In The Name of Almighty

Statistical Pattern Recognition- HW#1

Part 1

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Chapter 2: 2.2, 2.3, 2.5, 2.7, 2.10, 2.12,

Part 2

Problem 1

In a two-class problem, the conditional probability density functions of the one-dimensiona feature, x, have the following Cauchy distribution:

$$p(x \mid \omega_i) = \frac{1}{\pi b} \cdot \frac{1}{1 + \left(\frac{x - a_i}{b}\right)^2}, \quad i = 1, 2, \quad a_2 > a_1$$

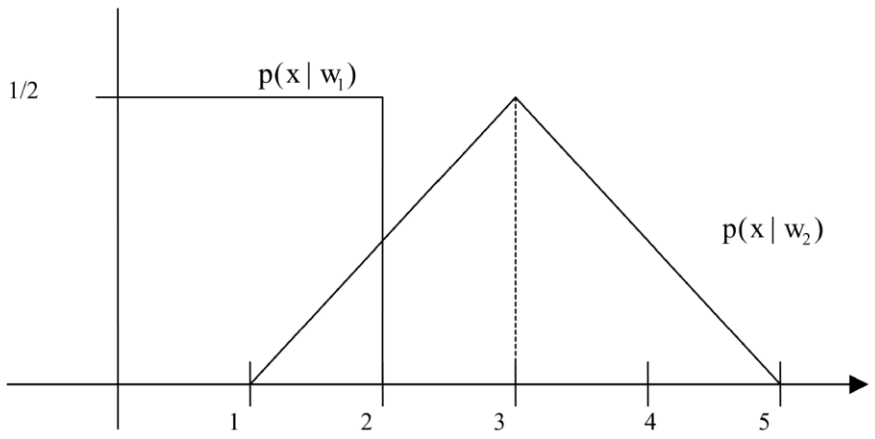
In each of the following cases, simplify your answer as much as possible.

a) Find the Bayes min-error (BME) classifier in terms of a_i and b given $P(\omega_1) = P(\omega_2)$
b) Find the decision boundary for the BME
c) Find the error associated with the BME
d) Repeat (a) and (b) for $P(\omega_1) = 1/3$ and $P(\omega_2) = 2/3$
e) Find the Bayes min-risk (BMR) classifier if $P(\omega_1) = ¾$, $P(\omega_2) = ¼$, and $\lambda_{11} = \lambda_{22} = 0 \quad \lambda_{21} = 2 \quad \lambda_{12} = 1$
f) Find the conditional risks, $R(\alpha_1|x)$ and $R(\alpha_2|x)$ for the BMR if $a_1=1$, $a_2=4$, and $b=1$.

Problem 2

The pdfs of a one-dimensional feature, x, for a two-class problem are shown in the figure next page. If $P(\omega_1) = 0.2$ and $P(\omega_2) = 0.8$,

a) Find the BME classifier
b) Find the error of BME
c) Find the BMR classifier using $\lambda_{11} = \lambda_{22} = 0 \quad \lambda_{21} = 2 \quad \lambda_{12} = 1$
d) Find the conditional risks, $R(\alpha_1|x)$ and $R(\alpha_2|x)$ for the BMR
e) Find the total risk, R for the BMR



Problem 3

There are three possible actions in a two-class problem. α_1 and α_2 indicate classification action to class1 and 2, respectively, whereas action α_3 is the rejection action, i.e., unrecognizable sample. Let

$\lambda_{11}= \lambda_{22}= 0 \quad \lambda_{21}= \lambda_{12}= a \quad \lambda_{31}= \lambda_{32}= b$

In other words, "a" is the loss incurred for making a substitution error and "b" is the loss of rejection.

a) Find the simplest form of $R(\alpha_i|x)$, i=1,2,3 in terms of $P(\omega_i|x)$, a, and b.
b) Find the simplest form of the BMR classifier in the form of conditions defined on $P(\omega_i|x)$.