40023494 آرین شکر - تکلیف چهارم درس شناسایی آماری الگوها

(1). $\hat{p}(x) = \dfrac{1}{h_N^l N} \sum_{i=1}^{N} \phi\left(\dfrac{x_i - x}{h_n}\right) \quad , h_N = \dfrac{\sqrt{11}}{\sqrt{11}} = 1 \quad , l = 1$

$$\phi(u) = \begin{cases} 1/2 & |u| \leqslant 1 \\ 0 & o.w \end{cases}$$

$$\Rightarrow \hat{p}(-7) = \frac{1}{11} \sum_{i=1}^{11} \phi\left(\frac{x_i + 7}{1}\right) = \frac{1}{22}$$

$$\hat{p}(-5) = \frac{1}{11} \sum_{i=1}^{11} \phi(x_i + 5) = \frac{1}{11} \left(\phi(-5+5) + \phi(-4+5)\right) = \frac{1}{11}$$

$$\hat{p}(-4) = \frac{1}{11} \left(\phi(-5+4) + \phi(-4+4) + \phi(-3+4)\right) = 3/22$$

$$\hat{p}(-3) = 3/22 \quad , p(-2) = \frac{1}{11} \quad , p(0) = \frac{1}{22}$$

Because of symmetry:

$$p(2) = p(-2) = \frac{1}{11} \quad , p(3) = p(-3) = \frac{3}{22} \quad , p(4) = p(-4) = \frac{3}{22}$$

$$p(5) = p(-5) = \frac{1}{11} \quad , p(7) = p(-7) = \frac{1}{22}$$

(2) $\hat{p}(x) = \dfrac{1}{h_N^l N} \sum_{i=1}^{N} \phi\left(\dfrac{x_i - x}{h_N}\right) \quad , h_N = \dfrac{2\sqrt{11}}{\sqrt{11}} = 2 \quad , l = 1$

$$\rightarrow p(-7) = \frac{1}{22} \sum_{i=1}^{N} \phi\left(\frac{x_i + 7}{2}\right) = \frac{1}{22} \left(\phi\left(\frac{-7+7}{2}\right) + \phi\left(\frac{-5+7}{2}\right)\right) = \frac{1}{22}$$

$$p(-5) = \frac{1}{22} \left(\phi\left(\frac{-7+5}{2}\right) + \phi\left(\frac{-5+5}{2}\right) + \phi\left(\frac{-4+5}{2}\right) + \phi\left(\frac{-3+5}{2}\right)\right) = \frac{1}{11}$$

$$p(-3) = \frac{1}{11} \quad , p(-2) = \frac{1}{11} \quad , p(0) = \frac{3}{44}$$

By symmetry: $p(2) = p(-2) \; , p(3) = p(-3) \; , p(4) = p(-4) \; , p(5) = p(-5)$

$p(7) = p(-7)$

③. $\hat{p}(\hat{x}) = \frac{1}{h_N^{\ell} N} \sum_{i=1}^{N} \phi\left(\frac{x_i - x}{h_N}\right)$, $\ell = 1$, $h_N = 1$

$$\phi(u) = \begin{cases} 1 - |u| , & |u| \leq 1 \\ 0 , & \text{o.w} \end{cases}$$

$p(-7) = \frac{1}{11} \phi(-7+7) = 1/11$

$p(-5) = \frac{1}{11} \left(\cancelto{1}{\phi(-5+5)} + \cancelto{0}{\phi(-4+5)} \right) = \frac{1}{11}$

$p(-4) = \frac{1}{11} \left(\cancelto{0}{\phi(-5+4)} + \cancelto{1}{\phi(-4+4)} + \cancelto{0}{\phi(-3+4)} \right) = \frac{1}{11}$

$p(-3) = \frac{1}{11}$, $p(-2) = \frac{1}{11}$, $p(0) = p(2) = p(3) = p(4) = p(5) = p(7) = \frac{1}{11}$

Because the samples are at least one unit apart, the density estimate for all of the is exactly $1/11$ as $\phi(u)$ evaluates to 0 when $|u| = 1$ and non zero ~~1~~ only if $|u| < 1$.

④.

$\hat{p}(x) = \frac{k_N}{N V(x)}$, $k_N = \frac{2}{\sqrt{11}} \times \sqrt{11} = 2$, $V(x) = 2r(x)$ (radius)

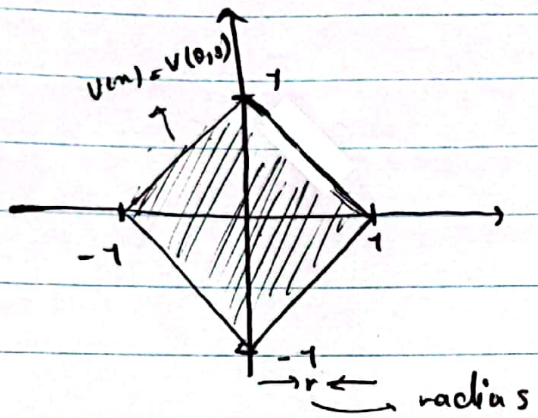
$\rightarrow \hat{p}(-7) = \frac{2}{11 \times (7-4) \times 2} = \frac{2}{66} = \frac{1}{33}$, $p(-5) = \frac{2}{11 \times 2 \times 2} = \frac{1}{22}$

$p(-4) = \frac{2}{11 \times 1 \times 2} = \frac{1}{11}$, $p(-3) = \frac{2}{11 \times 1 \times 2} = \frac{1}{11}$, $p(-2) = \frac{2}{11 \times 2 \times 2} = \frac{1}{22}$

$p(0) = \frac{2}{11 \times 2 \times 2} = \frac{1}{22}$, because of symmetry $p(2) = p(-2)$,

$p(3) = p(-3)$, $p(4) = p(-4)$, $p(5) = p(-5)$, $p(7) = p(-7)$

⑤. The key point to be aware of in this problem is that the $V(x)$ for manhattan-distance is the area of a tilted square centered at the point x. See below for a demonstration:



The reason for this is that every point on the perimeter of this square has the same manhattan distance from the center.
we will call this a mahattan square with radius (r).

Back to the problem:

Assuming X's correspond to ω_1 and ●'s to ω_2.

A mahattan square of at least radius 2 is required to house at least $k_n = 3$ w.r.t. manhattan distance for both ω_1 and ω_2. There both of the $V(x)$'s are equal. The classification rule therefore becomes: (assuming no risk and equiprobability)

assign $(0,0)$ to ω_1 (ω_2) if $\cancel{\frac{V_2}{V_1}}^{\,1} > (<) \frac{N_1}{N_2} = \frac{6}{9}$

$\longrightarrow$ $1 > 6/9 \Rightarrow (0,0)$ belongs to ω_1