Problem Set 4

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The economy has the following production function $F(K,N) = K^{\alpha}L^{1-\alpha}$.
Profit each period : $\pi(K,N;w_t,r_t) = F(K_t,N_t) - w_tN_t - r_tK_t + (1-\delta)K_t$
Discounted lifetime profits are: $max_{\{K^d_{t+1},N^d_t\}} \sum_{t=0}^{\infty}\left(\frac{1}{\Pi_{i=0}^{t} r_i}\right)\pi(K,N;w_t,r_t)$

The consumer's Budget constraint will be: $c_t + a_{t+1} = z_t w_t \bar{l} + r_t a_t$ where z_t is labor productivity and it's log follows an AR(1) process.

1. F.O.C.'s for the firm to get factor prices.

For Capital:

$$\begin{aligned}
\frac{\delta\pi(K,N;w_t,r_t)}{\delta K_t^d} &= \sum_{t=0}^{\infty}\left(\frac{1}{\Pi_{i=0}^{t} r_i}\right)\left(\alpha\left(\frac{N}{K_t^d}\right)^{1-\alpha} + (1-\delta) - r_t\right) \\
0 &= \left(\alpha\left(\frac{N}{K_t^d}\right)^{1-\alpha} + (1-\delta) - r_t\right)\sum_{t=0}^{\infty}\left(\frac{1}{\Pi_{i=0}^{t} r_i}\right) \\
&= \frac{1}{1-\frac{1}{r_t}}\left(\alpha\left(\frac{N}{K_t^d}\right)^{1-\alpha} + (1-\delta) - r_t\right) \\
&= \frac{r_t}{1-r_t}\left(\alpha\left(\frac{N}{K_t^d}\right)^{1-\alpha} + (1-\delta) - r_t\right) \\
r_t &= \alpha\left(\frac{N_t^d}{K_t^d}\right)^{1-\alpha} + (1-\delta)
\end{aligned}$$

For Labor:

$$\begin{aligned}
\frac{\delta\pi(K,N;w_t,r_t)}{\delta N_t^d} &= \sum_{t=0}^{\infty}\left(\frac{1}{\Pi_{i=0}^{t} r_i}\right)\left((1-\alpha)\left(\frac{K}{N_t^d}\right)^{\alpha} + (1-\delta) - r_t\right) \\
&= (1-\alpha)\left(\frac{K_t^d}{N_t^d}\right)^{\alpha}
\end{aligned}$$

2. State Recursive Problem for Household

Household Problem:

$$V(z,a) = max_{a' \in \Gamma(z,a)} U(zw\bar{l} + ra + a') + \beta E_{z'|z}[V(z,a)]$$

a. The control variable is tomorrow's asset holding. The state variables are the current asset holding and labor productivity

b. Utility is CRRA

3. Dscritize State Space

4. Time to run.

Value Function take 354.48 sec to sun. The Policy Function takes 377.56 seconds to run on my computer on the virtual desktop.

5. Rental Rate of capital

The model puts out a steady state r of 1.0072. The Euler equaltion would predict an rental rate of $r_t = \frac{1}{\beta} = 1.0101$, slightly higher than this rate.

Graphs:

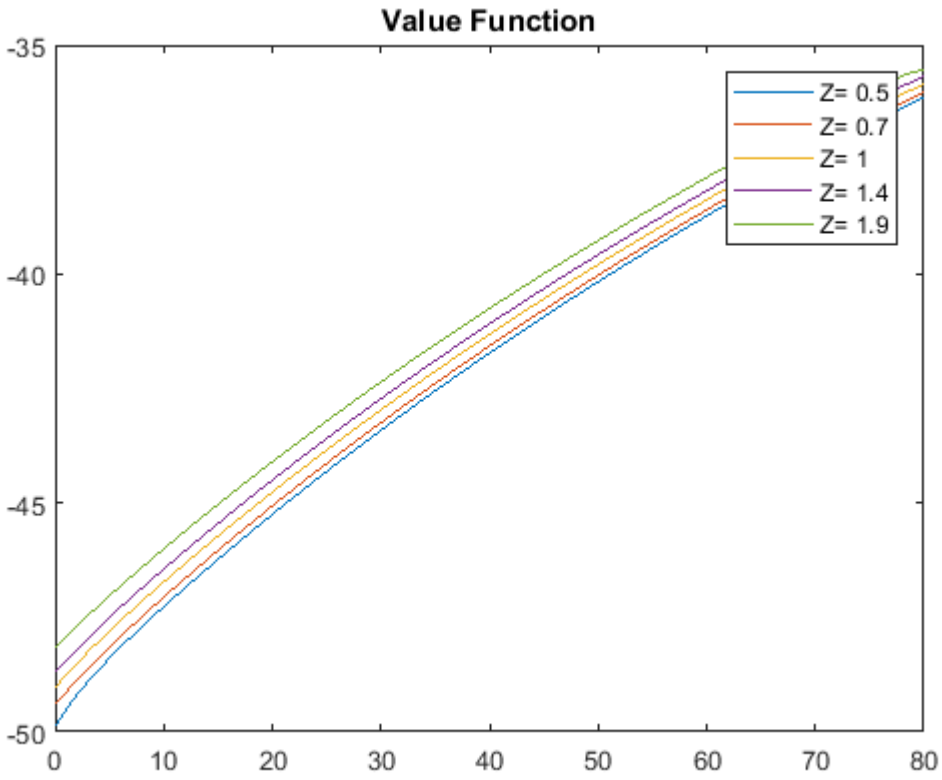
Value Function
-35
-40
-45
-50
0
10
20
30
40
50
60
70
80
Z= 0.5
Z= 0.7
Z= 1
Z= 1.4
Z= 1.9

Figure 1:

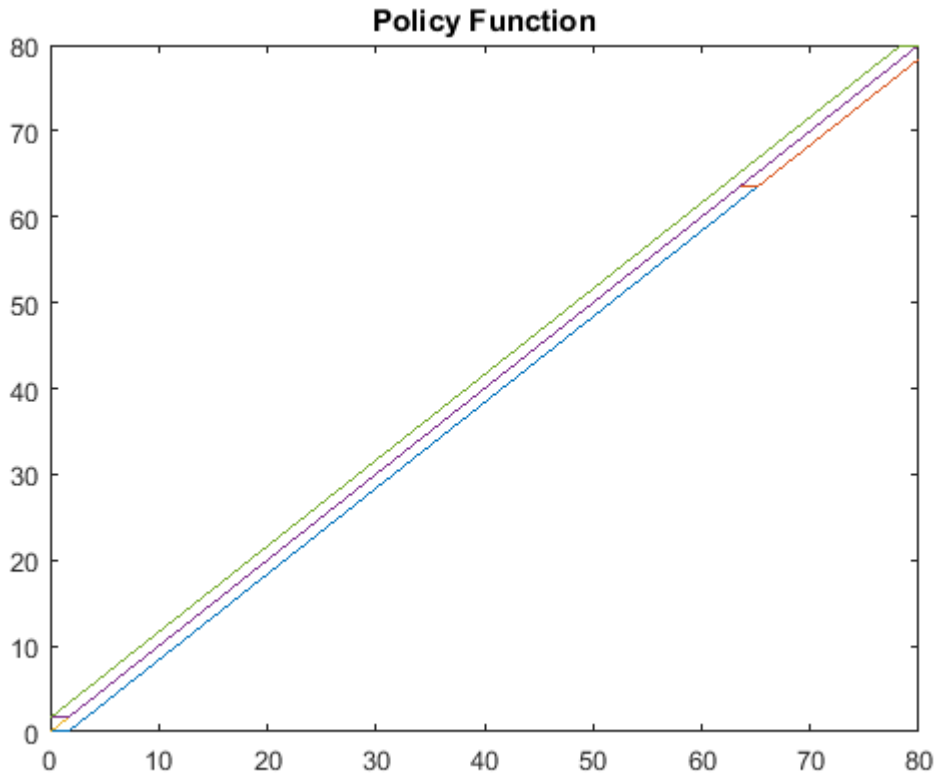

Figure 2:

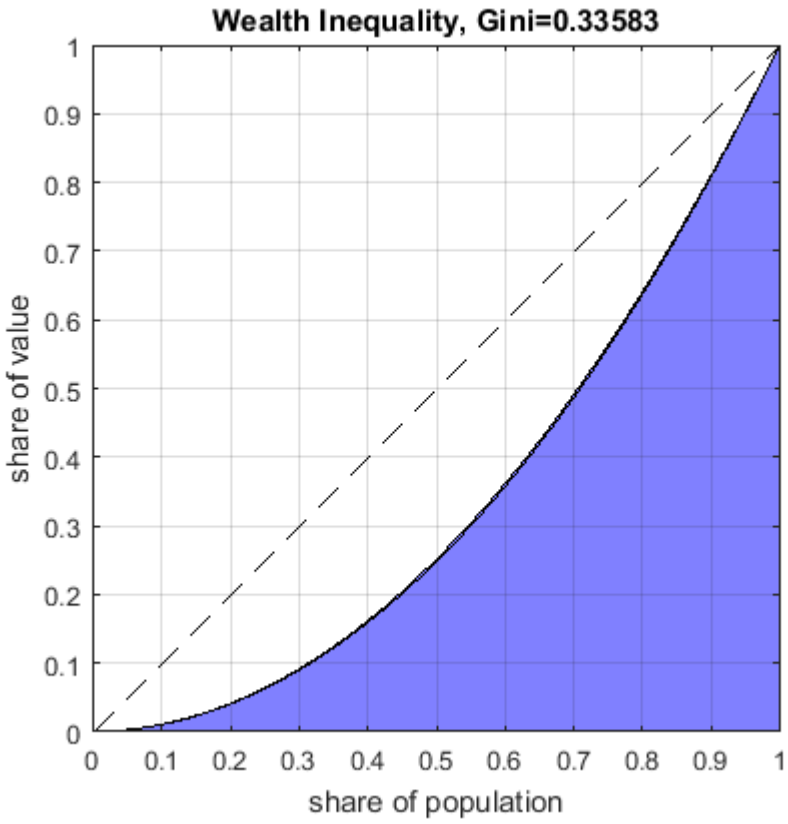
Wealth Inequality, Gini=0.33583
share of value
share of population
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 3:

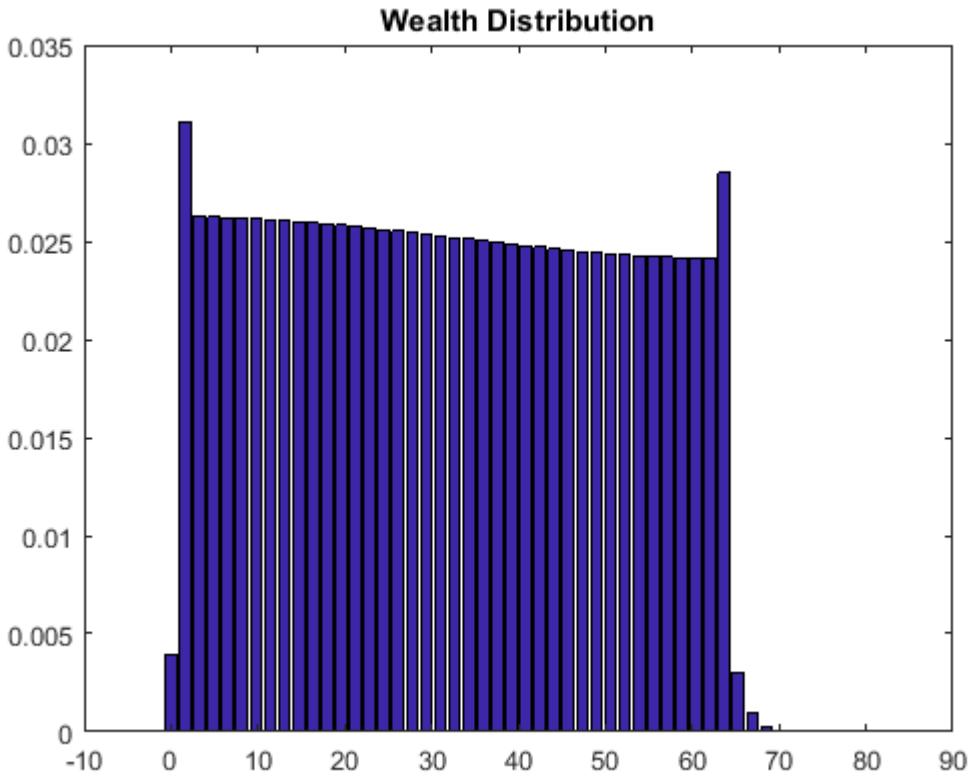
Wealth Distribution
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-10
0
10
20
30
40
50
60
70
80
90

Figure 4: