In The Name of Almighty

Statistical Pattern Recognition- HW#2

Part 1

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Chapter 2: 2.19, 2.20, 2.22, 2.30

Part 2

Problem 1

Let x_k , k=1, …,n, denote independent training samples from one of the following probability density functions. Find the maximum-likelihood estimate of θ.

(a) $p(x_k \mid \theta) = \theta\, e^{-\theta x_k}, \quad x_k \ge 0; \quad \theta > 0$ (Exponential Density)

(b) $p(x_k \mid \theta) = \theta\, c^{\theta}\, x_k^{-(\theta+1)}, \quad x_k \ge c;\ c \text{ constant} > 0; \quad \theta > 0$ (Pareto Density)

(c) $p(x_k \mid \theta) = c\,\theta^{c}\, x_k^{-(c+1)}, \quad x_k \ge \theta;\ c \text{ constant} > 0; \quad \theta > 0$ (Pareto Density)

(d) $p(x_k \mid \theta) = \sqrt{\theta}\, x_k^{\sqrt{\theta}-1}, \quad 0 \le x_k \le 1; \quad \theta > 0$ (Beta Density)

(e) $p(x_k \mid \theta) = \left(\dfrac{x_k}{\theta^2}\right) \exp\left\{\dfrac{-x_k^2}{2\theta^2}\right\}, \quad x_k > 0; \quad \theta > 0$ (Rayleigh Density)

(f) $p(x_k \mid \theta) = \theta c\, x_k^{\,c-1} \exp\left\{-\theta x_k^{\,c}\right\}, \quad x_k \ge 0;\ c \text{ constant} > 0;\ \theta > 0$ (Weibull Density)

(g) $p(x_k \mid \theta) = \begin{cases} 1 & \theta - \frac{1}{2} \le x_k \le \theta + \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$ (Uniform Density)

Problem 2

Let $x_1, \ldots, x_m$ and $y_1, \ldots, y_n$ be two independent sets of training samples from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, respectively. Find the maximum-likelihood estimate of $\theta = (\mu_1, \mu_2, \sigma^2)$.

Problem 3

One-dimensional features are used to represent classes in a two-class pattern recognition problem. It is assumed that features of both classes are normally distributed. Ten samples are available from each class according to the following:

ω_1 : {1.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 0.7, 1.5}

ω_2 : {1.0., 0.5, -0.5, 1.2, 0.9, 1.01, -0.9, -1.2, -1.0, -1.01}

It is known that the variance of the 2nd class, $\sigma_2^2 = 1$, and the mean of of the 2nd class is normally distributed as: $p(\mu_2) \sim N(0.5, 0.5)$.

a) Find the maximum-likelihood estimates of the mean and variance of the features of the 1st class and express $p(x \mid \omega_1)$ in terms of them.
b) Use the Bayesian Learning approach to find $p(x \mid \omega_2)$.
c) Using the above estimated densities, and $P(\omega_1) = P(\omega_2)$, design the BME classifier.