Fuzzy Systems - Assignment #2 - Arian Tashakkor - 40023494

Problem 1

Composition of two relations is done exactly like matrix multiplication with the exception that we replace "$\times$" with a T-norm and "$+$" with max. The result of the $R \circ S$ in both cases below is a 2-by-4 relation matrix mapping x to z.

A. Max-min composition $R \circ S$:

$R \circ S$	z_1	z_2	z_3	z_4
x_1	$\max\{\min\{0,0\}, \min\{0.3, 1\}, \min\{1, 0.25\}\}$	$\max\{\min\{0, 0.3\}, \min\{0.3, 0.1\}, \min\{1, 0\}\}$	$\max\{\min\{0,1\}, \min\{0.3, 0.4\}, \min\{1,0.5\}\}$	$\max\{\min\{0,0.32\}, \min\{0.3,0.45\}, \min\{1,0\}\}$
x_2	$\max\{\min\{0.2,0\}, \min\{0.7,1\}, \min\{0.5,0.25\}\}$	$\max\{\min\{0.2, 0.3\}, \min\{0.7,0.1\}, \min\{0.5, 0\}\}$	$\max\{\min\{0.2, 1\}, \min\{0.7,0.4\}, \min\{0.5,0.5\}\}$	$\max\{\min\{0.2,0.32\}, \min\{0.7, 0.45\}, \min\{0.5,0\}\}$

$R \circ S$	z_1	z_2	z_3	z_4
x_1	0.3	0.1	0.5	0.3
x_2	0.7	0.2	0.5	0.45

B. Max-product composition $R \circ S$:

$R \circ S$	z_1	z_2	z_3	z_4
x_1	$\max\{0,0.3,0.25\}$	$\max\{0,0.03,0\}$	$\max\{0,0.12,0.5\}$	$\max\{0,0.135,0\}$
x_2	$\max\{0,0.7,0.125\}$	$\max\{0.06,0.07,0\}$	$\max\{0.2,0.28,0.25\}$	$\max\{0.064,0.315,0\}$

$R \circ S$	z_1	z_2	z_3	z_4
x_1	0.3	0.03	0.5	0.135
x_2	0.7	0.07	0.28	0.315

Problem 2

Assuming T-norm T, S-norm S and complement N:

A. (x is close and x is not slow) or x is positive

$S(T(\mu_C, N(\mu_S)), \mu_P)$

B. (x is positive or x is slow) and x is close

$T(S(\mu_P, \mu_S), \mu_C)$

C. (x is not slow and x is not close) or (x is not positive or x is slow)

$S(T(N(\mu_S), N(\mu_C)), S(N(\mu_P), \mu_S))$

D. ((x is not slow and x is close) or (x is not positive or x is not slow)) and x is positive

$T(S(T(N(\mu_S), \mu_C), S(N(\mu_P), N(\mu_S)), \mu_P)$

Problem 3

We can define the membership functions mathematically as follows:

$$\mu_A(x) = \begin{cases} 1 - \left|\frac{x-4}{4}\right|, 0 \leq x \leq 8 \\ 0, otherwise \end{cases}$$

$$\mu_{A'}(x) = \begin{cases} 1 - \left|\frac{x-8}{4}\right|, 4 \leq x \leq 12 \\ 0, otherwise \end{cases}$$

$$\mu_B(y) = \begin{cases} 1 - \left|\frac{y}{4}\right|, -4 \leq y \leq 4 \\ 0, otherwise \end{cases}$$

$$\mu_{B'}(y) = \begin{cases} 1 - \left|\frac{y-6}{4}\right|, 2 \leq y \leq 10 \\ 0, otherwise \end{cases}$$

$$\mu_C(z) = \begin{cases} 1 - \left|\frac{z+4}{4}\right|, -8 \leq z \leq 0 \\ 0, otherwise \end{cases}$$

And the fuzzy rule is:

"if x is A and y is B then z is C"

We will first calculate the firing strength of A and B:

For A and A' we have:

$$\omega_A = S(A, A') = \max\{\min\{\mu_A(x), \mu_{A'}(x)\}\}$$

A and A' are both non-zero only for $4 \leq x \leq 8$ and the intersection of μ_A and $\mu_{A'}$ in this interval determines ω_A so we solve $\mu_A(x) = \mu_{A'}(x)$:

$$1 - \frac{x-4}{4} = 1 + \frac{x-8}{4} \rightarrow x - 4 = 8 - x \rightarrow x = 6, \mu_A(x) = \mu_{A'}(x) = 0.5 = \omega_A \ ,$$

similarly for the B and B' we have:

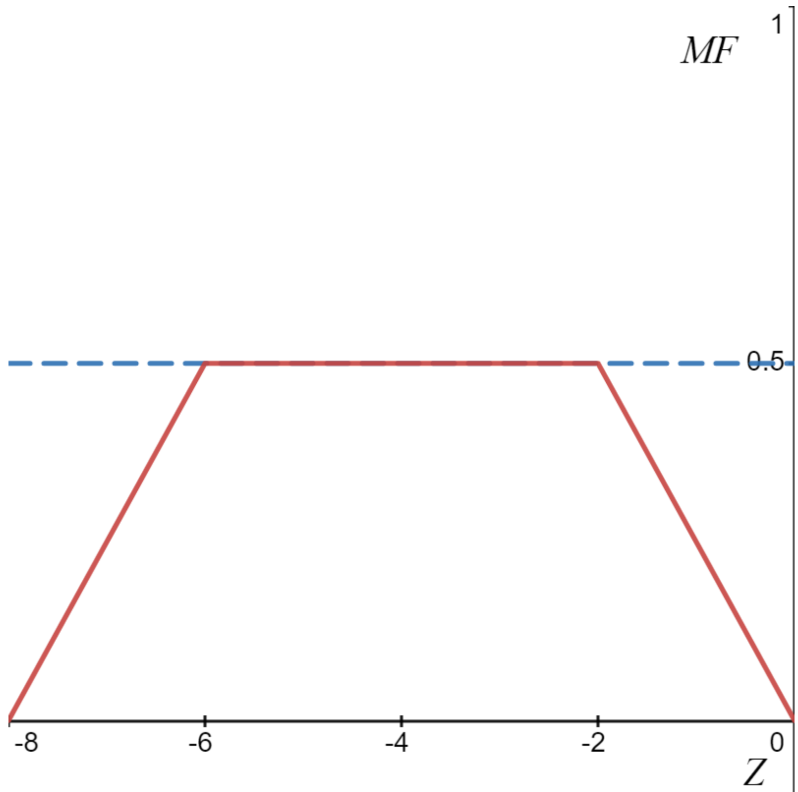
$$\omega_B = S(B, B') = \max\{\min\{mu_B(y), \mu_{B'}(y)\}\}$$

B and B' are both non-zero only for $2 \leq y \leq 4$ and the intersection of μ_B and $\mu_{B'}$ in this interval determines ω_B:

$$1 - \frac{y}{4} = 1 + \frac{y-6}{4} \rightarrow y = 6 - y \rightarrow y = 3 \rightarrow \mu_B(y) = \mu_{B'}(y) = 0.75 = \omega_B.$$

With both firing strengths calculated, we are ready to construct $\mu_{C'}$:

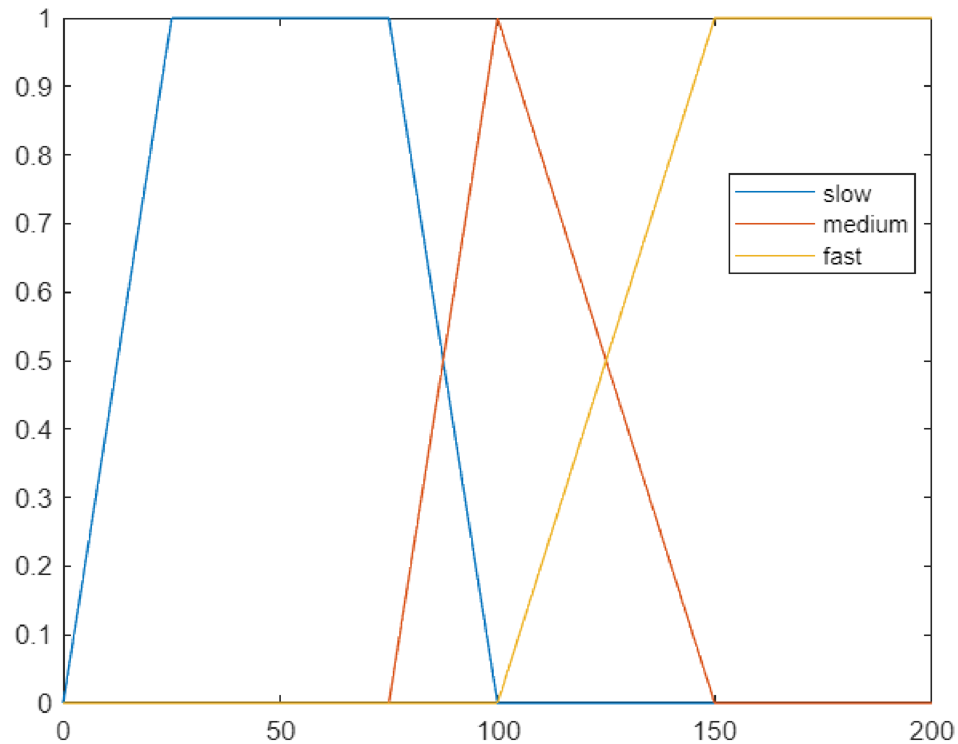
$\mu_{C'} = \min\{\omega_A, \omega_B, \mu_C\} = \min\{0.5, \mu_C\}$



Problem 4

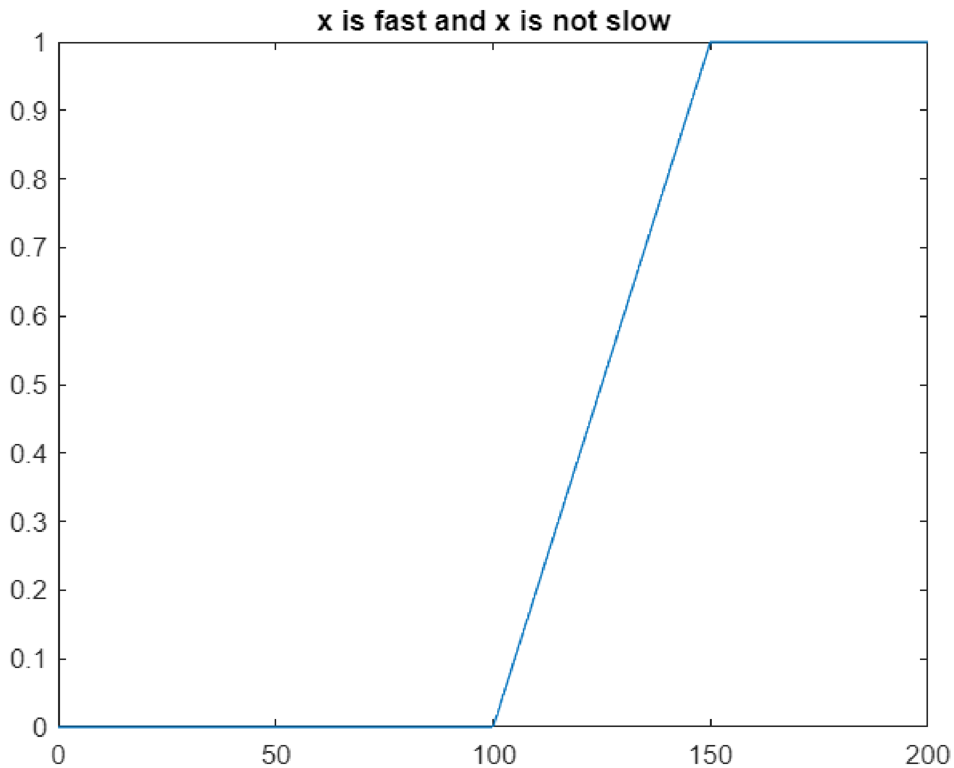
First we will define our fuzzy sets as per the instructions provided within the problem statement:

```
domain4 = 0:0.5:200;
slow = trapmf(domain4, [0, 25, 75, 100]);
medium = trimf(domain4, [75, 100, 150]);
fast = trapmf(domain4, [100, 150, 200, 200]);
plot(domain4, slow, domain4, medium, domain4, fast);
legend("slow", "medium", "fast", "Location","best");
hold off
```

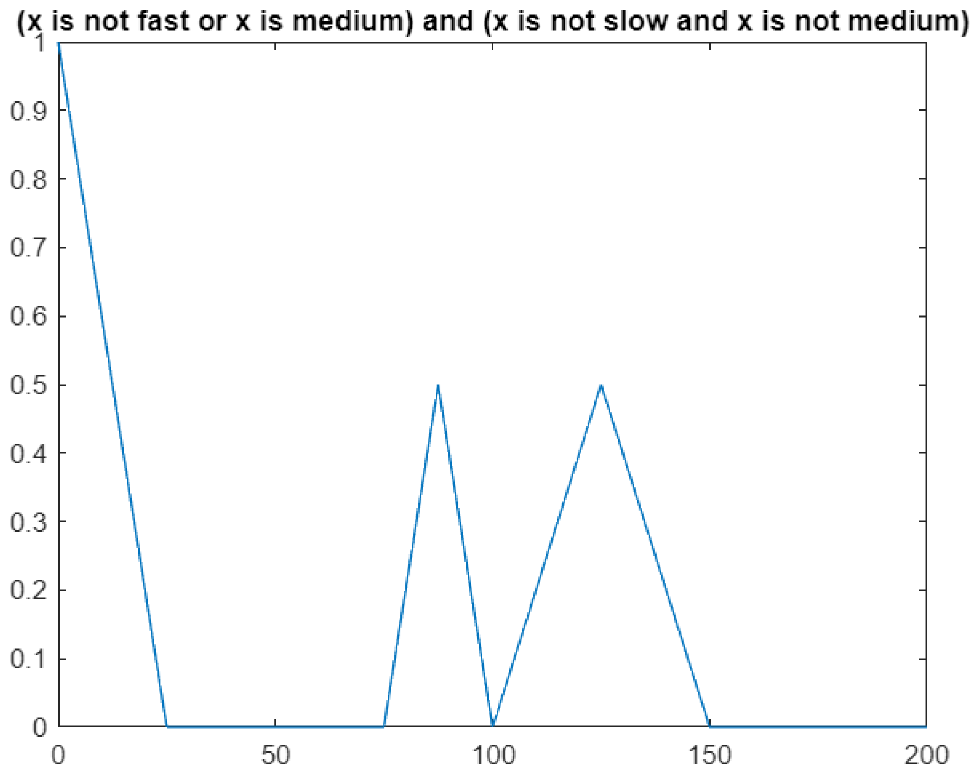


Now we can define our compound fuzzy statements and plot them:

```
fp1 = min(fast, 1 - slow);
plot(domain4, fp1);
title("x is fast and x is not slow");
hold off
```



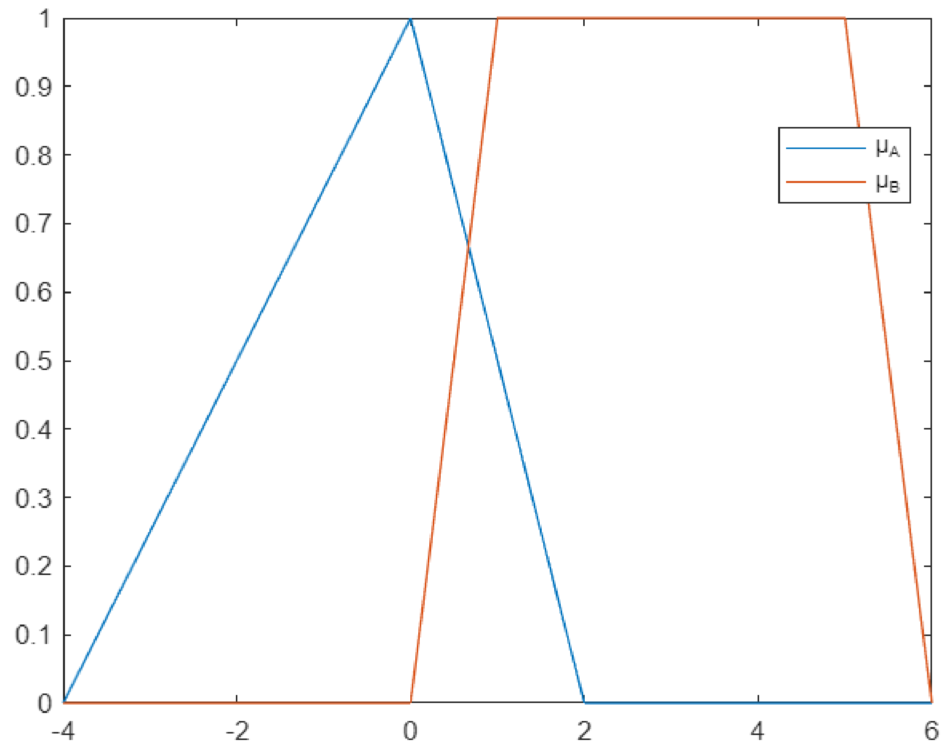

```
fp2 = min(max(1 - fast, medium), min(1 - slow, 1 - medium));
plot(domain4, fp2);
title("(x is not fast or x is medium) and (x is not slow and x is not medium)");
hold off
```



Problem 5

Similar to Problem 4, we will first define our fuzzy sets as instructed by the problem statement:

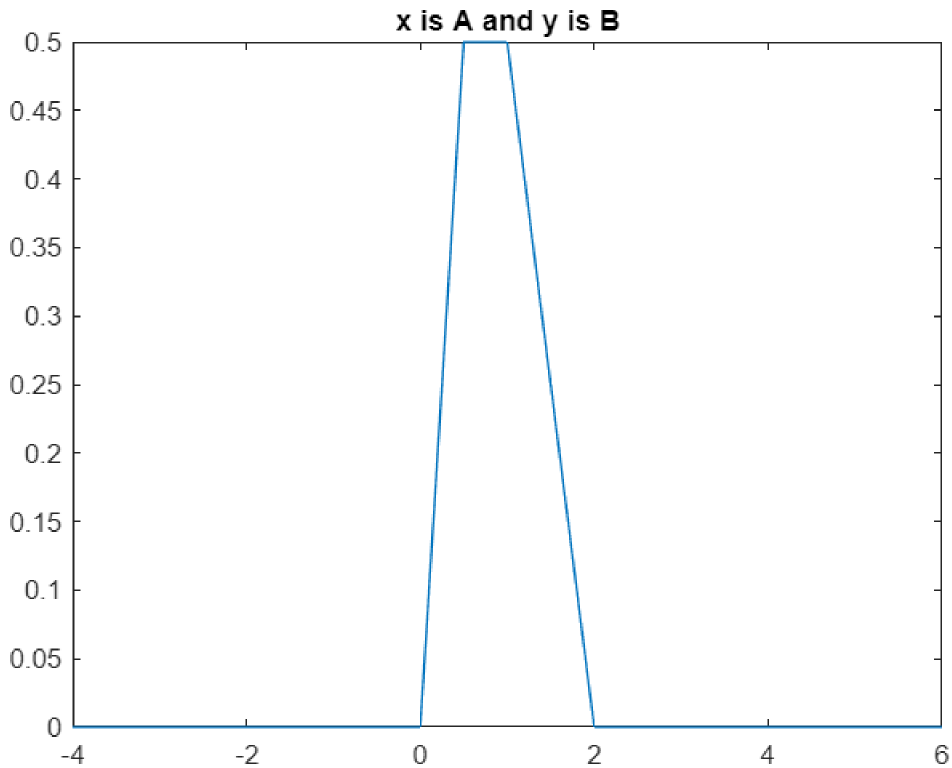
```
domain5 = -4:0.5:6;
A_5 = trimf(domain5, [-4, 0, 2]);
B_5 = trapmf(domain5, [0, 1, 5, 6]);
plot(domain5, A_5, domain5, B_5);
legend("\mu_A", "\mu_B", "Location","best");
hold off
```



We can now plot the results of the following compound fuzzy statements:

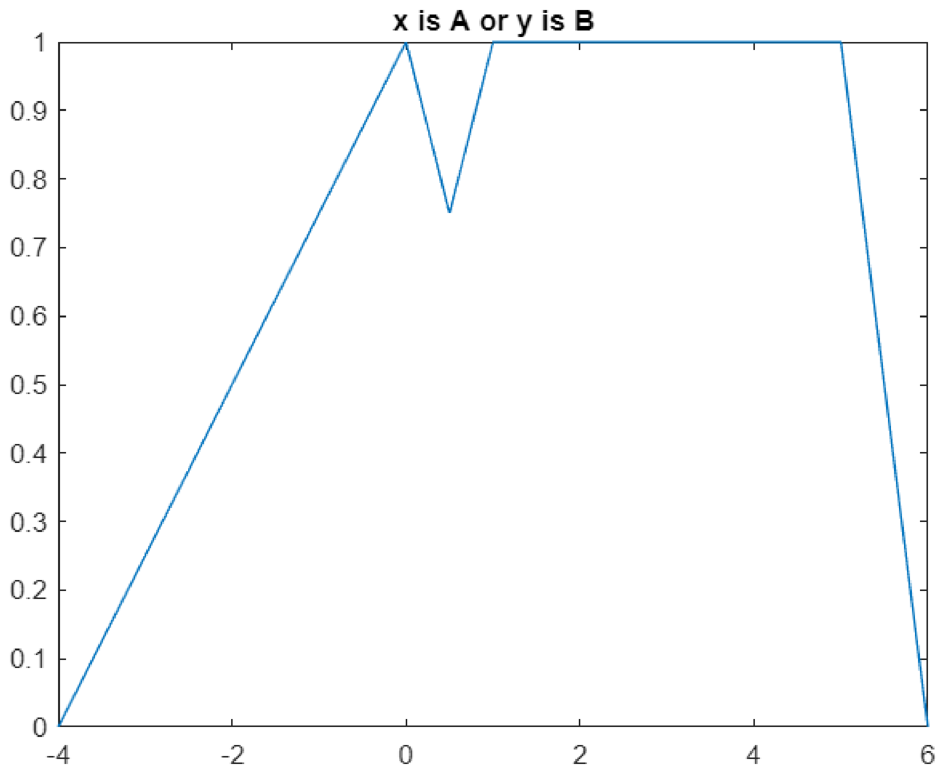
A. x is A and y is B

```
fpa = min(A_5, B_5);
plot(domain5, fpa);
title("x is A and y is B");
hold off
```



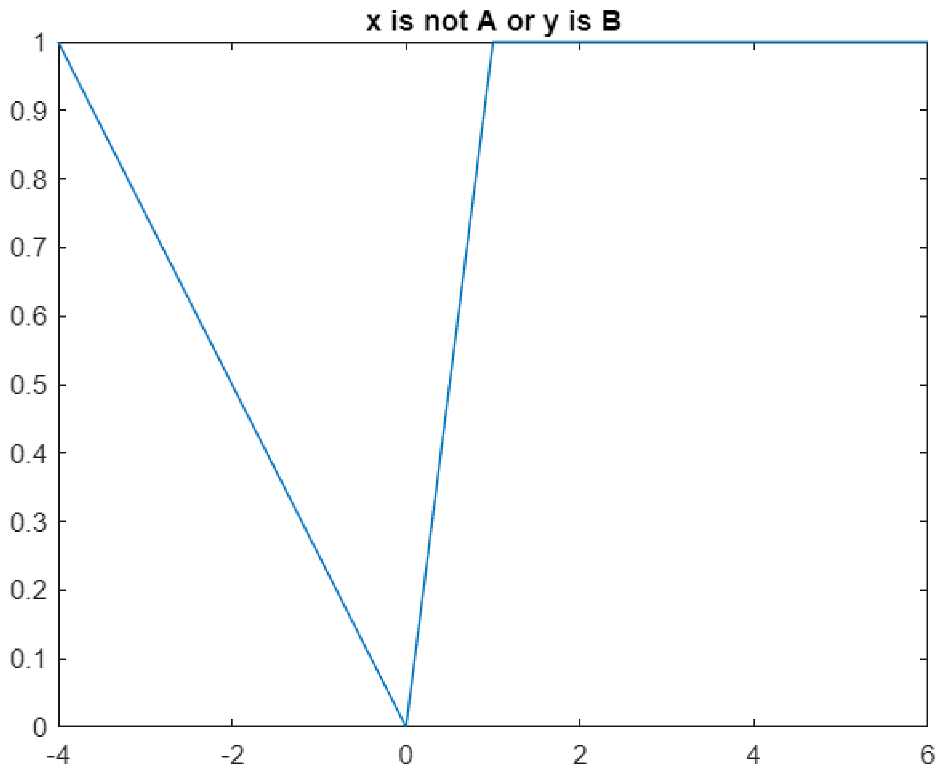
B. x is A or y is B

```
fpb = max(A_5, B_5);
plot(domain5, fpb);
title("x is A or y is B");
hold off
```



C. x is not A or y is B

```
fbc = max(1 - A_5, B_5);
plot(domain5, fbc);
title("x is not A or y is B");
hold off
```



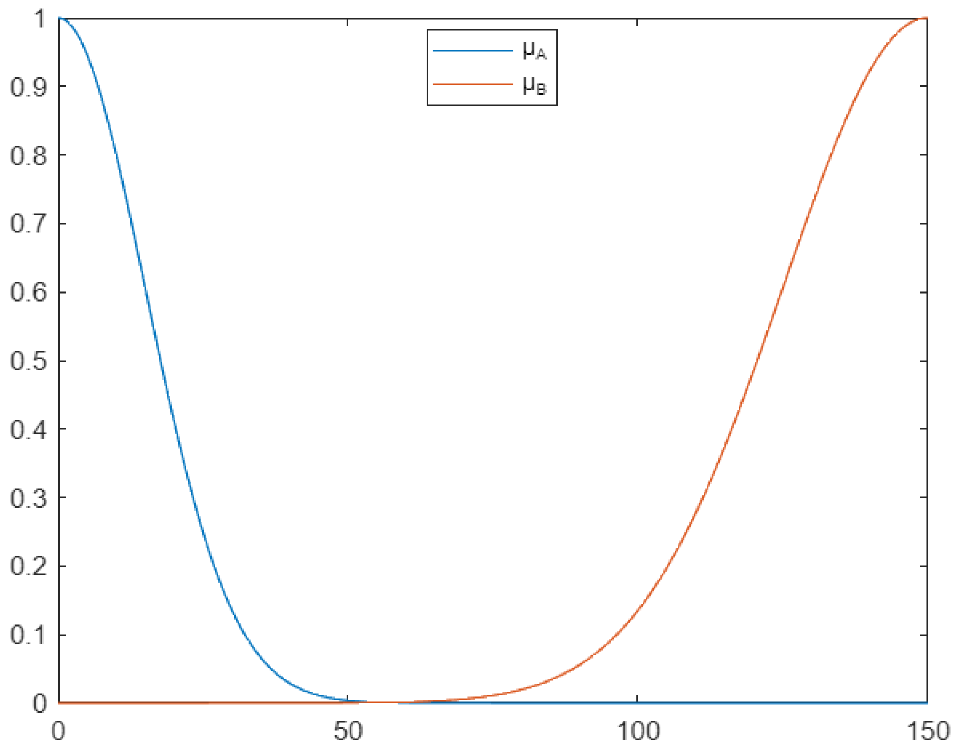
D. x is not A or y is not B

```
fbd = max(1 - A_5, 1 - B_5);
plot(domain5, fbd);
title("x is not A or y is not B");
hold off
```

Problem 6

First we will define A and B according the specifications given in the problem statement:

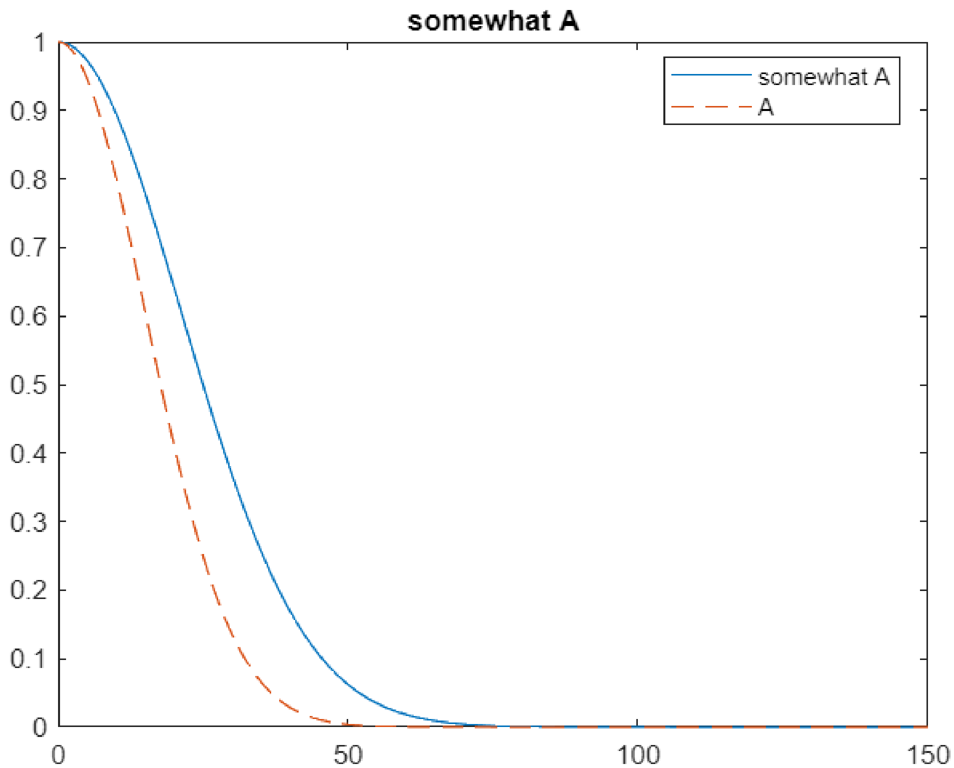
```
domain6 = 0:0.5:150;
A_6 = gaussmf(domain6, [15, 0]);
B_6 = gaussmf(domain6, [25, 150]);
plot(domain6, A_6, domain6, B_6);
legend("\mu_A", "\mu_B", "Location","best");
hold off
```



Now we can use the linguisitic hedges to alter these fuzzy sets:

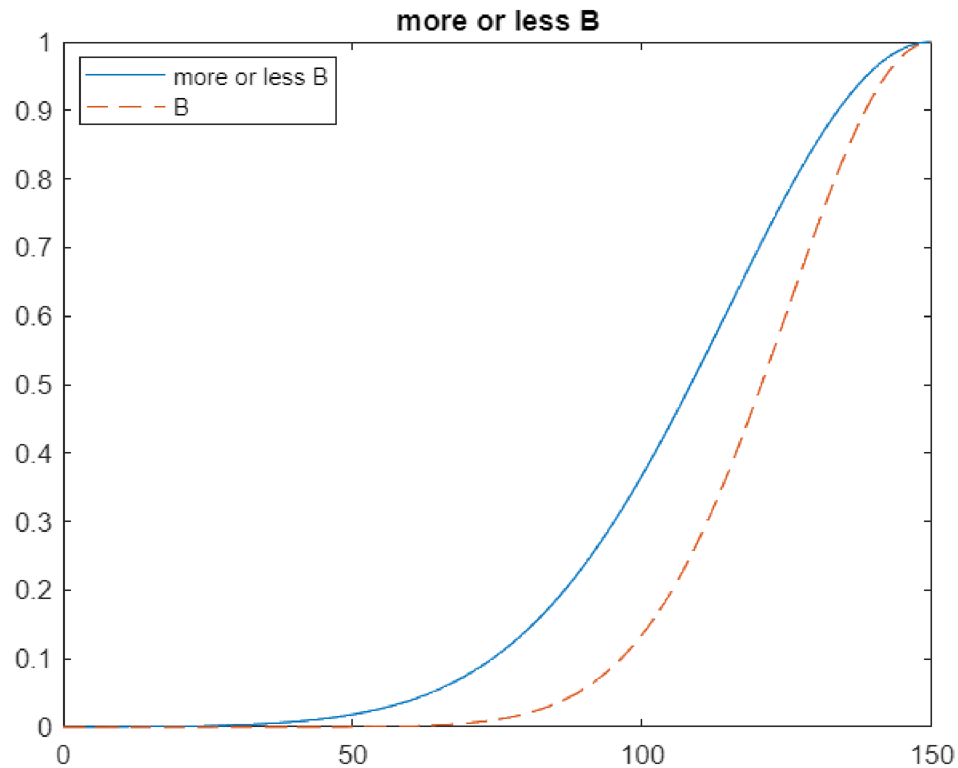
A. somewhat A

```
plot(domain6, A_6.^0.5, domain6, A_6, "--");
title("somewhat A");
legend("somewhat A", "A", "Location","best")
hold off
```



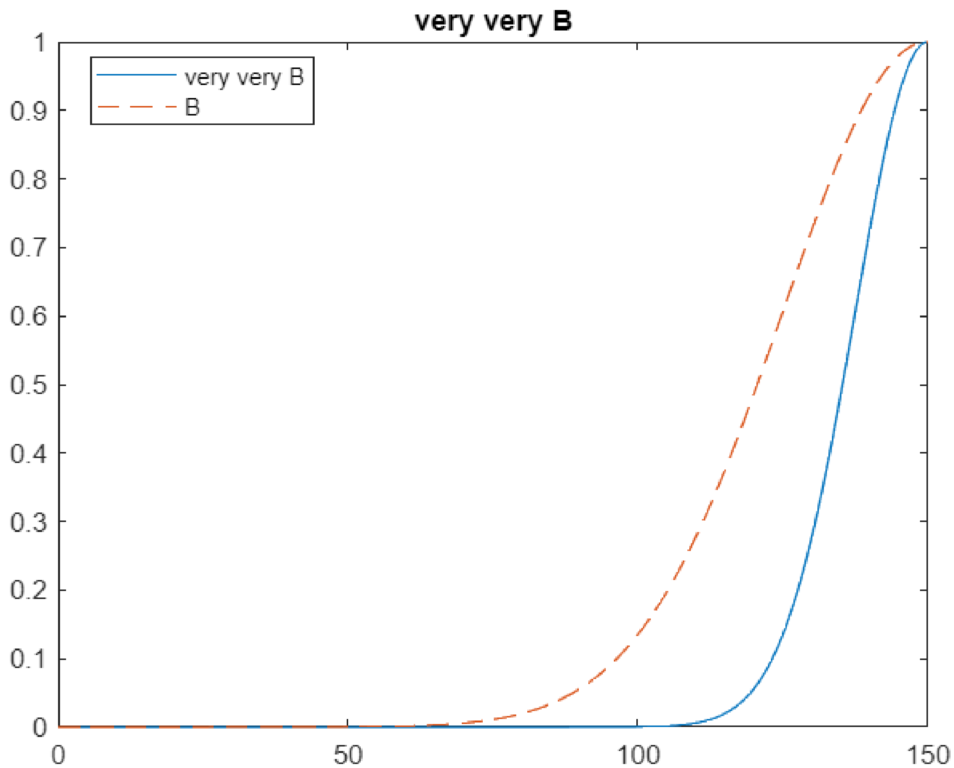
B. more or less B

```
plot(domain6, B_6.^0.5, domain6, B_6, "--");
title("more or less B");
legend("more or less B", "B", "Location","best");
hold off
```



C. very very B

```
plot(domain6, B_6.^4, domain6, B_6, "--");
title("very very B");
legend("very very B", "B", "Location","best");
hold off
```


very very B
very very B
B
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150