Homework 4

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Step 1: Frim's problem

Firm's problem now is:

$$\max_{\{K^d_{t+1},N^d_t\}} \sum_{t=0}^{\infty}(\frac{1}{\prod_{i=0}^{t} r_i})\pi(K,N;w_t,r_t) = \max_{\{K^d_{t+1},N^d_t\}} \sum_{t=0}^{\infty}(\frac{1}{\prod_{i=0}^{t} r_i})(K_t^{\alpha}N_t^{1-\alpha} - w_tN_t - r_tK_t + (1-\delta)K_t) \tag{1}$$

By doing the first order condition with respect to K_{t+1} and N_t, we can get the following conditions:

$$\alpha K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha} = r_{t+1} + \delta - 1 \tag{2}$$

$$(1-\alpha)K_t^{\alpha}N_t^{-\alpha} = w_t \tag{3}$$

Equation (2) also holds for period t, so we should also have that

$$\alpha K_t^{\alpha-1}N_t^{1-\alpha} = r_t + \delta - 1 \tag{4}$$

Step 2: Household value function

The household choose c_t and a_{t+1} to maximize the expected utlity, or we can write c_t as a function of a_{t+1} so that we can write down the value function as:

$$v(z_t, a_t) = \max_{a_{t+1}} U(c_t) + \beta E v(z_{t+1}, a_{t+1}) \tag{5}$$

where $c_t = z_t w_t \bar{l} + r_t a_t - a_{t+1}$, and the utility function is CRRA utility function.

Step 3: Discretize the exogenous state variable

For $\rho = 0.5$, $\sigma_{\epsilon} = 0.2$ and number of z equals to 5, we can get a discrete set of possible values for z:

$$z = \begin{bmatrix} 0.5002 & 0.7072 & 1.0000 & 1.4140 & 1.9993 \end{bmatrix} \tag{6}$$

we can also get the invatiant distribution from $m \times m$ transition matrix:

$$\pi^{inv}(z) = \begin{bmatrix} 0.0145 & 0.2189 & 0.5333 & 0.2189 & 0.0145 \end{bmatrix} \tag{7}$$

So the aggregate labor supply $N^s = 1.0338$

Step 4: Discretize the endogenous state variable

In this step, we discretize a into a grid with 500 points. At this time, I choose $a_{max} = 5$, and we will adjust this number later according to the binding constraint.

Step 5: Solving the model numerically

From Step 3, we know that $N^s = 1.0338$, thus $N_d = 1.0338$ due to market clear condition. Now let's guess $K = 2$, and using equation (3) and equation (4), we can calculate the value for r and w.

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} + 1 - \delta \tag{8}$$

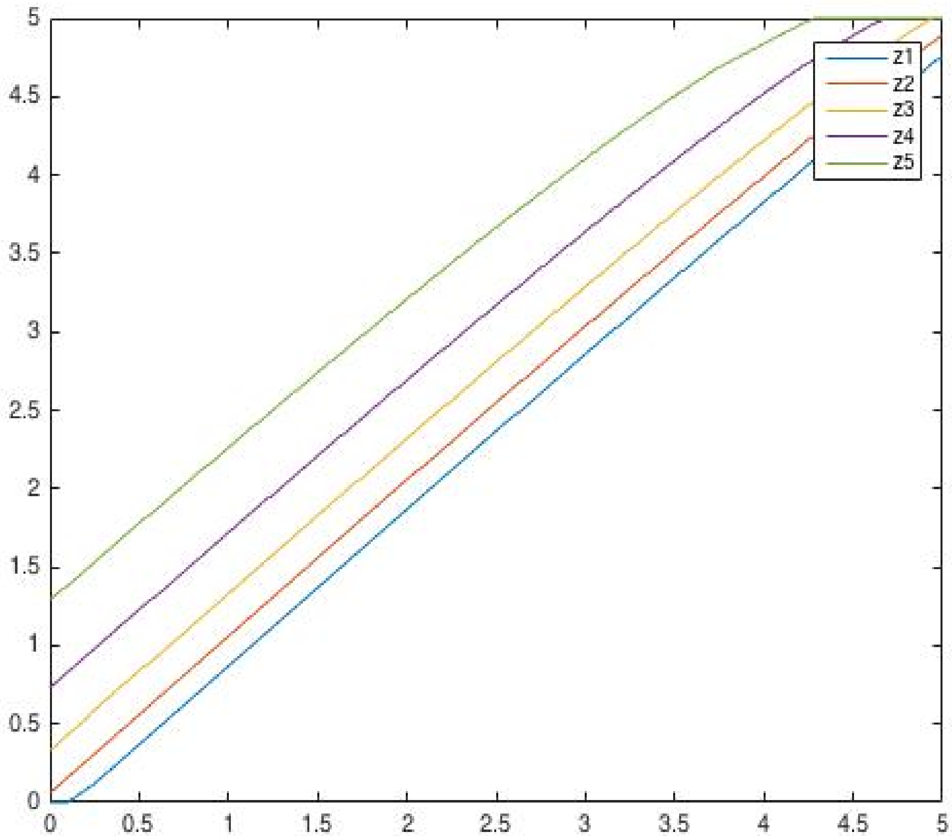
$$w_t = (1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{9}$$

Please see the Appendix for Matlab code.

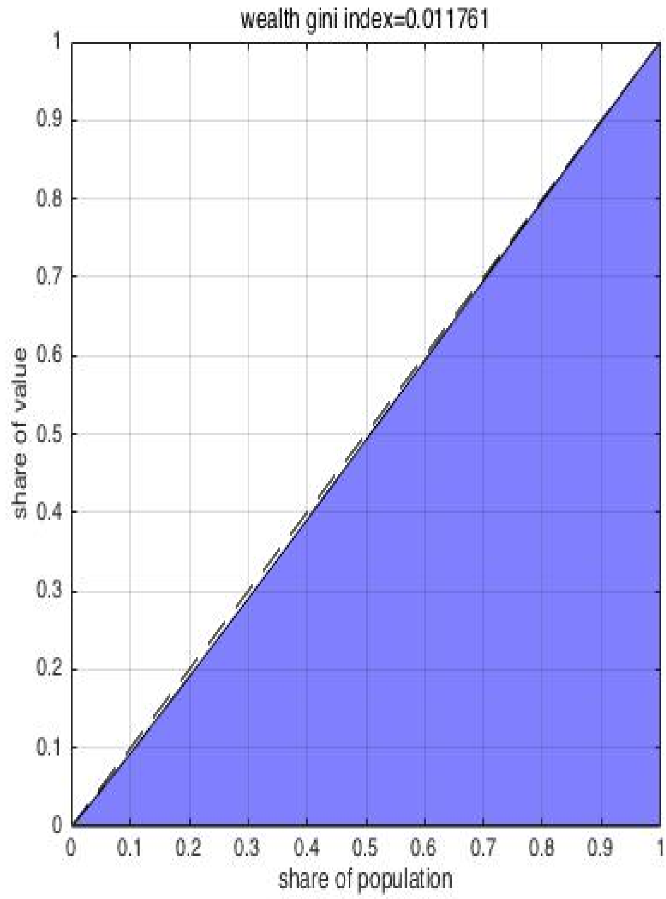
Step 6: Analyze the results

a. The steady state interest rate I get is $r = 1.0928$, while in the complete market case $r^{CM} = 1/\beta = 1.0101$. So interest rate in the incomplete market is higher. Why???

b. Below is the policy functions for m productivity series:



c. This time we take asset holding as wealth(why?), and the Lorenz curve is:



The gini index is smaller comparing with gini index in Hugget model.

Step 7: Alternative way of coding

Solving the value function on a coarse grid, and using the result as a starting value. Then use policy function iteration.

For this approach, runtime is .While the runtime is when using valur funtion iteration.

Please see appendix.

Appendix: Matlab code

Way 1

```
%%%%%%%%parameters%%%%%%%
```

```
alpha=1/3;
beta=0.99;
s=2;
delta=0.025;


%%%%%question 3%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m=5;
rho=0.5;
sigma=0.2;
d=3;

[z,zprob] =TAUCHEN(m,rho,sigma,d);
z=exp(z);
pi=zprob^1000;
pz=pi(1,:);
ns=pz*z;
%%%%%%%question 4%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a_min=0;
a_max=5;
a_num=500;
a=linspace(a_min,a_max,a_num);
%%%%%%question 5%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
nd=ns;
k_guess=2;

d=1;
while d>=0.001;
r=alpha*k_guess^(alpha-1)*nd^(1-alpha)+1-delta;
w=(1-alpha)*k_guess^alpha*nd^(-alpha);

%consumption fn
cons=bsxfun(@minus,r*a',a);
cons=bsxfun(@plus,cons,permute(w*z,[2 3 1]));
%return fn
ret=cons.^(1-s)/(1-s);
ret(cons<0)=-Inf;
%value fn and policy fn
v_guess=zeros(m,a_num);
v_tol = 1;
while v_tol >.0001;
% CONSTRUCT RETURN + EXPECTED CONTINUATION VALUE

vf=bsxfun(@plus,ret,permute(beta*zprob*v_guess,[3,2,1]));
```

```
% CHOOSE HIGHEST VALUE (ASSOCIATED WITH a' CHOICE)
[vfn,pol_indx]=max(vf,[],2);
v_tol=[max(abs(vfn(:,:,1)' - v_guess(1,:))) ; max(abs(vfn(:,:,2)' - v_guess(2,:)));
max(abs(vfn(:,:,3)' - v_guess(3,:)));max(abs(vfn(:,:,4)' - v_guess(4,:)));...
max(abs(vfn(:,:,5)' - v_guess(5,:)))];
v_guess=[vfn(:,:,1)';vfn(:,:,2)';vfn(:,:,3)';vfn(:,:,4)';vfn(:,:,5)'];
end;
% KEEP DECSISION RULE
pol_indx=permute(pol_indx, [3 1 2]);
pol_fn=a(pol_indx);
% SET UP INITITAL DISTRIBUTION
Mu=ones(m,a_num)/(m*a_num);

% ITERATE OVER DISTRIBUTIONS
m_tol=1;
while m_tol>0.0001
[z_ind, a_ind, mass] = find(Mu > 0); % find non-zero indices

MuNew = zeros(size(Mu));


for ii = 1:length(z_ind)
apr_ind = pol_indx(z_ind(ii), a_ind(ii)); % which a prime does the policy fn prescr
MuNew(:, apr_ind) = MuNew(:, apr_ind) +[zprob(z_ind(ii),1)*Mu(z_ind(ii),a_ind(ii));
% which mass of households goes to which exogenous state?
end
m_tol=max(max(abs(MuNew-Mu)));
Mu=MuNew;
end

aggsav=Mu(1,:)*a'+Mu(2,:)*a'+Mu(3,:)*a'+Mu(4,:)*a'+Mu(5,:)*a';
d=abs(k_guess-aggsav);
if k_guess<aggsav
k_guess=k_guess+abs(k_guess-aggsav)/2;
else k_guess=k_guess-abs(k_guess-aggsav)/2;
end
k_guess;
d;
end

%%%%%%%%question 6
plot(a,pol_fn(1,:),a,pol_fn(2,:),a,pol_fn(3,:),a,pol_fn(4,:),a,pol_fn(5,:)),legend(

%gini
p=[Mu(1,:);Mu(2,:);Mu(3,:);Mu(4,:);Mu(5,:)];
```

```
wealth=[a;a;a;a;a];
wg=gini(p,wealth,true);
title(['wealth gini index=',num2str(wg)]);
```