HOME 6. BAYESIAN OLS – REPORT

HUAYAN GENG

Q1. OLS Estimation

The linear model is

$$\begin{aligned} log(wage) &= \beta_0 + \beta_{educ} Educ + \beta_{exp} Expr + \beta_{SMSA} D_{SMSA} + \beta_{black} D_{race} + \beta_{south} D_{region} + \varepsilon \\ &\text{or} \\ Y &= X\beta + \varepsilon \end{aligned}$$

where $\varepsilon \sim \boldsymbol{N}(0, \sigma^2)$.

Given dataset card.csv, Table 1 shows the OLS estimation output and standard error of all coefficients.

Table 1. OLS Estimation Output

Variables	$\hat{\beta}$	s.e
Constant (β_0)	4.913331	0.063121
Education (β_{educ})	0.073807	0.003534
Experience (β_{exp})	0.039313	0.002196
SMSA (β_{SMSA})	0.164741	0.015692
Race (β_{black})	-0.188223	0.017768
Region (β_{south})	-0.129053	0.015229

Considering homoskedasticity, the $\hat{\sigma}_\varepsilon^2 = \frac{RSS}{n-k} = 0.1423$ which is unbiased estimator of σ^2. As we assumed , the $(n-k)\hat{\sigma}_\varepsilon^2/\sigma^2 \xrightarrow{d} \chi^2_{n-k}$ whose variance is $2(n-k)$. Therefore, the

$$\begin{aligned} Var\left((n-k)\hat{\sigma}_\varepsilon^2/\sigma^2\right) &= 2(n-k) \\ \left(\frac{n-k}{\sigma^2}\right)^2 Var\left(\hat{\sigma}_\varepsilon^2\right) &= 2(n-k) \\ Var\left(\hat{\sigma}_\varepsilon^2\right) &= 2(n-k)\left(\frac{\sigma^2}{n-k}\right)^2 \\ Var\left(\hat{\sigma}_\varepsilon^2\right) &= 2\sigma^4/(n-k) \end{aligned}$$

. Given data, $Var\left(\hat{\sigma}_\varepsilon^2\right) = 1.3479 \times 10^{-5}$.

Q2. Bayesian Estimation (Metropolis-Hastings)

The parameter vector I am supposed to estimate is $\theta = \begin{bmatrix} \beta_0 & \beta_{educ} & \beta_{exp} & \beta_{SMSA} & \beta_{black} & \beta_{south} & \sigma_\varepsilon^2 \end{bmatrix}^T$ and our proposal $\tilde{\theta}_{n+1} = \hat{\theta}_n + v$ where v follows multivariate normal distribution $\boldsymbol{N}(\mu_0, \Sigma)$.

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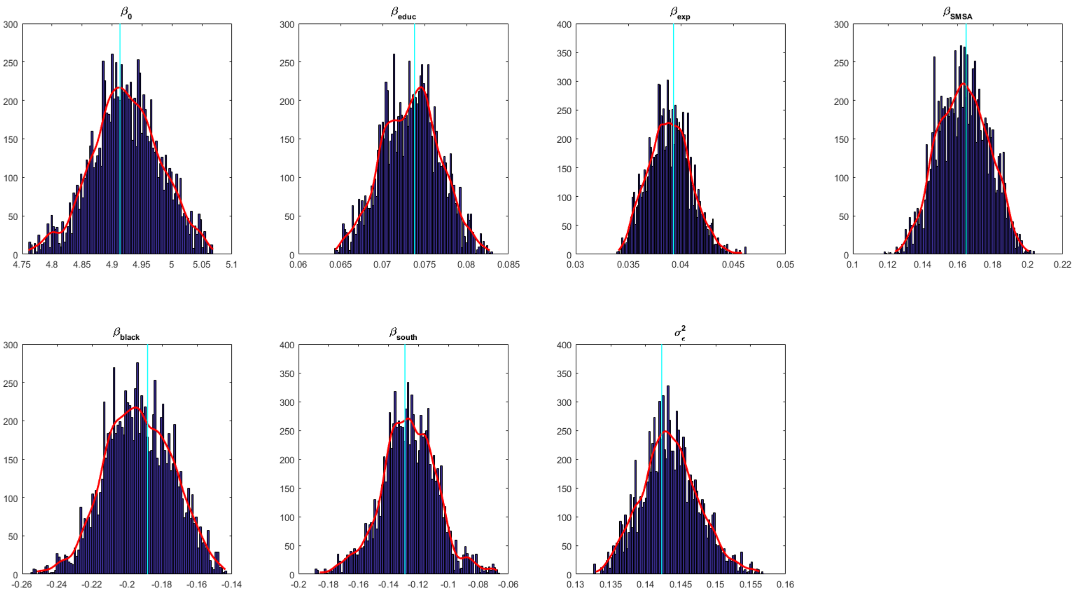
$\mu_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and

$$\Sigma \quad = r \times \begin{bmatrix} Var(\hat{\beta}_0) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Var(\hat{\beta}_{educ}) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Var(\hat{\beta}_{exp}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Var(\hat{\beta}_{SMSA}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Var(\hat{\beta}_{black}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Var(\hat{\beta}_{south}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Var(\hat{\sigma_\varepsilon^2}) \end{bmatrix}$$

where r is the adjusted scalar of the variance-covariance matrix. The likelihood of $Y \equiv log(wage)$ follows normal distribution because $\varepsilon \sim \boldsymbol{N}(0, \sigma^2)$. Also, $L(\theta) \equiv P(y|\theta) \times P(\theta)$. When $\frac{L(\tilde{\theta}_{n+1})}{L(\hat{\theta}_n)} > u$, we accept proposal $\hat{\theta}_{n+1} = \tilde{\theta}_{n+1}$, otherwise $\hat{\theta}_{n+1} = \hat{\theta}_n$ where u is a random number between 0 and 1.

A. Flat Priors. Due to the flat priors of all parameters, $\frac{L(\tilde{\theta}_{n+1})}{L(\hat{\theta}_n)} = \frac{P(y|\tilde{\theta}_{n+1})}{P(y|\hat{\theta}_n)}$. Figure 1 shows posterior distribution using flat priors where red line indicates the kernel distribution of estimators and vertical line indicates the OLS point estimates.

FIGURE 1. Posterior Distributions with Flat Prior v.s OLS Estimates

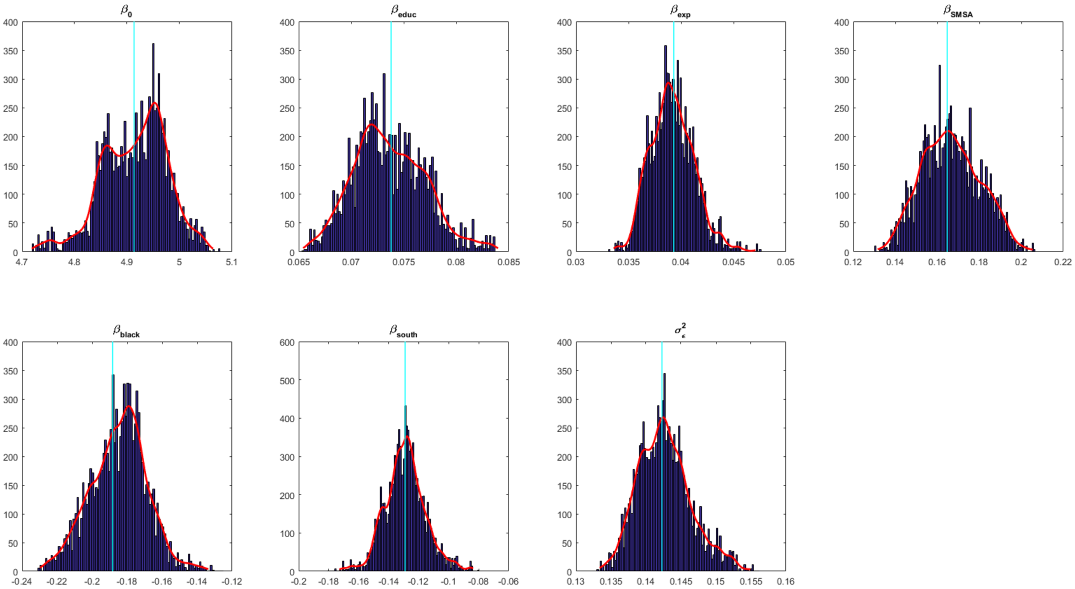


B. Given Prior of β_{educ}. Given the information about β_{educ} and known the flat prior distributions for other parameters, the joint prior distribution of θ is the same as $\beta_{educ} \sim \boldsymbol{N}(0.06, 0.0255^2)$. In this case, $\frac{L(\tilde{\theta}_{n+1})}{L(\hat{\theta}_n)} = \frac{P(y|\tilde{\theta}_{n+1})P(\tilde{\beta}_{educ,n+1})}{P(y|\hat{\theta}_n)P(\hat{\beta}_{educ,n})}$. Figure 2 shows posterior distribution given β_{educ} prior distribution where red line indicates the kernel distribution of estimators and vertical line indicates the OLS point estimates.

Q3. SUMMARY

1. Acceptance Rate. To get reasonably fast convergence properties, I tried and obtain $r = 0.12$ to the acceptance rate as 23.95% for Section 2.A and 22.44% for given-prior Section 2.B.

Figure 2. Posterior Distributions with Priors v.s OLS Estimates



2. Bayesian Estimation v.s. OLS Estimation of Linear Equation. OLS estimation gives point estimates of parameters but Bayesian estimation offers distribution of parameters. The most distributions of parameters provided by the Bayesian estimation looks like symmetric and centering around OLS point estimates. Also this situation is slightly significant in flat-prior case. However, in general, comparing two kinds of prior distribution, the parameters distributions do not change much.