E10.13

vi.

We will use the LMS algorithm to train the weights of this ADALINE network. The implementation is adopted from E10_4.mlx.

First we will use a stable learning rate, $\alpha = 0.3 < 1.11$

We cannot use the same convergence condition as before because the squared error value fluctuates and the algorithm may terminate before it has actually converged.

We will use the following in the upcoming implementation of the LMS algorithm:

$$y(k) = \sin\left(\frac{k\pi}{5}\right)$$

$$t(k) = y(k)$$

$$p(k) = \begin{bmatrix} y(k-1) \\ y(k-2) \end{bmatrix}$$

```
% LMS Implementation
% initial guess
x_star = [0; 0];
x_hist = zeros([2, 41]);
x_hist(:, 1) = x_star;
error_hist = zeros([1, 40]);
num_steps = 40;
alpha = 0.3;
for iteration=1:num_steps
    iteration
    k = iteration - 1;
    % calculate y(k), y(k-1) and y(k-2) to construct the target value and
    % the input vectors.
    y_k = sin(k*pi/5);
    y_k_1 = sin((k-1)*pi/5);
    y_k_2 = sin((k-2)*pi/5);
    % target and inputs
    target = y_k;
    input = [y_k_1; y_k_2];
    error = target - x_star'*input
    error_hist(iteration) = error^2;
    x_star = x_star + 2*alpha*error*input
    x_hist(:, iteration+1) = x_star;
end
```

```
iteration = 1
error = 0
x_star = 2×1
     0
     0
iteration = 2
error = 0.5878
x_star = 2×1
         0
```

```
   -0.2073
iteration = 3
error = 0.9511
x_star = 2×1
    0.3354
   -0.2073
iteration = 4
error = 0.7539
x_star = 2×1
    0.7656
    0.0586
iteration = 5
error = -0.1961
x_star = 2×1
    0.6537
   -0.0533
iteration = 6
error = -0.3336
x_star = 2×1
    0.5361
   -0.2436
iteration = 7
error = -0.4446
x_star = 2×1
    0.5361
   -0.4004
iteration = 8
error = -0.6360
x_star = 2×1
    0.7604
   -0.4004
iteration = 9
error = -0.4633
x_star = 2×1
    1.0247
   -0.2370
iteration = 10
error = 0.1613
x_star = 2×1
    0.9327
   -0.3291
iteration = 11
error = 0.2352
x_star = 2×1
    0.8497
   -0.4633
iteration = 12
error = 0.3154
x_star = 2×1
    0.8497
   -0.5746
iteration = 13
error = 0.4516
x_star = 2×1
    1.0090
   -0.5746
iteration = 14
error = 0.3292
x_star = 2×1
    1.1968
   -0.4585
iteration = 15
error = -0.1144
x_star = 2×1
```

```
    1.1315
   -0.5238
iteration = 16
error = -0.1670
x_star = 2×1
    1.0727
   -0.6190
iteration = 17
error = -0.2239
x_star = 2×1
    1.0727
   -0.6980
iteration = 18
error = -0.3206
x_star = 2×1
    1.1857
   -0.6980
iteration = 19
error = -0.2337
x_star = 2×1
    1.3190
   -0.6156
iteration = 20
error = 0.0812
x_star = 2×1
    1.2727
   -0.6620
iteration = 21
error = 0.1185
x_star = 2×1
    1.2309
   -0.7296
iteration = 22
error = 0.1589
x_star = 2×1
    1.2309
   -0.7856
iteration = 23
error = 0.2276
x_star = 2×1
    1.3112
   -0.7856
iteration = 24
error = 0.1659
x_star = 2×1
    1.4058
   -0.7271
iteration = 25
error = -0.0577
x_star = 2×1
    1.3729
   -0.7600
iteration = 26
error = -0.0841
x_star = 2×1
    1.3432
   -0.8080
iteration = 27
error = -0.1128
x_star = 2×1
    1.3432
   -0.8478
iteration = 28
error = -0.1615
```

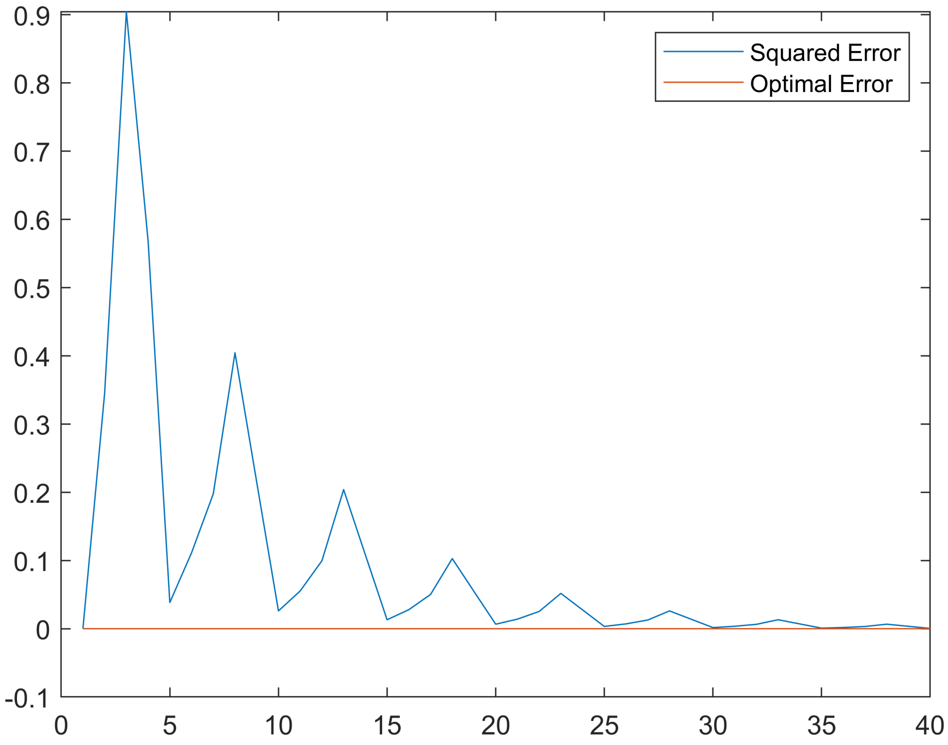
```
x_star = 2×1
    1.4002
   -0.8478
iteration = 29
error = -0.1177
x_star = 2×1
    1.4674
   -0.8063
iteration = 30
error = 0.0409
x_star = 2×1
    1.4440
   -0.8297
iteration = 31
error = 0.0597
x_star = 2×1
    1.4230
   -0.8637
iteration = 32
error = 0.0801
x_star = 2×1
    1.4230
   -0.8920
iteration = 33
error = 0.1147
x_star = 2×1
    1.4634
   -0.8920
iteration = 34
error = 0.0836
x_star = 2×1
    1.5111
   -0.8625
iteration = 35
error = -0.0291
x_star = 2×1
    1.4945
   -0.8791
iteration = 36
error = -0.0424
x_star = 2×1
    1.4796
   -0.9033
iteration = 37
error = -0.0568
x_star = 2×1
    1.4796
   -0.9233
iteration = 38
error = -0.0814
x_star = 2×1
    1.5083
   -0.9233
iteration = 39
error = -0.0593
x_star = 2×1
    1.5421
   -0.9024
iteration = 40
error = 0.0206
x_star = 2×1
    1.5304
   -0.9142
```

```
fprintf('#####\n\n Final Weights: [%f, %f]. Expected Weights: [%f, %f]', x_star(1), x_star(2),
```

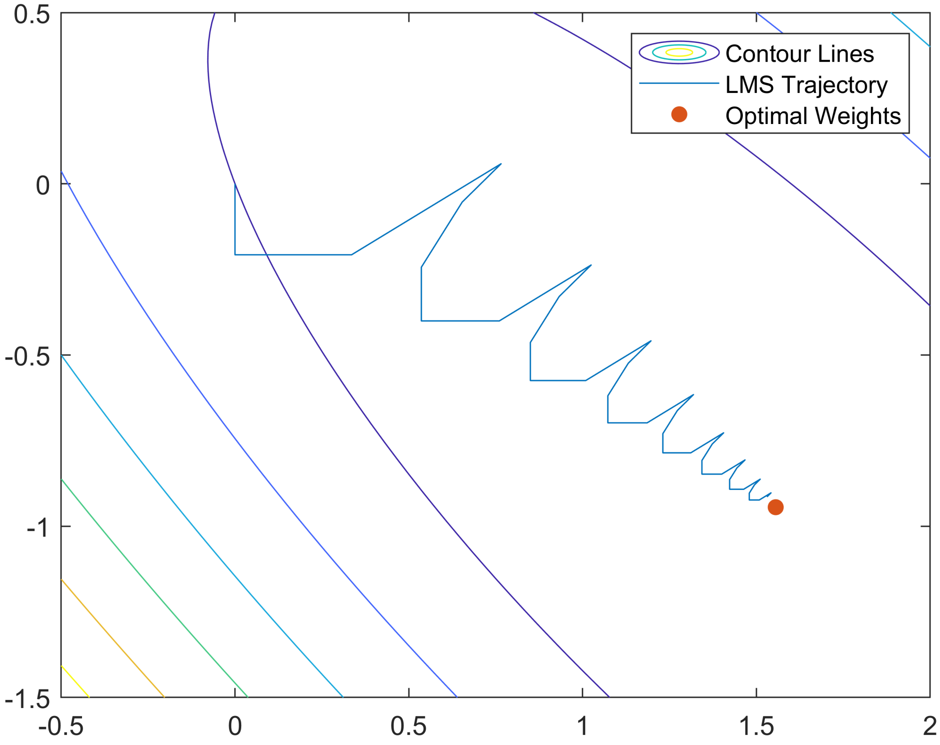
```
#####

 Final Weights: [1.530358, -0.914176]. Expected Weights: [1.555556, -0.944444]
```

```
% plotting error history
plot(error_hist)
hold on
plot(zeros([1, 40]))
ylim([-0.1 max(error_hist)])
legend('Squared Error', 'Optimal Error')
hold off
```



```
% sketching the trajectory on the contour plot
syms x1 x2 real
X = [x1; x2];
fx = 0.5 - 2*X'*[0.4; 0.15] + X'*[0.5 0.4; 0.4 0.5]*X;
fcontour(fx,[-0.5 2 -1.5 0.5])
hold on
plot(x_hist(1,:), x_hist(2,:));
hold on
scatter(14/9, -17/18, 'filled');
legend('Contour Lines', 'LMS Trajectory', 'Optimal Weights')
hold off
```



As we can see from the trajectory, the algorithm is slowly closing in on the optimal weights.

vii.

Here we will run the same exact experiment except we will set the learning rate to an intentionally unstable value like 1.12 which is only slightly greater than the max stable learning rate.

```
% LMS Implementation
% initial guess
x_star = [0; 0];
x_hist = zeros([2, 41]);
x_hist(:, 1) = x_star;
error_hist = zeros([1, 40]);
num_steps = 40;
alpha = 1.12;
for iteration=1:num_steps
    iteration
    k = iteration - 1;
    % calculate y(k), y(k-1) and y(k-2) to construct the target value and
    % the input vectors.
    y_k = sin(k*pi/5);
    y_k_1 = sin((k-1)*pi/5);
    y_k_2 = sin((k-2)*pi/5);
    % target and inputs
    target = y_k;
    input = [y_k_1; y_k_2];
    error = target - x_star'*input
    error_hist(iteration) = error^2;
    x_star = x_star + 2*alpha*error*input
    x_hist(:, iteration+1) = x_star;
end
```

```
iteration = 1
error = 0
x_star = 2×1
     0
     0
iteration = 2
error = 0.5878
x_star = 2×1
         0
   -0.7739
iteration = 3
error = 0.9511
x_star = 2×1
    1.2522
   -0.7739
iteration = 4
error = 0.2150
x_star = 2×1
    1.7103
   -0.4908
iteration = 5
error = -0.5720
x_star = 2×1
    0.4916
   -1.7094
iteration = 6
error = 1.3368
x_star = 2×1
    2.2517
    1.1384
iteration = 7
error = -1.2569
x_star = 2×1
    2.2517
   -0.5165
iteration = 8
error = 0.3725
x_star = 2×1
    1.7613
   -0.5165
iteration = 9
error = 0.4205
x_star = 2×1
    0.8656
   -1.0701
iteration = 10
error = -0.7823
x_star = 2×1
    2.5321
    0.5965
iteration = 11
error = 2.0556
x_star = 2×1
   -0.1744
   -3.7828
iteration = 12
error = -1.6357
x_star = 2×1
   -0.1744
   -1.6292
iteration = 13
error = 1.0536
x_star = 2×1
    1.2128
   -1.6292
iteration = 14
error = 0.7552
x_star = 2×1
    2.8217
   -0.6348
iteration = 15
error = -1.4921
x_star = 2×1
   -0.3570
   -3.8136
iteration = 16
error = 3.8368
x_star = 2×1
    4.6946
    4.3602
iteration = 17
error = -3.1506
x_star = 2×1
    4.6946
    0.2119
iteration = 18
error = 1.8084
x_star = 2×1
    2.3136
    0.2119
iteration = 19
error = 1.3739
x_star = 2×1
   -0.6133
   -1.5970
iteration = 20
error = -2.6899
x_star = 2×1
    5.1172
    4.1335
iteration = 21
error = 6.9390
x_star = 2×1
   -4.0190
  -10.6492
iteration = 22
error = -5.6716
x_star = 2×1
   -4.0190
   -3.1817
iteration = 23
error = 3.3133
x_star = 2×1
    0.3435
   -3.1817
iteration = 24
error = 2.4945
x_star = 2×1
    5.6577
    0.1027
iteration = 25
error = -4.8907
x_star = 2×1
```

```
   -4.7612
  -10.3162
iteration = 26
error = 12.6099
x_star = 2×1
   11.8415
   16.5474
iteration = 27
error = -10.3141
x_star = 2×1
   11.8415
    2.9675
iteration = 28
error = 6.0092
x_star = 2×1
    3.9295
    2.9675
iteration = 29
error = 4.5304
x_star = 2×1
   -5.7218
   -2.9974
iteration = 30
error = -8.8803
x_star = 2×1
   13.1965
   15.9209
iteration = 31
error = 22.8983
x_star = 2×1
  -16.9524
  -32.8610
iteration = 32
error = -18.7274
x_star = 2×1
  -16.9524
   -8.2037
iteration = 33
error = 10.9154
x_star = 2×1
   -2.5807
   -8.2037
iteration = 34
error = 8.2275
x_star = 2×1
   14.9469
    2.6289
iteration = 35
error = -16.1278
x_star = 2×1
  -19.4112
  -31.7292
iteration = 36
error = 41.5859
x_star = 2×1
   35.3424
   56.8640
iteration = 37
error = -34.0116
x_star = 2×1
   35.3424
   12.0830
iteration = 38
error = 19.8227
```

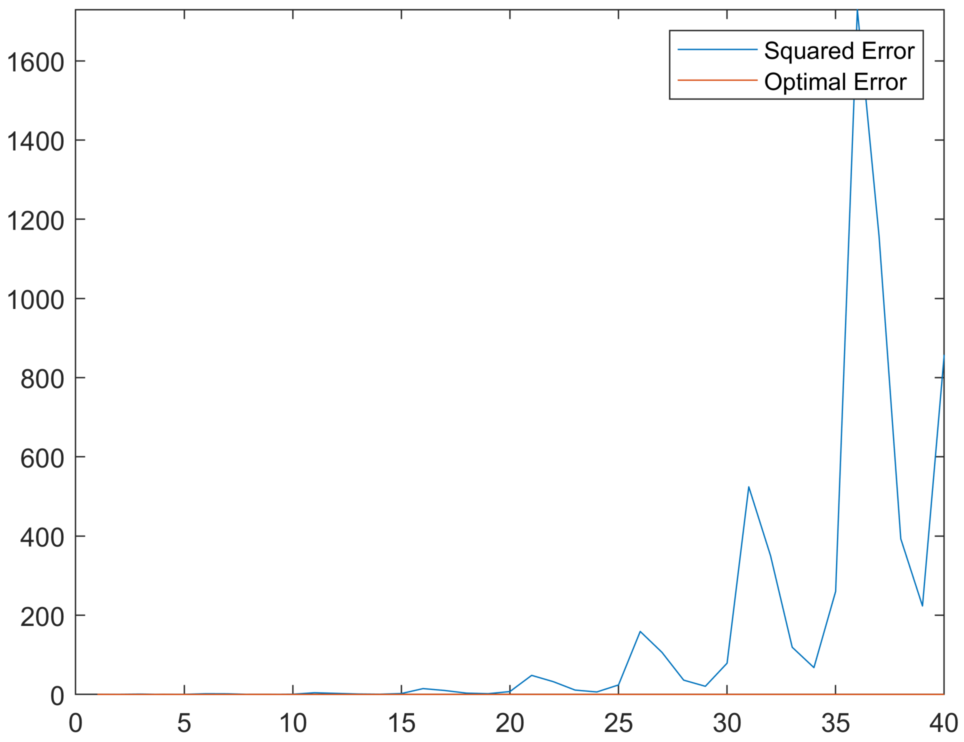
```
x_star = 2×1
    9.2431
   12.0830
iteration = 39
error = 14.9418
x_star = 2×1
  -22.5885
   -7.5900
iteration = 40
error = -29.2892
x_star = 2×1
   39.8083
   54.8068
```

```
fprintf('#####\n\n Final Weights: [%f, %f]. Expected Weights: [%f, %f]', x_star(1), x_star(2),
```

```
#####

 Final Weights: [39.808335, 54.806827]. Expected Weights: [1.555556, -0.944444]
```

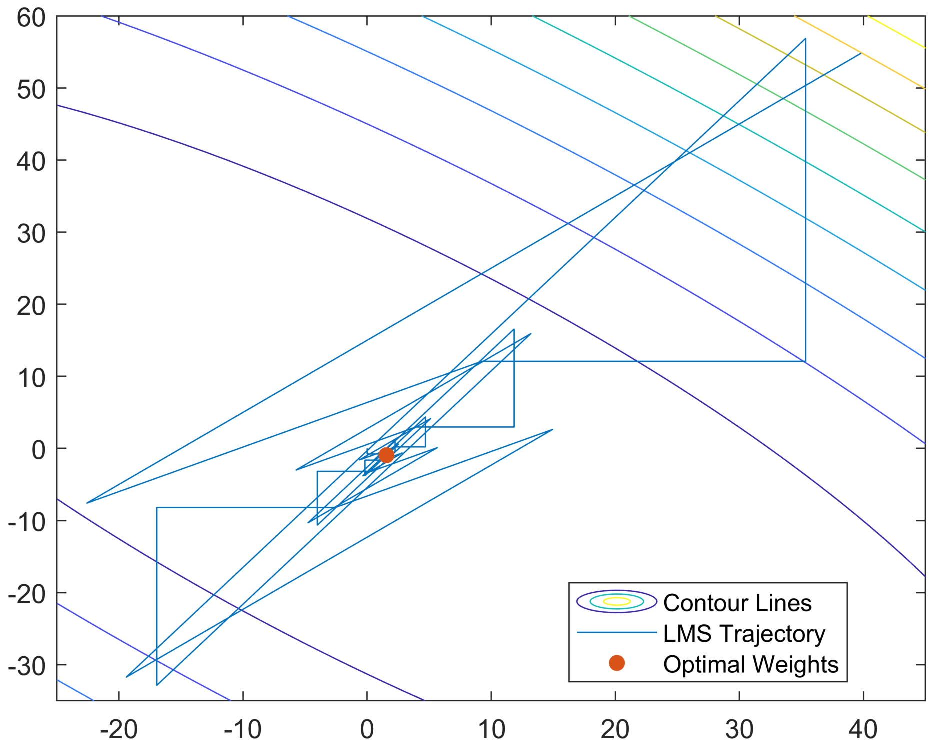
```
% plotting error history
plot(error_hist)
hold on
plot(zeros([1, 40]))
ylim([-0.1 max(error_hist)])
legend('Squared Error', 'Optimal Error')
hold off
```



As we can see the error is getting greater the further we go.

```
% sketching the trajectory on the contour plot
```

```
syms x1 x2 real
X = [x1; x2];
fx = 0.5 - 2*X'*[0.4; 0.15] + X'*[0.5 0.4; 0.4 0.5]*X;
fcontour(fx,[-25 45 -35 60])
hold on
plot(x_hist(1,:), x_hist(2,:));
hold on
scatter(14/9, -17/18, 'filled');
legend('Contour Lines', 'LMS Trajectory', 'Optimal Weights', 'location', 'best')
hold off
```



It is also obvious from the trajectory that the algorithm fails to converge, resulting in a messy trajectory that does not seem to be headed in any specific direction.