ECON 634 Homework 4

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Question 1

The firms want to solve the optimization function

$$\begin{aligned}&\max_{\{K_{t+1}^d,\ N_t^d\}} \sum_{t=0}^{\infty} \left(\frac{1}{\prod_{i=0}^{t} r_i}\right) \pi(K,\ N;\ w_t,\ r_t) \\ &= \max_{\{K_{t+1}^d,\ N_t^d\}} \sum_{t=0}^{\infty} \left(\frac{1}{\prod_{i=0}^{t} r_i}\right) \left[K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - r_t K_t + (1-\delta) K_t\right]\end{aligned}$$

and the first order condition $w.r.t\ K_t$ and N_t will be

$$\begin{aligned}\left(\frac{1}{\prod_{i=0}^{t} r_i}\right) \left[\alpha K_t^{\alpha-1} N_t^{1-\alpha} - r_t + (1-\delta)\right] = 0 \\ \left(\frac{1}{\prod_{i=0}^{t} r_i}\right) \left[(1-\alpha) K_t^{\alpha} N_t^{-\alpha} - w_t\right] = 0.\end{aligned}$$

Given K and N, these two functions show the factor prices are

$$\begin{aligned}r_t &= \alpha K_t^{\alpha-1} N_t^{1-\alpha} + (1-\delta) \\ w_t &= (1-\alpha) K_t^{\alpha} N_t^{-\alpha}.\end{aligned}$$

Question 2

The households want to solve the optimization function

$$v(z_t,\ a_t) = \max_{a_{t+1}} \frac{(z_t w_t \bar{l} + r_t a_t - a_{t+1})^{1-\sigma}}{1-\sigma} + \beta E_t\left[v(z_{t+1},\ a_{t+1})\right].$$

Question 3

By setting the grid number $nz = 5$ and the scale $m = 3$, the grid is $Z = \begin{pmatrix}0.5002 & 0.7072 & 1.0000 & 1.4140 & 1.9993\end{pmatrix}$ and the invariant distribution of productivity states are $\pi^{inv}(z) = \begin{pmatrix}0.0145 & 0.2189 & 0.5333 & 0.2189 & 0.0145\end{pmatrix}$. The aggregate effective labor supply $N^s = 1.0338$.

Question 4

Included in the *Matlab* code.

Question 5

I set the tolerance level as 0.01. First, use $\bar{a} = 80$ as the upper bound of a so that it will not bound households' saving. The equilibrium gives interest rate 0.99%, capital level $K = 30.4610$.

Question 6

The interest rate is 0.99%. For complete market, $r^{CM} = 1.01\%$. So the interest rate in lower than complete market interest rate. The policy functions are shown in figure 1.

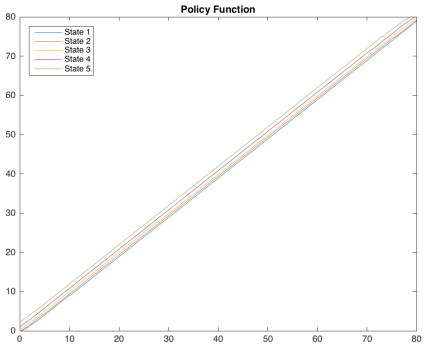
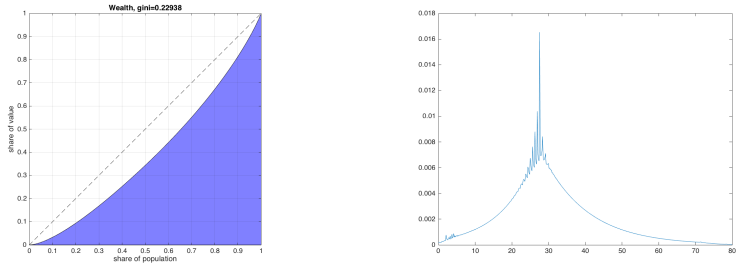


Figure 1: Policy Function

The Lorenz curve and the wealth distribution are shown below, and I get Gini coefficient 0.22938. Comparing with Huggett model, Aiyagari model has higher level of equality with lower Gini coefficient, and the wealth distribution gradually increases and suddenly goes to the peak around $K = 30$ and decreases gradually to zero. Overall, it is very different from the empirical wealth distribution.



(a) Lorenz Curve (b) Wealth Distribution

Figure 2: Wealth Distribution

Question 7

Given a fixed $r = 1.0099$, the Euler equation for household is

$$c_t^{-\sigma} = \beta E\left[c_{t+1}^{-\sigma} r\right]$$

The running time is 642.370s using VFI only. Using Coase Grid can reduce the running time to 344.740s, and PFI takes 525.154s to get the result. I use the weighted Euler equatoin error to measure the accuacy.

The accuracy is 0.00497 for VFI. After doing the linear interpolation, the error is 0.00495, which is slightly more accurate. But the running time ($4848.917s$) is much longer.