Course: Econ 634, Fall 2017
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Homework No 6. Bayesian Estimation.

1. **OLS Estimation.**

Table (1) reports the OLS results for the model[1],

$$y_i \quad = \quad \beta_1 + \beta_2 * Edu_i + \beta_3 * Exp_i + \beta_4 * \mathbb{D}_{SMSA,i} + \beta_5 * \mathbb{D}_{black,i} + \beta_6 * \mathbb{D}_{south,i} + \varepsilon_i \quad (1)$$

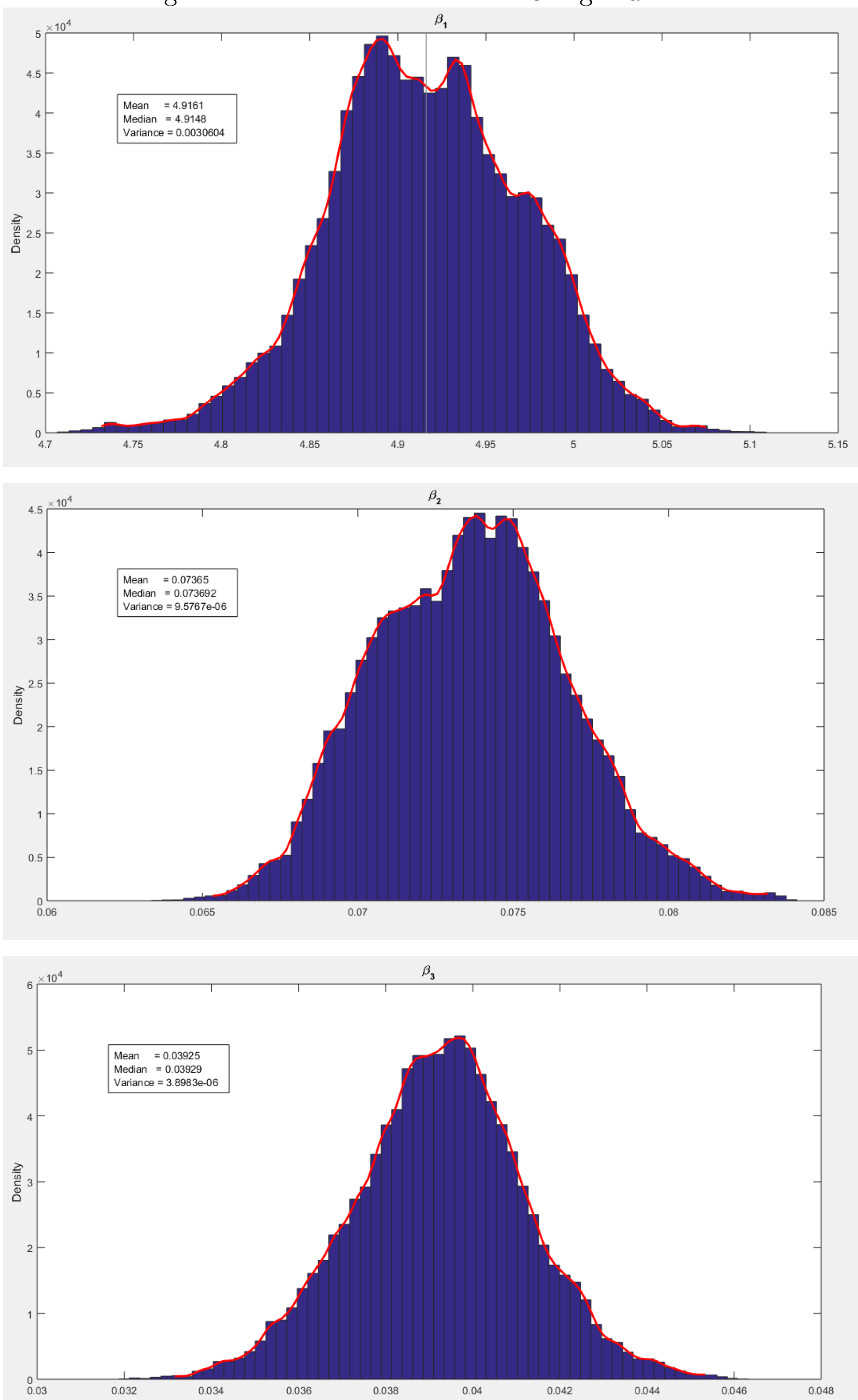
Table 1: OLS Estimation.

Variable	$\hat{\beta}_j$	s.e.	t	p-value
Constant (β_1)	4.913331	0.0631212	77.84	0.000
Education (β_2)	0.073807	0.0035336	20.89	0.000
Experience (β_3)	0.0393134	0.0021955	17.91	0.000
SMSA (β_4)	0.1647411	0.0156919	10.50	0.000
Black (β_5)	-0.1882225	0.0177678	-10.59	0.000
South (β_6)	-0.1290528	.0152285	-8.47	0.000
$\hat{\sigma}^2_\varepsilon = 0.14228$				

[1]See the appendix for a copy of the Matlab code.

2. **Bayesian using Flat Priors.**
   Using the Metropolis-Hastings algorithm, we obtain the following results for the vector of parameters $\theta = (\beta, \sigma_\varepsilon^2)' = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \sigma_\varepsilon^2)'$:

Figure 1: Posterior Distribution Using Flat Priors



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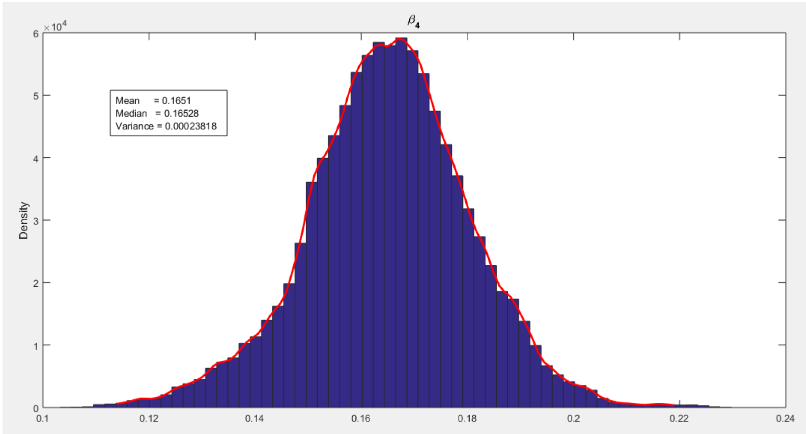

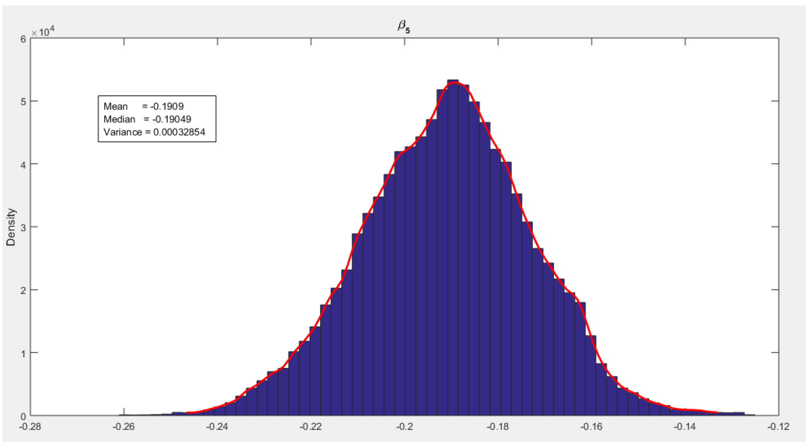

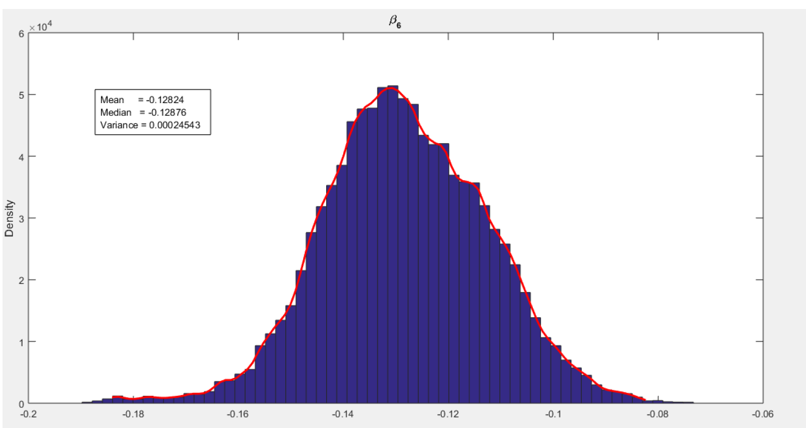

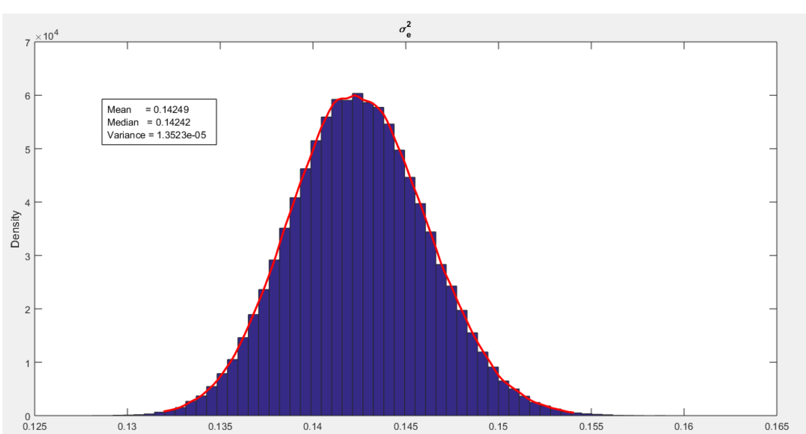

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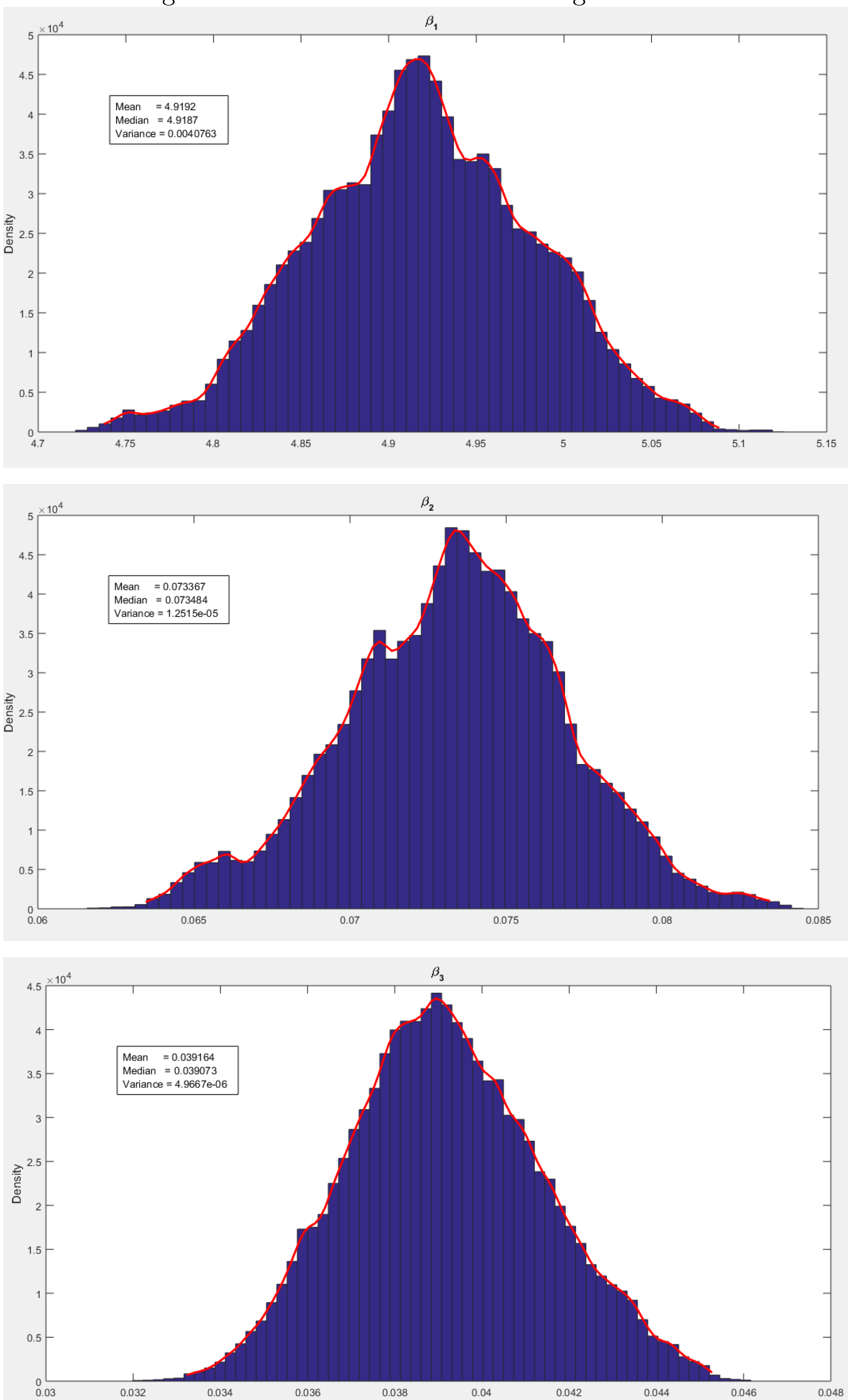
3. **Bayesian using a Given Prior.**
   If we use the information that $V(\beta_e du) = \epsilon_{edu}^2/4$, with $\epsilon_{edu} = (0.085 - 0.06)$, we obtain the following results,

Figure 2: Posterior Distribution Using Flat Priors



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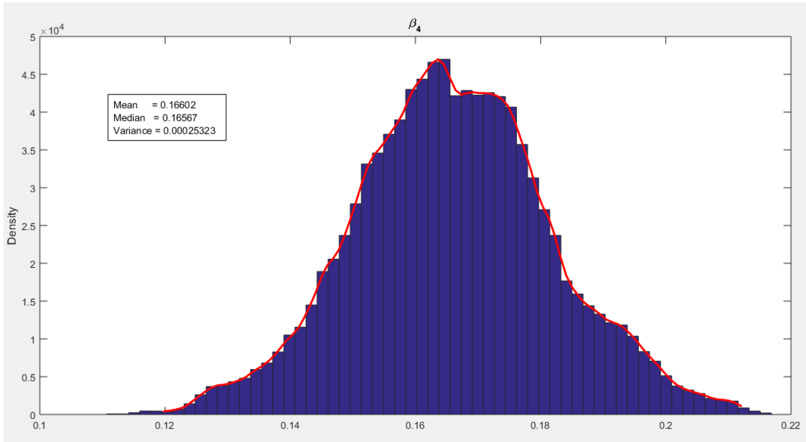
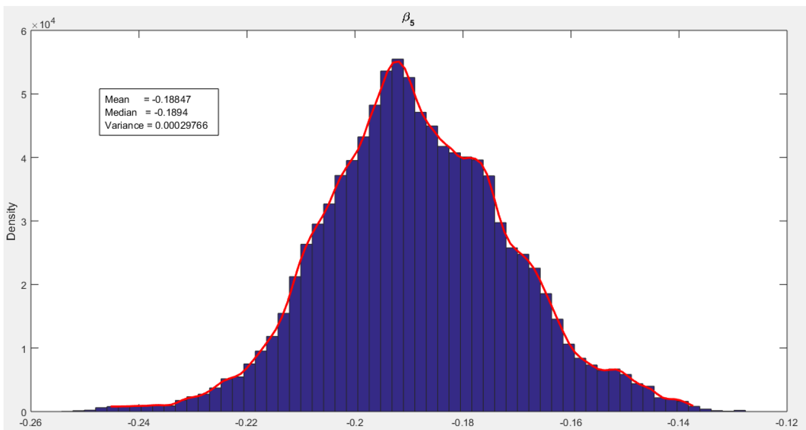
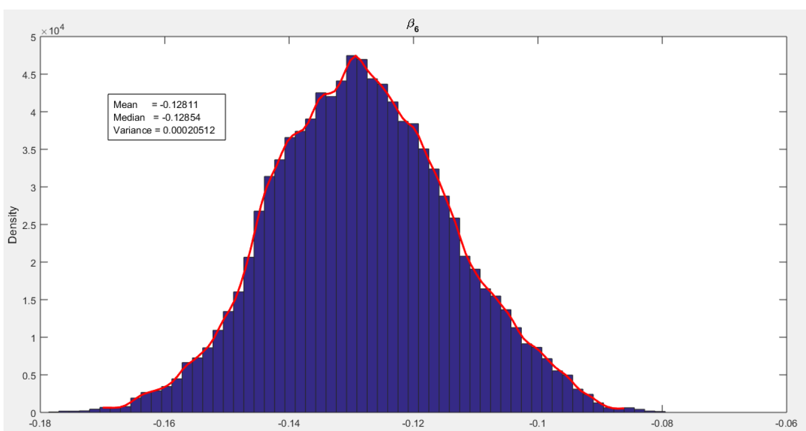
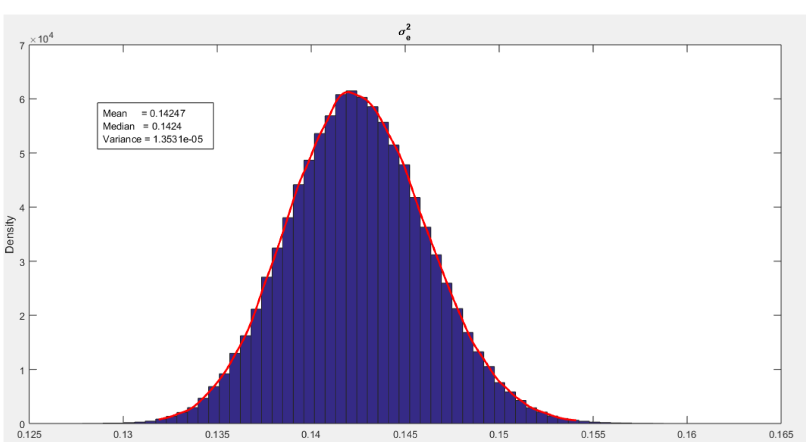

β_4
Mean = 0.16602
Median = 0.16567
Variance = 0.00025323
Density


β_5
Mean = -0.18847
Median = -0.1894
Variance = 0.00029766
Density


β_6
Mean = -0.12811
Median = -0.12854
Variance = 0.00020512
Density


σ_e^2
Mean = 0.14247
Median = 0.1424
Variance = 1.3531e-05
Density

4. **Final Comments.**

- If we compare the estimates from OLS and the posterior distributions in 2.(a) and 2.(b), we can notice that the values of first moments (i.e. see the mean, median and variance in each graph) for the posterior distributions are very close to the OLS estimates (table 1). On one hand, OLS gives a point estimate (or the 'maximum' vector of parameters θ) from minimizing SSR. On the other hand, under the assumptions of normality, we got a whole distribution (the posterior distribution) for parameters. Moreover, notice that from OLS we got that all the parameters were statistically significant at 1%, which under normality can be interpreted as a relevant role of the data used. This result is also related to the fact that the more information is contained in the data, the less influential is the prior for β_{edu}. In other words, given the relevance of the data and the large number of samples drawn, the prior played a minor role.

- For the Metropolis-Hastings algorithm, I drew the random parameters using[2]

$$\theta^{(*)} \sim \left(\boldsymbol{\theta}^{(0)}, \boldsymbol{\Sigma}\right)$$

with,

$$\boldsymbol{\Sigma} = \boldsymbol{\kappa} * \begin{pmatrix} Var(\beta_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Var(\beta_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Var(\beta_3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Var(\beta_4) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Var(\beta_5) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Var(\beta_6) & 0 \\ 0) & 0 & 0 & 0 & 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

In this case, I played with the value of κ to get an acceptance rate between 20% and 25%. Table (2) shows the accpetance rate fort the value of κ that I used.

Table 2: Acceptance Rate for $\kappa = 0.002$. Rate in percentage (%)

Prior	Acceptance Rate
Flat	24.3400
Prior for β_{edu}	23.8200

[2]The number of samples I drew was 1'000,000.

Appendix. Matlab Codes

Main Code: Hw6_Chanci.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Binghamton University                          %
% PhD in Economics                               %
% ECON634 Advanced Macroeconomics                %
% Fall 2017                                      %
% Luis Chancí (lchanci1@binghamton.edu)          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all; close all; clc;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data:
cd 'C:\Users\Chanci\Dropbox\PhD\III. Third Year\1. ECON-634 Advanced
    Macroeconomics\Hw\hw6'
data   = csvread('data1.csv');
[n k]  = size(data);
Y      = data(:,1);
X      = [ones(n,1) data(:,2:end)];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Target D. and Other functions
L = @(T,V,E)(-(T/2)*log(2*pi)-(T/2)*log(V)-inv(2*V)*(E'*E));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 1. OLS
b  = (inv(X'*X)*(X'*Y));
s  = inv(n-k)*(Y-X*b)'*(Y-X*b);
Vb = s*diag(inv(X'*X));



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 2. Flat Prior
r      = 0.002; % I played with this number to have acceptance rate 20-25%
Sigma  = r*[[bsxfun(@times,Vb,eye(k)) zeros(k,1)];[zeros(1,k) s]];

% 2.1. Firt, track accept-reject status
B      = [b;s];
acc0     = [0,0];
for i  = 1:1e4                            % MH routine;
    [B,a] = MHstep_Ch(B,Sigma,Y,X,L,1); % Option 1: flat prior
    acc0   = acc0 + [a 1];                 % track accept-reject status
end
    acc0(1)/acc0(2)*100      % Acceptance rate. IF it is low, increase r.

% 2.2. Second, MH routine (after the burn-in)
lag    = 1;
nsamp  = 1e6;                  % number of samples to draw
Theta  = zeros(nsamp,k+1); % storage
acc1     = [0,0];
for i = 1:nsamp
    for j=1:lag
        [B,a] = MHstep_Ch(B,Sigma,Y,X,L,1); % Option 1: flat prior
        acc1 = acc1 + [a 1];
    end
    Theta(i,:) = B';
end




```

```


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 3.   Using the Prior for Beta - Edu

% 3.1. Firt, track accept-reject status
B      = [b;s];
acc2   = [0,0];
for i  = 1:1e4                          % MH routine;
    [B,a] = MHstep_Ch(B,Sigma,Y,X,L,2); % Option 2: Prior
    acc2  = acc2 + [a 1];               % track accept-reject status
end
    acc2(1)/acc2(2)*100  % Acceptance rate. IF it is low, increase r.

% 3.2. Second, MH routine
Theta2 = zeros(nsamp,k+1);  % storage
acc3   = [0,0];
for i = 1:nsamp
    for j=1:lag
        [B,a] = MHstep_Ch(B,Sigma,Y,X,L,2); % Option 2: Prior
        acc3  = acc3 + [a 1];
    end
    Theta2(i,:) = B';
end
%%%%%%%%%%%%%%    Now we can Plot  %%%%%%%%%%%%%%
```

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