SUNY at Binghamton

Econ 634 Advanced Macroeconomics

Homework 4: Aiyagari

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1 Question 1

The firm's maximization problem is:

$$\max_{(K^d_{t+1},N^d_t)} \sum_{t=0}^{\infty} \left(\frac{1}{\Pi^t_{i=0}}\right) \pi(K, N, w_t, r_t). \tag{1}$$

$$= \max_{(K^d_{t+1},N^d_t)} \sum_{t=0}^{\infty} \left(\frac{1}{\Pi^t_{i=0}}\right) [K^{\alpha}_t N^{1-\alpha}_t - w_t N_t - r_t K_t + (1-\delta)K_t]$$

F.O.C for firm w.r.t K_{t+1} and N_t:

$$\begin{aligned} K_{t+1} &: \alpha K^{\alpha-1}_{t+1} N^{1-\alpha} - r_t + (1-\delta) = 0 \\ N_t &: (1-\alpha) K^{\alpha}_{t+1} N^{-\alpha} - w_t = 0 \end{aligned}$$

Therefore the factor prices will be:

$$r_t = \alpha \left(\frac{N}{K_{t+1}}\right)^{1-\alpha} + (1-\delta).$$
$$w_t = (1-\alpha) \left(\frac{K_{t+1}}{N}\right)^{\alpha}.$$

2 Question 2

The household recursive problem is:

$$V(K_t, Z_t) = \max_{a_{t+1}} \frac{(z_t w_t \bar{l} + r_t a_t - a_{t+1})^{1-\alpha}}{1-\sigma} + \beta E_{z_{t+1}} V(K_{t+1}, Z_{t+1}). \tag{2}$$

3 Question 3

By setting $nz = 5$ and max +− std.devs $m = 3$ we have:
The grid of Z = (0.5002 0.7072 1.000 1.4140 1.9993)
The Invariant Distribution of productivity states is $\pi^{inv}(Z)$ = (0.0145 0.2189 0.533 0.219 0.0145)
The aggregate labor supply is $N_s = 1.033$.

4 Question 4 5 6

There's some error in my code that I havent figure it out yet so i will try to fix it asap and update my submission.