a steady-state bond price q.

B) Given q, solve for the households' policy function: $a' = g(s,a;q)$.

C) Given the policy function $g(\cdot;q)$, find the stationary distribution over employment and assets $\mu(s,a;q)$.

D) Check if the market clears under the distribution $\mu(\cdot;q)$ by looking at excess demand for assets. If it is far from 0, go back to step A) and adjust the guess.

Take the model and the calibration as discussed in class. To summarize, households maximize utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

subject to the budget constraint $c_t + q_t a_{t+1} = y(s_t) + a_t$ and the borrowing constraint $a_{t+1} \geq \underline{a}$. The exogenous variable $s_t \in (e,u)$ indicates the employment status of a household, with earnings $y(e) = 1$ and $y(u) = b$. The transition probabilities are given by the matrix $\Pi = \begin{pmatrix} \pi_{ee} & \pi_{eu} \\ \pi_{ue} & \pi_{uu} \end{pmatrix}$ where $\pi_{ee} = .97$ and $\pi_{uu} = 0.5$. Let's set $\underline{a} = -2$, and as an upper bound for the discretization of the state space you can use $a^{\max} = 5$. The discount factor is $\beta = 0.994$ and the CRRA coefficient is $\sigma = 1.5$.

1. State the recursive problem, and define all elements (i.e., designate state and control variables, and define the state space and the constraint correspondence).

2. What is the risk-free interest rate in the economy in steady-state? To answer this, find the steady-state equilibrium:

   - For a given guess of $q \in [0,1]$, find the households' decision rule $a' = g(s,a;q)$ through value function iteration.
   - Approximate the stationary distribution μ^* for the current guess q by implementing the operator $\tilde{T}$, defined as

     $$\left(\tilde{T}\mu\right)(s',a') = \int_{S,A}\left(\mathbf{1}_{\{a'=g(s,a)\}}(s,a)\right)\pi(s'|s)\,\mu(ds,da)$$

     . Start with an arbitrary distribution μ_0 over (s,a) (for example, the uniform distribution), and then apply $\tilde{T}$ repeatedly to get $\mu_n =$

$\left(\tilde{T}\right)^n \mu_0$, until μ_{n+1} and μ_n are very similar (up to a very small numerical tolerance).

- See if the asset market clears in the stationary distribution. That is, use the policy function and the mass of households over the states to see if demand for assets equals supply of assets. Loosely speaking, see if there are as many borrowers as there are lenders. Formally, check if $\int_{(s,a)} g(s,a;q)\,\mu^*(ds,da;q) \approx 0$ up to numerical error. If not, adjust q and repeat the steps.

3. Plot the Lorenz curve for earnings and wealth, and compute the respective Gini coefficients. (For the purposes of this problem set, define "wealth" as asset holdings plus income $a_t + y(s_t)$ in order to avoid division by zero for the Gini coefficient since assets are in zero net supply.)

4. Extra credit: What are the welfare effects of incomplete markets? Consider the following questions:

- If everyone started out with zero assets, and there was perfect insurance against all income shocks, what would the allocation be? What would be expected utility W^{FB} of that allocation?

- Assume that a government proposes to "implement the first-best" by making a complete set of state-contingent bonds available and suggests a plan to reset everyone's assets to zero. Which fraction of households would benefit from that plan? (I.e. find the share of households for who $W^{FB} > v(s,a)$ where v is the value function.)

- How much would households be willing to pay in order to have the government's plan implemented? The so-called "consumption equivalent" measures the percentage that a household's consumption needs to be increased (or reduced) permanently so that they are indifferent between the reduced consumption and the alternative plan. Compute the consumption equivalent $\lambda(s,a)$ in the following way:

$$\begin{aligned}
W^{FB} &= E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left(\left[1+\lambda(s,a)\right]c(s,a)\right)^{1-\sigma}}{1-\sigma}\right] \\
&= \left[1+\lambda(s,a)\right]^{1-\sigma} E\left[\sum_{t=0}^{\infty} \beta^t \frac{c(s,a)^{1-\sigma}}{1-\sigma}\right] \\
&= \left[1+\lambda(s,a)\right]^{1-\sigma} v(s,a) \\
\lambda(s,a) &= \left(\frac{W^{FB}}{v(s,a)}\right)^{\frac{1}{1-\sigma}} - 1
\end{aligned}$$

What is the economy-wide welfare gain $\int_{(s,a)} \lambda(s,a)\mu^*(ds,da)$?