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Advanced Macro 2

HW#4

Aiyagari Model

1) Firm Problem:

$$\max_{K_t, N_t} K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - r_t K_t + (1-\delta) K_t$$

FOC:

$$K_t: \ \alpha \left(\frac{N_t}{K_t}\right)^{1-\alpha} - r_t + (1-\delta) = 0$$

$$\Rightarrow \quad \boldsymbol{r_t = \alpha \left(\frac{N_t}{K_t}\right)^{1-\alpha} + (1-\delta)}$$

$$N_t: \ (1-\alpha) \left(\frac{K_t}{N_t}\right)^{\alpha} - w_t = 0$$

$$\Rightarrow \quad \boldsymbol{w_t = (1-\alpha) \left(\frac{K_t}{N_t}\right)^{\alpha}}$$

2) Household Problem:

$$V(z,a) = \max_{a' \in \Gamma(z,a)} U(\, zw\bar{l} + ra - a') + \beta E_{z'|z}[V(s', a')]$$

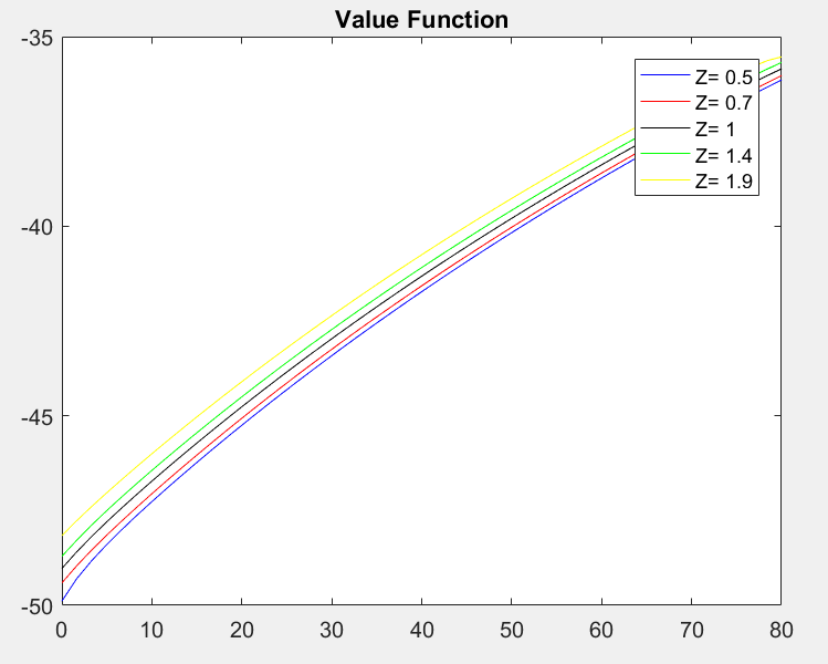
   a. The control variable is asset holding for tomorrow. The state variables are the current labor productivity and current asset holding.
   b. Utility is CRRA.

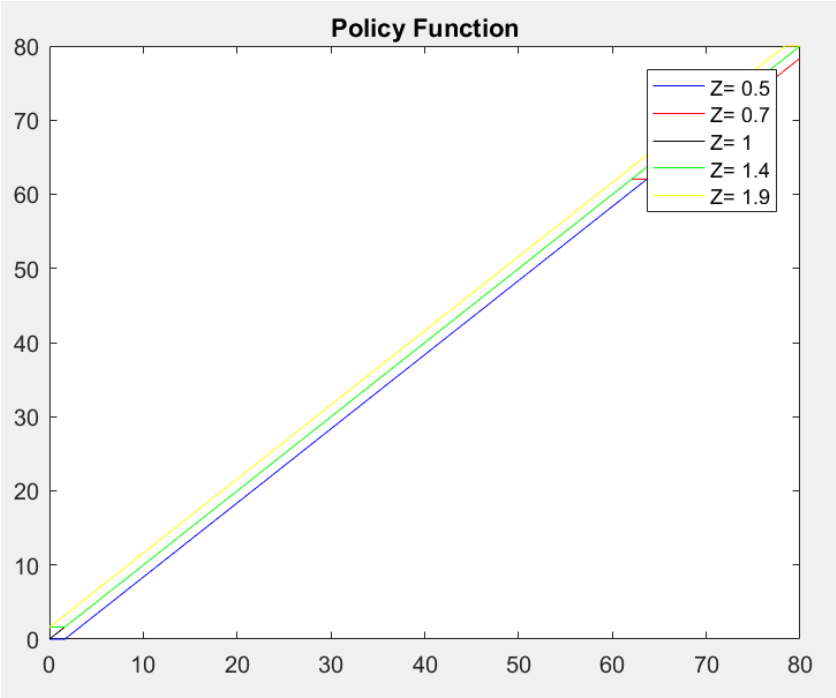
3) Productivity space was discretized into 5 points with a 5x5 transition matrix. The invariant distribution was calculated by normalizing the 1st eigenvector of the transition matrix. The effective aggregate labor supply is the discrete sum (across 5 Z-states) of the product $Z_i * \pi_i^{inv}$.

4) The value function iteration runtime was 158.39 seconds, whereas the policy function runtime was slightly faster at 153.99

5) The Aiyagari model has a steady state interest rate of 1.0089. The competitive market rate using the Euler is $r_t = \frac{1}{\beta}$, which is 1.0101. The Aiyagari rate is lower than the competitive equilibrium.

6) Policy and Value Functions:





7) Lorenz, Gini and Wealth Distribution

The wealth distribution is much less positively skewed and the