Solid State

Electrons in a Magnetic field (1)

For a Classical Electromagnetic field, with scalar and vector potentials ϕ and A.

$E = -\nabla\phi - \frac{\partial A}{\partial t}$, $B = \nabla \times A$.

The Lagrangian, $L = \frac{1}{2}m\dot{x}^2 + q(\dot{x}.A - \phi)$

For, $E = 0$ and $B = B_z\hat{k}$,

$$(x(t), y(t)) = (R\sin(\omega_B(t-t_0)), R\cos(\omega_B(t-t_0)))$$

Using Lagrange's equation and the Legendre transformation,

$$p = \frac{\partial L}{\partial \dot{x}} = m\dot{x} + qA \; H = \frac{1}{2m}(p-qA)^2 + q\phi$$

With poisson brackets,
$x_i, p_j = \delta_{ij}, x_i, x_j = p_i, p_j = 0$

Gauge: $\phi \to \phi - \frac{\partial\alpha}{\partial t}, A \to A + \nabla\alpha$

This leads to the Schrodinger's equation,

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - qA)^2\psi + q\phi\psi$$

For the above case $(E = 0, B = B_z\hat{k})$,

$$H = \frac{1}{2m}(p_x^2 + (p_y - qBx)^2 + p_z^2) \; \psi = e^{i(k_y y + k_z z)}X(x) \; \hat{p_y}\psi = \hbar k_y\psi, \; \hat{p_z}\psi = \hbar k_z\psi$$

$$H = \frac{1}{2m}p_x^2 + \frac{m\omega_B^2}{2}(x - k_y l_B^2)^2$$

Where, $l_B = \sqrt{\frac{\hbar}{qB}}$

This is a harmonic oscillator, therefore the eigenvalues and eigenvectors are,

$$E = \hbar\omega_B(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m} \; \psi_{n,k}(x,y,z) = e^{i(k_y y + k_z z)}H_n(x - k_y l_B^2)e^{-\frac{(x-k_y l_B^2)^2}{2l_B^2}}$$

These are **Landau levels**, There exists a large degeneracy because of dependence on n and k. Say, we study a finite region on the (x, y) plane., with side lengths L_x & L_y.

$$\psi(x, y + L_y, z) = \psi(x, y, z) \implies e^{ik_y L_y} = 1$$

and the total degeneracy, As the x direction has the whole harmonic oscillator thing, it does not have translation invariance under the gauge transform. So as $x = k_y l_B^2$ is where the exponent localises, for $0 \leq x \leq L_x \implies 0 \leq k_y \leq L_x/l_B^2$

Degeneracy for each level is, $\mathcal{N} = \frac{\mathcal{L}_y}{2\pi}\int_0^{\mathcal{L}_x/l_\mathcal{B}^2} dk = \frac{q\mathcal{B}\mathcal{A}}{2\pi\hbar}$. Where $\mathcal{A}$ is the area of the sample.