ECON 634 Problem Set 3

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1. The recursive problem is

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$s.t.\ c_t + q_t a_{t+1} = y(s_t) + a_t$$

$$a_{t+1} \in \Gamma(s_t, a_t)$$

$$a_{t+1} \geq \underline{a},\ \forall t,\ \text{and } a_0 \text{ given.}$$

The functional equation of the dynamic programming problem is

$$V(s,a) = \max_{a' \in \Gamma(s,a)} \left\{ \frac{[y(s) + a - qa']^{(1-\sigma)}}{1-\sigma} + \beta \mathbb{E}_{s'|s} V(s', a') \right\}$$

where s and a are the state variables and s' and a' are the control variables.
The state space is the combination of the possible states of asset

$$\mathscr{A} = \{a_1, a_2, ..., a_n\}$$

and the possible states of employment

$$\mathscr{S} = \{e, u\}.$$

The constraint correspondence is

$$\Gamma(a,s) = \left\{ a' : a' \geq 0,\ a' \leq \frac{y(s) + a}{q} \right\}.$$

2. Yes, according to the plot of the value function over K, it is concave and increasing in K.

3. Yes, according to the plot of the policy function over K, it is concave and increasing in K.