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Two state systems :-

Classically $\Rightarrow \quad \mu = \frac{q}{2m} S$

QM $\Rightarrow \quad \mu = g \frac{q}{2m} S$

$$\hat{H} = -\mu \cdot \vec{B}$$
$$= -\gamma (S \cdot B) \qquad \gamma = g\frac{q}{2m}$$

$$= -\gamma \left(B_x S_x + B_y S_y + B_z S_z \right)$$

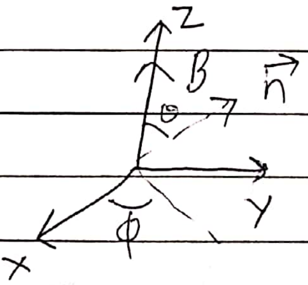
For $\vec{B} = |B| \hat{z}$

$$H = -\gamma |B| \hat{S}_z$$

$$U(t,0) = \exp\left(-\frac{i}{\hbar} H_s t\right) = \exp\left(\frac{i \gamma B t}{\hbar} (\hat{S}_z)\right)$$

For rotation operator,

$$R_{\vec{n}}(\alpha) = \exp\left(\frac{-i \alpha \cdot \hat{S}_{\vec{n}}}{\hbar}\right) \qquad \hat{S}_{\vec{n}} = n \cdot \hat{S}$$



$$|\psi; 0\rangle = \cos\left(\frac{\theta}{2}\right)|+\rangle + \sin\frac{\theta}{2} e^{i\phi} |-\rangle$$

$$H|+\rangle = -\gamma B . S_z |+\rangle = -\frac{\gamma B \hbar}{2} |+\rangle$$

$$H|-\rangle = -\gamma B S_z |-\rangle = +\frac{\gamma B \hbar}{2} |-\rangle$$

$$|\psi, t\rangle = e^{-\frac{iHt}{\hbar}} |\psi, 0\rangle$$

$$= \cos\frac{\theta}{2} e^{+\frac{i \gamma B \hbar}{2\hbar} t} . |+\rangle + \sin\frac{\theta}{2} . e^{-i\left(\frac{\gamma B \hbar t}{2\hbar}\right)} |-\rangle e^{i\phi}$$

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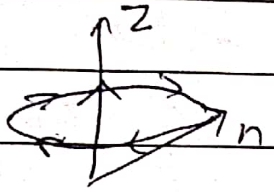
$$|\Psi, t\rangle = e^{i\frac{\gamma B t}{2}} \left(\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right) e^{i(\phi_0 - \gamma B t)} |-\rangle \right)$$

$$\phi(t) = \phi_0 - (\gamma B)t$$



2 X 2 Hermitian Hamilton.

$$H = \begin{pmatrix} g_0 + g_3 & g_1 - i g_2 \\ g_1 + i g_2 & g_0 - g_3 \end{pmatrix} \qquad g_0, g_1, g_2, g_3 \in \mathbb{R}.$$

$$= g_0 \mathbb{1} + g_1 \sigma_1 + g_2 \sigma_2 + g_3 \sigma_3$$

$$= g_0 \mathbb{1} + \vec{g} \cdot \vec{\sigma}$$

$$n \cdot \sigma |n; \pm\rangle = \pm |n; \pm\rangle$$

$$H = g_0 \mathbb{1} + \frac{2}{\hbar} g \cdot S$$

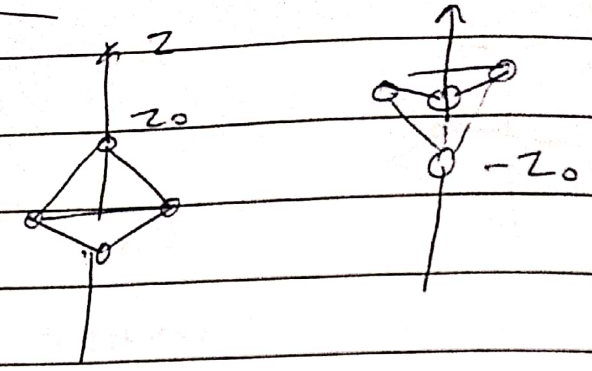
$$H_S = \cancel{\omega\,} \omega \cdot S$$

$$\omega = \frac{2}{\hbar} g$$

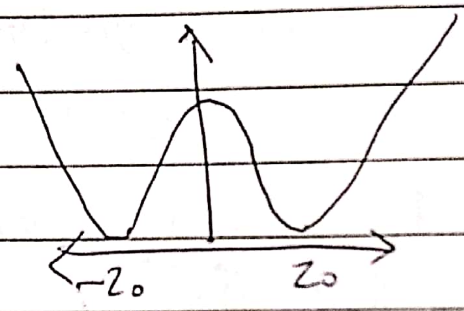
Ammonia molecule

Without Electric field :—

2 states.



For $V(z) =$



Δ if $\Delta \to \infty$

$$H = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix}$$

let bais be $|1\rangle, |2\rangle$.

For finite barrier

$$H = \begin{pmatrix} E_0 & -\Delta \\ -\Delta & E_0 \end{pmatrix}$$

$\Rightarrow \lambda = E_0 \pm \Delta$

Two energy states

$E_0 - \Delta$ $\quad |G\rangle \to$ ground $= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

$E_0 + \Delta$ $\quad |E\rangle \to$ exited $= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$

Time evolution

say $|\Psi(0)\rangle = |\uparrow\rangle = \frac{1}{\sqrt{2}}(|G\rangle + |E\rangle)$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i(E-\Delta)t/\hbar}|G\rangle + e^{-i(E+\Delta)t/\hbar}|E\rangle\right)$$

$$= e^{-iEt/\hbar}\left(\cos\left(\frac{\Delta t}{\hbar}\right)|\uparrow\rangle + i\sin\left(\frac{\Delta t}{\hbar}\right)|\downarrow\rangle\right)$$

$$P_\uparrow(t) = |\langle\uparrow|\Psi(t)\rangle|^2 = \cos^2\left(\frac{\Delta t}{\hbar}\right)$$

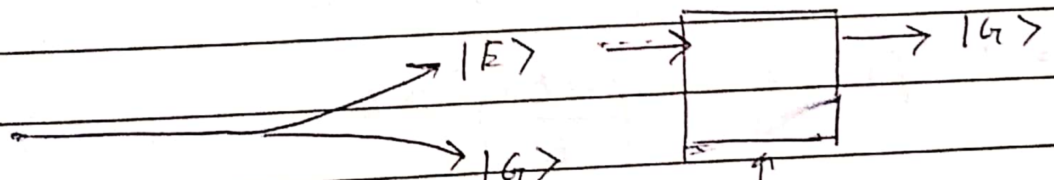
$$P_\downarrow(t) = |\langle\downarrow|\Psi(t)\rangle|^2 = \sin^2\left(\frac{\Delta t}{\hbar}\right)$$

With E - field :— $H(E) = -\mu \cdot \mathcal{E}$

$$H = \begin{pmatrix} E_0 + \mu\mathcal{E} & -\Delta \\ -\Delta & E_0 - \mu\mathcal{E} \end{pmatrix}$$

$$\Rightarrow \quad E_G(\mathcal{E}) = E_0 - \sqrt{\mu^2\mathcal{E}^2 + \Delta^2}$$

$$E_{Excited}(\mathcal{E}) = E_0 - \sqrt{\mu^2\mathcal{E}^2 + \Delta^2}$$



Construct Hamiltonian in

$|E\rangle = |1'\rangle, \quad |G\rangle = |2'\rangle$

$$H' = \begin{pmatrix} E_0 + \Delta & \mu\mathcal{E} \\ \mu\mathcal{E} & E_0 - \Delta \end{pmatrix}$$

$$\Psi(t) = \begin{pmatrix} C_E(t) \\ C_G(t) \end{pmatrix}$$

$$i\hbar \frac{\partial}{\partial t}\begin{bmatrix} C_E \\ C_G \end{bmatrix} = \begin{pmatrix} \Delta & \mu\mathcal{E} \\ \mu\mathcal{E} & -\Delta \end{pmatrix}\begin{pmatrix} C_E \\ C_G \end{pmatrix}$$

$E_0 = 0$, constant

Ansatz:
$$\begin{pmatrix} C_E \\ C_G \end{pmatrix} = \begin{pmatrix} e^{-\frac{i\Delta t}{\hbar}} \beta_E(t) \\ e^{\frac{i\Delta}{\hbar}t} \beta_G(t) \end{pmatrix}$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} \beta_E \\ \beta_G \end{pmatrix} = \begin{pmatrix} 0 & e^{i\omega_0 t}\mu\mathcal{E} \\ e^{-i\omega_0 t}\mu\mathcal{E} & 0 \end{pmatrix}\begin{pmatrix} \beta_E \\ \beta_G \end{pmatrix}$$

$$\omega_0 = \frac{2\Delta}{\hbar}$$

let $\mathcal{E}(t) = 2\mathcal{E}_0 \cos(\omega_0 t)$

$$i\dot{\beta}_E = \frac{\mu\mathcal{E}_0}{\hbar}\left(1 + e^{2i\omega_0 t}\right)\beta_G$$

$$i\dot{\beta}_G = \frac{\mu\mathcal{E}_0}{\hbar}\left(1 + e^{-2i\omega_0 t}\right)\beta_E$$

small

$$i\dot{\beta}_E = \frac{\mu\mathcal{E}_0}{\hbar}\beta_G, \qquad i\dot{\beta}_G = \frac{\mu\mathcal{E}_0}{\hbar}\beta_E$$

$$\Rightarrow \ddot{\beta}_E = -\left(\frac{\mu\mathcal{E}_0}{\hbar}\right)^2 \beta_E \ldots$$

$$\beta_E = \cos\left(\frac{\mu\mathcal{E}_0 t}{\hbar}\right)$$

$$P_E(t) = \cos^2\left(\frac{\mu\mathcal{E}_0 t}{\hbar}\right)$$

$$\frac{\mu\mathcal{E}_0 T}{\hbar} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

$— |E\rangle \ldots |$ $\quad | \rightarrow |G\rangle$

$\longleftarrow T \longrightarrow$

time to traverse