634 HW4

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1. Firms' Problem Firms' problem is expressed as:

$$\max_{K_{t+1}^d, N_t^d} \sum_{t=0}^{\infty} \left(\frac{1}{\prod_{i=0}^{t} r_i} \right) \pi(K, N; w_t, r_{t-1}) \tag{1}$$

Where $\pi(K, N; w_t, r_{t-1}) = K^\alpha N^{1-\alpha} - w_t N_t - r_{t-1} K_t + (1-\delta) K_t$. Taking the first-order condition with respect to K_{t+1}^d,N_t^d, we have:

$$\begin{aligned} \alpha \left(\frac{N_{t+1}^d}{K_{t+1}^d} \right)^{1-\alpha} - r_t + (1-\delta) = 0 \\ (1-\alpha) \left(\frac{K_t^d}{N_t^d} \right)^{\alpha} - w_t = 0 \end{aligned} \tag{2}$$

Solve the equations, we have:

$$\begin{aligned} r_t &= \alpha \left(\frac{N_{t+1}^d}{K_{t+1}^d} \right)^{1-\alpha} + (1-\delta) \\ w_t &= (1-\alpha) \left(\frac{K_t^d}{N_t^d} \right)^{\alpha} \end{aligned} \tag{3}$$

2. Households' Recursive Problem Households rental price r_t and wage w_t as given to maximize their lifetime utility, hence the recursive form can be expressed as:

$$\begin{aligned} v(z, a) &= \max_{a' \in \Gamma(z,a)} \frac{c^{1-\sigma}}{1-\sigma} + \beta E[v(z', a')] \\ s.t \quad c + a' &= zw\bar{l} + ra \\ z' &= \rho \ln z + \epsilon_t, \quad \epsilon \sim \mathbf{N}(0, \sigma_\epsilon^2) \\ a' &> \underline{a} \end{aligned} \tag{4}$$

3. Discretization of Exogenous State Variable Using Tauchen method in MATLAB, we acquire a set of discrete labor efficiency and a 5-by-5 transition matrix. We can calculate the aggregate effective labor supply as:

$$\begin{aligned} N^s &= \int_Z z\bar{l}\pi^{inv}(dz) \\ &= 1.0338 \end{aligned} \tag{5}$$

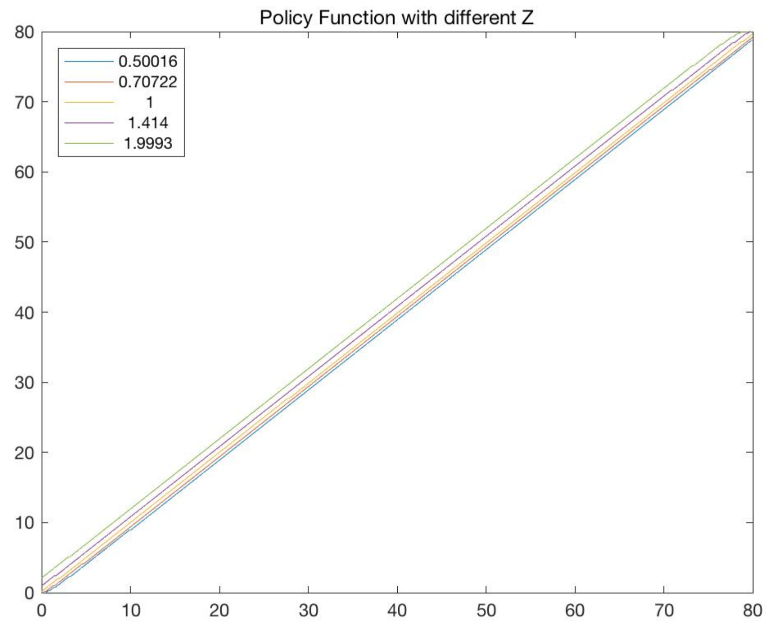
4. Discretization of Endogenous State Variable We set $a_{min} = \underline{a} = 0$, and guess $a_{max} = 7$ for now.

5. VFI By guessing an equilibrium value for the aggregate capital stock K, as well as the equilibrium in the labor market ($N^d = N^s$), we can find out the factor prices using equation(3):

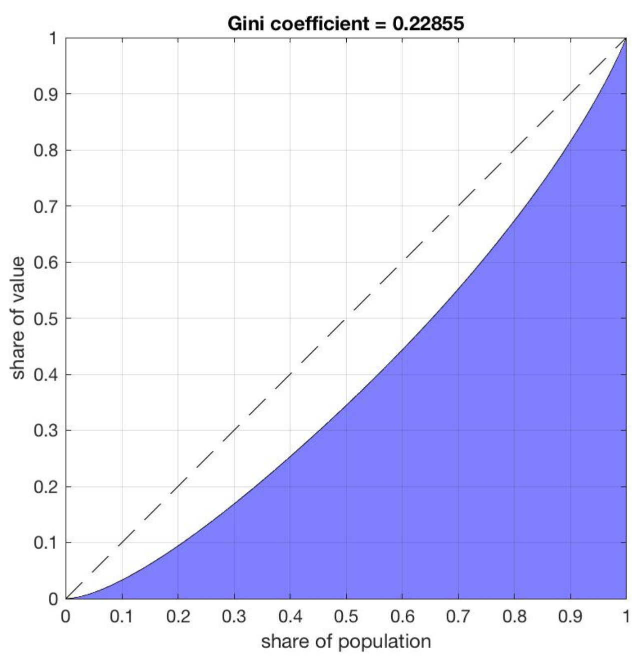
$$\begin{aligned} r &= \alpha \left(\frac{N^s}{K}\right)^{1-\alpha} + (1-\delta) \\ w &= (1-\alpha)\left(\frac{K}{N^s}\right)^{\alpha} \end{aligned} \tag{6}$$

Then we can do a value function iteration similar to the one we did in the Huggett model, and adjust K to satisfy the market clearing condition. Hence we calculate that the steady state aggregate capital stock is $K = 6.8223$.

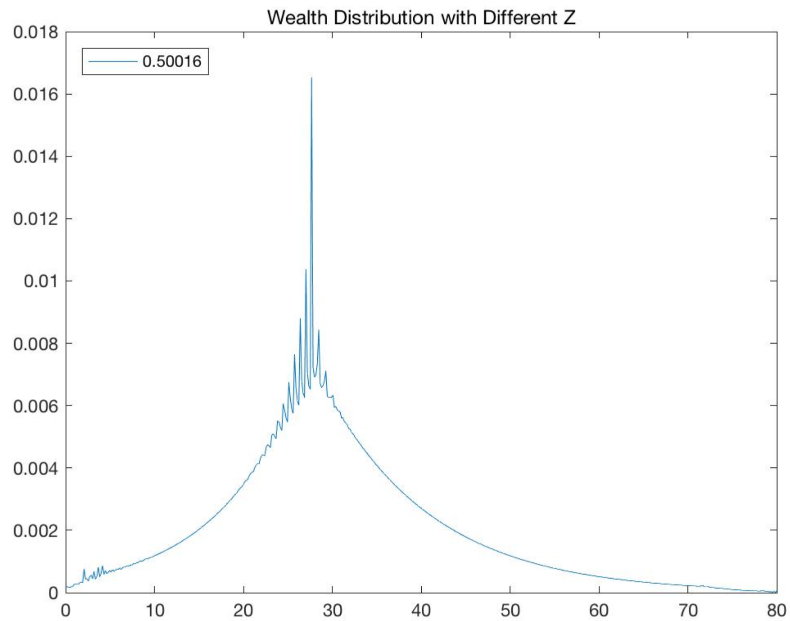
6. Result Analysis The steady state interest rate we got from VFI is $r = 1.0099$, which is approximately the same as the interest rate in complete market: $r^{CM} = 1/\beta = 1.0101$.
The policy functions for different productivity states are plotted below:



The Lorenz curve are plotted below. The wealth gini coefficient is 0.2286.



The wealth distribution is plotted below. Comparing with the distribution in the Huggett model, there is no sudden cut off at certain wealth level. But it's less smooth than the empirical data. There is a spike around 30.



7. Various Methods The baseline coarse grid VFI yields a run time of 181.074 seconds and the Euler equation error is 0.0386.
The policy function iteration with $k = 30$ yields a run time of 489.480 seconds and the Euler equation error is 0.0049.
The linear interpolation method yields a run time of 238.925 seconds and the Euler equation error is 0.0050.
The cubic interpolation method yields a run time of 237.105 seconds and the Euler equation error is 0.0050.
We can see that the policy function iteration has better accuracy over the coarse grid VFI, but it takes a lot more times. Both interpolation methods have a relative short run time and same accuracy as PFI.