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Advanced Macro II

HW#6

Bayes Estimation using Metropolis Hastings

#1) Estimate the following using OLS:

$$\ln wage = \beta_0 + \beta_1 Educ + \beta_2 Exp + \beta_3 I(MSA) + \beta_4 I(Black) + \beta_5 I(South) + \epsilon$$

Where  $\epsilon$  is assumed to come from a normal distribution with mean zero with a constant variance.

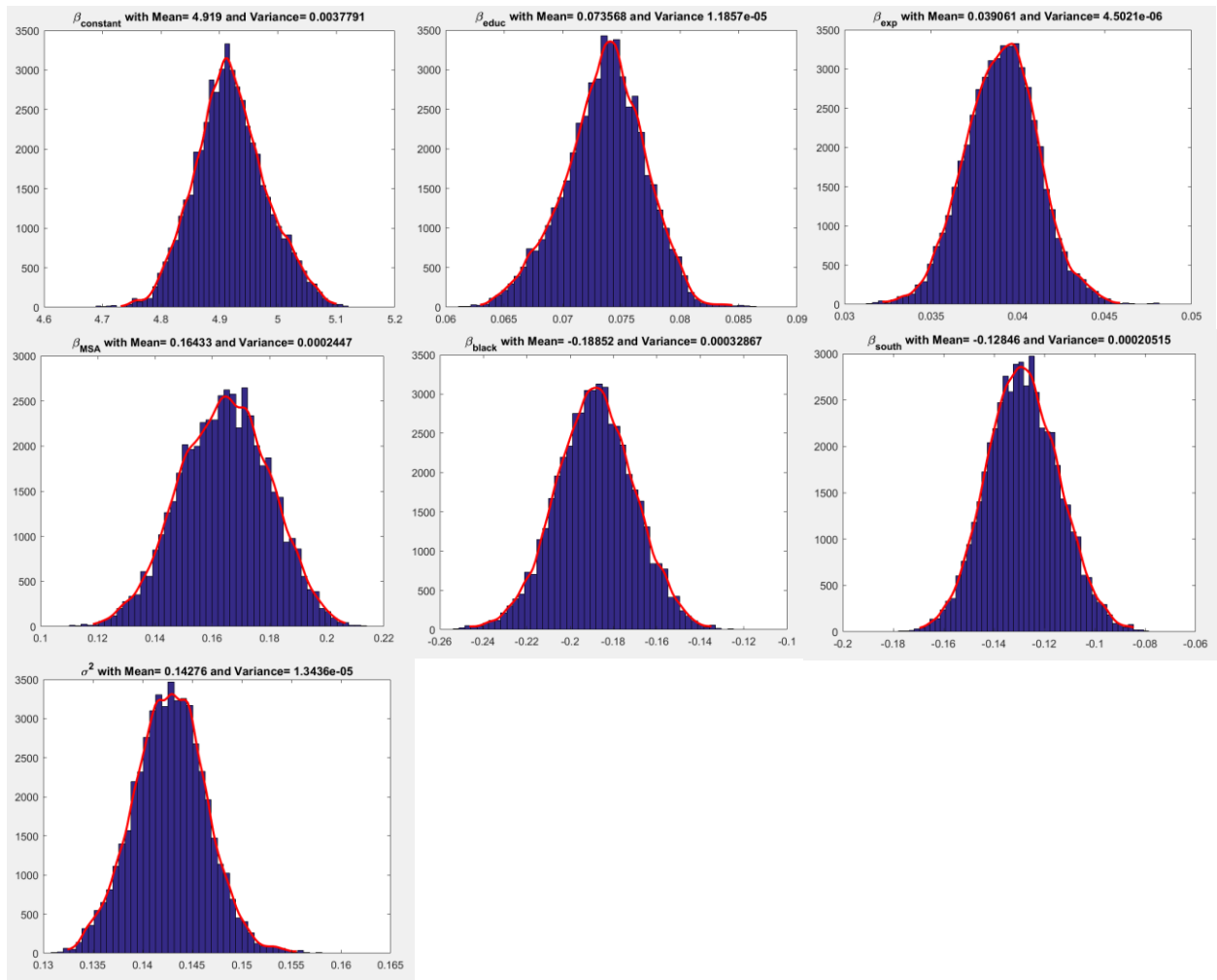
It is straight forward to obtain the sample sigma squared and variances of the coefficients, as well as the variance of sigma squared.

#2 A) Estimate posterior using Metropolis Hastings Algorithm under a flat prior

Steps:

- i. Scale the variances of betas and sigma squared
- ii. MH\_flat.m: uses current vector of parameters  $(\mathbf{B}, \sigma^2)$  to draw the proposed guess.
  - a. The program then constructs the log likelihood functions for the proposed and current theta vectors. We use the normal distribution based on the assumption of normal errors.
  - b. Next, it compares the ratio of Likelihoods (or Difference in Log Likelihoods) to a random draw from a uniform distribution. If the ratio is greater than the random draw, the function accepts the proposal and outputs a new Theta vector. If not, the loop carries the old theta vector onto the next iteration.

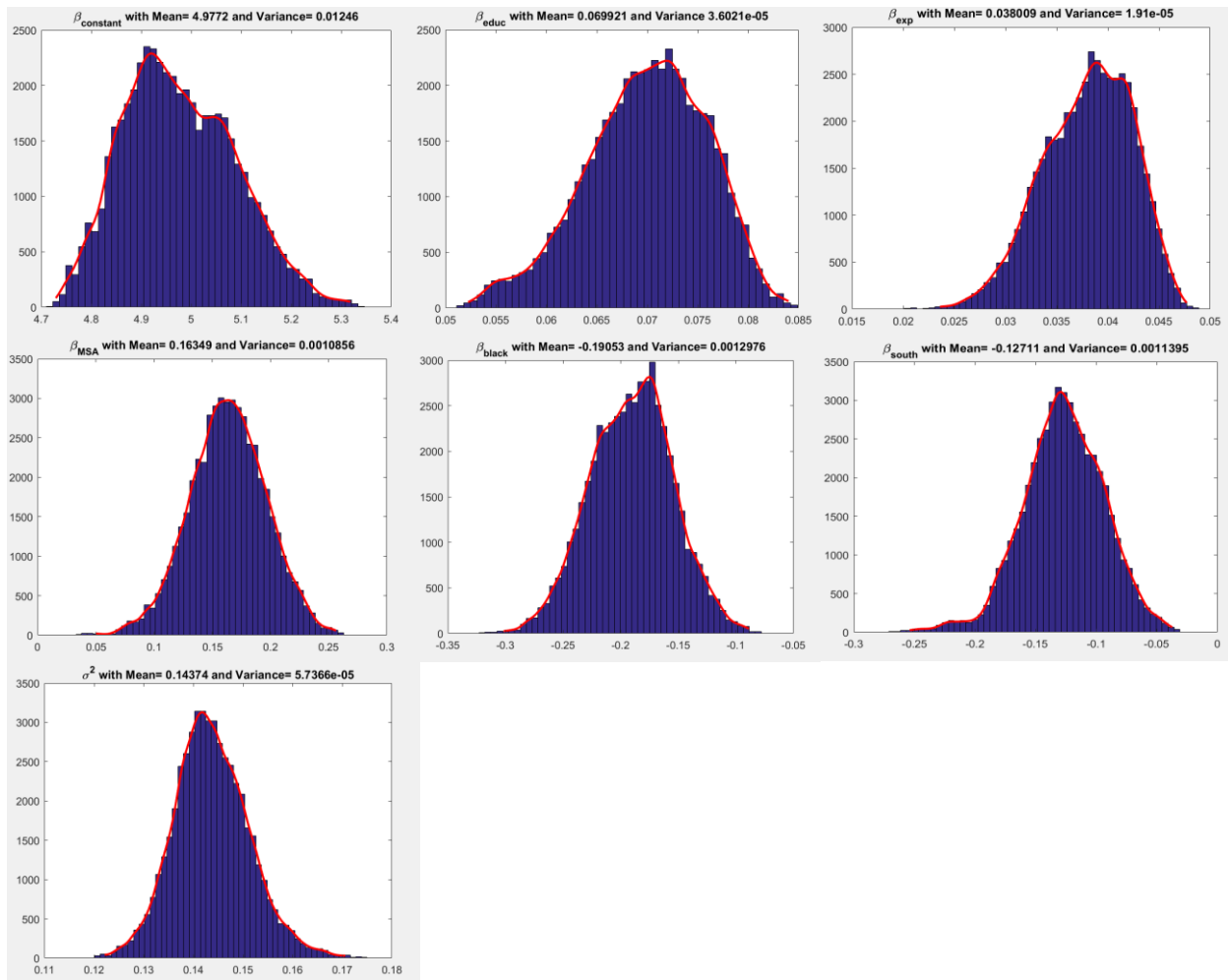
iii. Plot histograms with flat prior:



## #2 B) Estimate posterior using Metropolis Hastings Algorithm using prior for education parameter

Steps:

- i. Same process as with flat prior, except now the log likelihood is the sum individual log likelihoods including the education prior.
  - a. Prior for Education coefficient is 0.06. Theta will follow a normal distribution with a mean of 0.06 and standard deviation of 0.007. The standard deviation here is twice the standard deviation of the education coefficient from the OLS estimation.
- ii. Plot histograms with education prior:



The main difference between OLS and Bayes is that OLS provides point estimates of parameters, whereas the Bayes estimation provides distributions of parameters. From the OLS perspective, parameters are fixed with a 95% confidence interval. But from the Bayes perspective, parameters lie within a certain interval with a 95% chance.