Def: A graph G is a pair of sets (V, E) where

V is a nonempty set of vertices or nodes

E is a set of 2-item subsets of V called edges.

E can be ϕ

Def: Two nodes x_i, x_j are <u>adjacent</u> if $\{x_i, x_j\} \in E$

Def. an edge $e = \{x_i, x_j\}$ <u>incident</u> to its end points x_i, x_j.

Def: The number of edges incident to a node is called a <u>degree</u> of a node

Def: A graph is <u>simple</u> if it has no loops or multiple edges

Graph Coloring Problem

Given a graph G, k colors, assign a color to each node so adjacent nodes get different colors.

Def: The minimum value of k for which such a coloring exits is the

<u>Chromatic Number</u> of G

$\chi(G)$

<u>Basic Coloring Alg</u> for $G(V, E)$

<u>Greedy Alg</u>

1. order the nodes $v_1, v_2, \dots v_n$
2. order the colors $C_1, C_2 \dots$
3. For $i = 1, 2, \dots n$ assign the lowest legal color to v_i

Thm: If every node in an n-node G has degree $\le d$.

Basic Alg uses at most $d+1$ colors.

Pf: By induction.

The first to do with a G is usually putting an "N" in the statement.

I.H. $P(n)$

Base Case. $n=1 \stackrel{\text{simple}}{\Longrightarrow} 0$ edges $\to d=0$ $\to$ 1 color ✓

Ind Step: Assume $P(n)$ is true.

Let $G=(V,E)$ be any $(n+1)$-node graph. let d be the max degree

Order the node $v_1, v_2, \dots, v_n, v_{n+1}$

Remove v_{n+1} from G to create $G'=(V',E')$, G' has max degree $\le d$.

$P(n)$ says Basic Alg uses $\le d+1$ colors for $v_1, v_2 \dots v_n$

v_{n+1} has $\le d$ neighbors $\Rightarrow$ use the left color(s) $\square$.

K_n = n-node complete graph

Def: A graph $G=(V,E)$ is bipartite if V can be split into V_L, V_R

so that all the edges connect a node in V_L to a node in V_R