

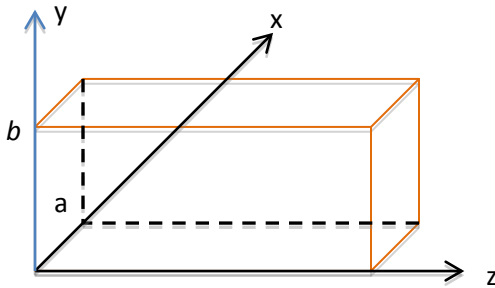
Wave Guides

Lecture37: Electromagnetic Theory

Professor D. K. Ghosh, Physics Department, I.I.T., Bombay

Rectangular Waveguide

In the last lecture, we have considered the case of guided wave between a pair of infinite conducting planes. In this lecture, we consider a rectangular wave guide which consists of a hollow pipe of infinite extent but of rectangular cross section of dimension $a \times b$. The long direction will be taken to be the z direction.



Unlike in the previous case $\partial/\partial y$ is not zero in this case. However, as the propagation direction is along the z direction, we have $\frac{\partial}{\partial z} \rightarrow -\gamma$ and $\frac{\partial}{\partial t} \rightarrow i\omega$. We can write the Maxwell's curl equations as

$$\begin{aligned}\frac{\partial H_z}{\partial y} + \gamma H_y &= i\omega\epsilon E_x \\ -\gamma H_x - \frac{\partial H_z}{\partial x} &= i\omega\epsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= i\omega\epsilon E_z\end{aligned}$$

The second set of equations are obtained from the Faraday's law, (these can be written down from above by $E \leftrightarrow H, \epsilon \rightarrow -\mu$)

$$\begin{aligned}\frac{\partial E_z}{\partial y} + \gamma E_y &= -i\omega\mu H_x \\ -\gamma E_x - \frac{\partial E_z}{\partial x} &= -i\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -i\omega\mu H_z\end{aligned}$$

As in the case of parallel plate waveguides, we can express all the field quantities in terms of the derivatives of E_z and H_z . For instance, we have,

$$\begin{aligned} i\omega\epsilon E_x &= \gamma H_y + \frac{\partial H_z}{\partial y} \\ &= \frac{\gamma}{i\omega\mu} \left(\gamma E_x + \frac{\partial E_z}{\partial x} \right) + \frac{\partial H_z}{\partial y} \end{aligned}$$

which gives,

$$\left(i\omega\epsilon - \frac{\gamma^2}{i\omega\mu} \right) E_x = \frac{\gamma}{i\omega\mu} \frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}$$

so that

$$E_x = -\frac{\gamma}{k^2} \frac{\partial E_z}{\partial x} - \frac{i\omega\mu}{k^2} \frac{\partial H_z}{\partial y} \quad (1)$$

where

$$k^2 = \gamma^2 + \omega^2\mu\epsilon$$

The other components can be similarly written down,

$$E_y = -\frac{\gamma}{k^2} \frac{\partial E_z}{\partial y} - \frac{i\omega\mu}{k^2} \frac{\partial H_z}{\partial x} \quad (2)$$

$$H_x = \frac{i\omega\epsilon}{k^2} \frac{\partial E_z}{\partial y} - \frac{\gamma}{k^2} \frac{\partial H_z}{\partial x} \quad (3)$$

$$H_y = -\frac{i\omega\epsilon}{k^2} \frac{\partial E_z}{\partial x} - \frac{\gamma}{k^2} \frac{\partial H_z}{\partial y} \quad (4)$$

As before, we will look into the TE mode in detail. Since $E_z = 0$, we need to solve for H_z from the Helmholtz equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 \right) H_z(x, y) = 0 \quad (5)$$

Remember that the complete solution is obtained by multiplying with $e^{-\gamma z + i\omega t}$. We solve equation (5) using the technique of separation of variables which we came across earlier. Let

$$H_z(x, y) = X(x)Y(y)$$

Substituting this in (5) and dividing by XY throughout, we get,

$$\frac{1}{X} \frac{d^2 X}{dx^2} + k^2 = -\frac{1}{Y} \frac{d^2 Y}{dy^2} \equiv k_y^2$$

where k_y is a constant. We further define $k_x^2 = k^2 - k_y^2$

We now have two second order equations,

$$\frac{d^2 X}{dx^2} + k_x^2 X = 0$$

$$\frac{d^2 Y}{dy^2} + k_y^2 Y = 0$$

The solutions of these equations are well known

$$X(x) = C_1 \cos k_x x + C_2 \sin k_x x$$

$$Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$$

This gives,

$$H_z(x, y) = C_1 C_3 \cos k_x x \cos k_y y + C_1 C_4 \cos k_x x \sin k_y y$$

$$+ C_2 C_3 \sin k_x x \cos k_y y + C_2 C_4 \sin k_x x \sin k_y y$$

The boundary conditions that must be satisfied to determine the constants is the vanishing of the tangential component of the electric field on the plates. In this case, we have two pairs of plates. The tangential direction on the plates at $x = 0$ and $x = a$ is the y direction, so that the y component of the electric field

$$E_y = 0 \text{ at } x = 0, a$$

. Likewise, on the plates at $y = 0$ and $y = b$,

$$E_x = 0 \text{ at } y = 0, b$$

We need first to evaluate E_x and E_y using equations (1) and (2) and then substitute the boundary conditions. Since $E_z = 0$, we can write eqn. (1) and (2) as

$$E_x = -\frac{i\omega\mu}{k^2} \frac{\partial H_z}{\partial y}$$

$$E_y = -\frac{i\omega\mu}{k^2} \frac{\partial H_z}{\partial x}$$

$$E_x = -\frac{i\omega\mu}{k^2} [-C_1 C_3 k_y \cos k_x x \sin k_y y + C_1 C_4 k_y \cos k_x x \cos k_y y$$

$$- C_2 C_3 k_y \sin k_x x \sin k_y y + C_2 C_4 k_y \sin k_x x \cos k_y y]$$

$$E_y = -\frac{i\omega\mu}{k^2} [-C_1 C_3 k_x \sin k_x x \cos k_y y - C_1 C_4 k_x \sin k_x x \sin k_y y$$

$$+ C_2 C_3 k_x \cos k_x x \cos k_y y + C_2 C_4 k_x \cos k_x x \sin k_y y]$$

Since $E_y = 0$ at $x = 0$, we must have $C_2 = 0$ and then we get,

$$E_y = -\frac{i\omega\mu}{k^2} [-C_1 C_3 k_x \sin k_x x \cos k_y y - C_1 C_4 k_x \sin k_x x \sin k_y y]$$

Further, since $E_x = 0$ at $y=0$, we have $C_4 = 0$. Combining these, we get, on defining a constant $C = C_1 C_3$

$$E_x = \frac{i\omega\mu}{k^2} C k_y \cos k_x x \sin k_y y$$

$$E_y = \frac{i\omega\mu}{k^2} C k_x \sin k_x x \cos k_y y$$

and

$$H_z = C \cos k_x x \cos k_y y$$

We still have the boundary conditions, $E_x = 0$ at $y = b$ and $E_y = 0$ at $x = a$ to be satisfied. The former gives $k_y = \frac{n\pi}{b}$ while the latter gives $k_x = \frac{m\pi}{a}$, where m and n are integers. Thus we have,

$$H_z = C \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right)$$

and

$$k^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

However, $k^2 = \omega^2 \mu \epsilon - \gamma^2$, so that,

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

For propagation to take place, γ must be imaginary, so that the cutoff frequency below which propagation does not take place is given by

$$\omega_c = \frac{1}{\sqrt{\mu \epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

The minimum cutoff is for $TE_{1,0}$ (or $TE_{0,1}$) mode which are known as *dominant mode*. For these modes E_x (or E_y) is zero.

TM Modes

We will not be deriving the equations for the TM modes for which $H_z = 0$. In this case, the solution for E_z , becomes,

$$E_z = E_{z0} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right)$$

As the solution is in terms of product of sine functions, neither m nor n can be zero in this case. This is why the lowest TE mode is the dominant mode.

For propagating solutions, we have,

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \sqrt{\mu \epsilon} \sqrt{\omega^2 - \omega_c^2}$$

where,

$$\omega_c = \frac{1}{\sqrt{\mu \epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

We have, $\omega^2 = \frac{\beta^2}{\mu\epsilon} + \omega_c^2$. Differentiating both sides, we have,

$$\omega \frac{d\omega}{d\beta} = \frac{1}{\mu\epsilon} \beta$$

The group velocity of the wave is given by

$$v_g = \frac{d\omega}{d\beta} = \frac{\beta}{\omega\mu\epsilon} = \frac{\sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}}{\omega\mu\epsilon} = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

which is less than the speed of light. The phase velocity, however, is given by

$$v_\phi = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}} = \frac{1}{\sqrt{\mu\epsilon}} \frac{1}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

It may be noted that $v_\phi v_g = \frac{1}{\mu\epsilon}$, which in vacuum equal the square velocity of light.

For propagating TE mode, we have, from (1) and (4) using $E_z = 0$

$$\frac{E_x}{H_y} = \frac{i\omega\mu}{\gamma} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}} \equiv \eta_{TE}$$

where η_{TE} is the characteristic impedance for the TE mode. It is seen that the characteristic impedance is resistive. Likewise,

$$\frac{E_y}{H_x} = -\eta_{TE}$$

Power Transmission

We have seen that in the propagating mode, the intrinsic impedance is resistive, indicating that there will be average flow of power. The Poynting vector for TE mode is given by

$$\begin{aligned} \langle S \rangle &= \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) \\ &= \frac{1}{2} \text{Re}(E_x H_y^* - E_y H_x^*) \\ &= \frac{1}{2\eta_{TE}} (|E_x|^2 + |E_y|^2) \end{aligned}$$

Substituting the expressions for the field components, the average power flow through the x y plane is given by

$$\begin{aligned} \langle P \rangle &= \int \langle S \rangle dx dy = \frac{1}{2\eta_{TE}} \int_0^a \int_0^b (|E_x|^2 + |E_y|^2) dx dy \\ &= \frac{1}{2\eta_{TE}} \frac{C^2 \omega^2 \mu^2}{k^4} \left[\left(\frac{n\pi}{b} \right)^2 \int_0^a \int_0^b \cos^2 \left(\frac{m\pi}{a} x \right) \sin^2 \left(\frac{n\pi}{b} y \right) dx dy \right. \\ &\quad \left. + \left(\frac{m\pi}{a} \right)^2 \int_0^a \int_0^b \sin^2 \left(\frac{m\pi}{a} x \right) \cos^2 \left(\frac{n\pi}{b} y \right) dx dy \right] \end{aligned}$$

$$= \frac{1}{2\eta_{TE}} \frac{C^2 \omega^2 \mu^2}{k^4} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right] \frac{ab}{4}$$

Impossibility of TEM mode in Rectangular waveguides

We have seen that in a parallel plate waveguide, a TEM mode for which both the electric and magnetic fields are perpendicular to the direction of propagation, exists. This, however is not true of rectangular wave guide, or for that matter for any hollow conductor wave guide without an inner conductor.

We know that lines of H are closed loops. Since there is no z component of the magnetic field, such loops must lie in the x-y plane. However, a loop in the x-y plane, according to Ampere's law, implies an axial current. If there is no inner conductor, there cannot be a real current. The only other possibility then is a displacement current. However, an axial displacement current requires an axial component of the electric field, which is zero for the TEM mode. Thus TEM mode cannot exist in a hollow conductor. (for the parallel plate waveguides, this restriction does not apply as the field lines close at infinity.)

Resonating Cavity

In a rectangular waveguide we had a hollow tube with four sides closed and a propagation direction which was infinitely long. We will now close the third side and consider electromagnetic wave trapped inside a rectangular parallelepiped of dimension $a \times b \times d$ with walls being made of perfect conductor. (The third dimension is taken as d so as not to confuse with the speed of light c).

Resonating cavities are useful for storing electromagnetic energy just as LC circuit does but the former has an advantage in being less lossy and having a frequency range much higher, above 100 MHz.

The Helmholtz equation for any of the components E_α of the electric field can be written as

$$\nabla^2 E_\alpha = -\omega^2 \mu \epsilon E_\alpha$$

We write this in Cartesian and use the technique of separation of variables,

$$E_\alpha(x, y, z) = X_\alpha(x)Y_\alpha(y)Z_\alpha(z)$$

Introducing this into the equation and dividing by $X_\alpha(x)Y_\alpha(y)Z_\alpha(z)$, we get,

$$\frac{1}{X_\alpha} \frac{\partial^2 X_\alpha}{\partial x^2} + \frac{1}{Y_\alpha} \frac{\partial^2 Y_\alpha}{\partial y^2} + \frac{1}{Z_\alpha} \frac{\partial^2 Z_\alpha}{\partial z^2} = -\omega^2 \mu \epsilon$$

Since each of the three terms on the right is a function of an independent variable, while the right hand side is a constant, we must have each of the three terms equaling a constant such that the three constants add up to the constant on the right. Let,

$$\frac{\partial^2 X_\alpha}{\partial x^2} = -k_x^2 X_\alpha$$

$$\frac{\partial^2 Y_\alpha}{\partial y^2} = -k_y^2 Y_\alpha$$

$$\frac{\partial^2 Z_\alpha}{\partial z^2} = -k_z^2 Z_\alpha$$

such that $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon$.

As each of the equations has a solution in terms of linear combination of sine and cosine functions, we write the solution for the electric field as

$$E_\alpha = (A_\alpha \cos k_x x + B_\alpha \sin k_x x)(C_\alpha \cos k_y y + D_\alpha \sin k_y y)(F \cos k_z z + G_\alpha \sin k_z z)$$

The tangential component of above must vanish at the metal boundary. This implies,

1. At $x = 0$ and at $x = a$, E_y and $E_z = 0$ for all values of y, z ,
2. At $y = 0$ and $y = b$, E_x and $E_z = 0$ for all values of x, z ,
3. At $z = 0$ and $z = d$, E_x and $E_y = 0$ for all values of x, y

Let us consider E_x which must be zero at $y = 0, y = b, z = 0, z = d$. This is possible if

$$E_x = E_{x0}(A_x \cos k_x x + B_x \sin k_x x) \sin k_y y \sin k_z z$$

with $k_y = \frac{m\pi}{b}$ and $k_z = \frac{n\pi}{d}$. Here m and n are non-zero integers.

In a similar way, we have,

$$E_y = E_{y0} \sin k_x x (C_y \cos k_y y + D_y \sin k_y y) \sin k_z z$$

$$E_z = E_{z0} \sin k_x x \sin k_y y (F_z \cos k_z z + G_z \sin k_z z)$$

with $k_x = \frac{l\pi}{a}$, with l being non-zero integer.

We now use

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

This relation must be satisfied for **all values of x, y, z** within the cavity. This requires,

$$\begin{aligned} E_{x0}(-A_x k_x \sin k_x x &+ B_x k_x \cos k_x x) \sin k_y y \sin k_z z \\ &+ E_{y0} \sin k_x x (-C_y k_y \sin k_y y \\ &+ D_y k_y \cos k_y y) \sin k_z z + E_{z0} \sin k_x x \sin k_y y (-F_z k_z \sin k_z z \\ &+ G_z k_z \cos k_z z) = 0 \end{aligned}$$

Let us choose some special points and try to satisfy this equation. Let $x = 0, y, z$ arbitrary,

This requires $B_x = 0$. Likewise, taking $y = 0, x, z$ arbitrary, we require $D_y = 0$ and finally, $z = 0, x, y$ arbitrary gives $G_z = 0$.

With these our solutions for the components of the electric field becomes,

$$E_x = E_{x0} \cos k_x x \sin k_y y \sin k_z z$$

$$E_y = E_{y0} \sin k_x x \cos k_y y \sin k_z z$$

$$E_z = E_{z0} \sin k_x x \sin k_y y \cos k_z z$$

where we have redefined our constants E_{x0}, E_{y0} and E_{z0} .

We also have,

$$k_x = \frac{l\pi}{a}, k_y = \frac{m\pi}{b}, k_z = \frac{n\pi}{d}$$

the integers l, m, n cannot be simultaneously zero for then the field will identically vanish.

Note further that since $\nabla \cdot \vec{E} = 0$ must be satisfied everywhere within the cavity, we must have, calculating the divergence explicitly from the above expressions,

$$k_x E_{x0} + k_y E_{y0} + k_z E_{z0} = 0$$

This requires that \vec{k} is perpendicular to \vec{E} .

One can see that the modes within a cavity can exist only with prescribed “resonant” frequency corresponding to the set of integers l, m, n

$$\omega = \frac{1}{\sqrt{\mu\epsilon}} \left[\left(\frac{l\pi}{a} \right)^2 + \left(\frac{m\pi}{b} \right)^2 + \left(\frac{n\pi}{d} \right)^2 \right]^{\frac{1}{2}}$$

Though there is nothing like a propagation direction here, one can take the longest dimension (say d) to be the propagation direction.

We can have modes like $TE_{l,m,n}$ corresponding to the set l, m, n for which $E_z = 0$. For this set we get,

$$E_x = E_{x0} \cos k_x x \sin k_y y \sin k_z z$$

$$E_y = E_{y0} \sin k_x x \cos k_y y \sin k_z z$$

with $k_x E_{x0} + k_y E_{y0} = 0$. The magnetic field components can be obtained from the Faraday’s law,

$$H_x = \frac{1}{-i\omega\mu} \left(-\frac{\partial E_y}{\partial z} \right) = \frac{k_z}{i\omega\mu} E_{y0} \sin k_x x \cos k_y y \cos k_z z$$

$$H_y = \frac{1}{-i\omega\mu} \left(\frac{\partial E_x}{\partial z} \right) = -\frac{k_z}{i\omega\mu} E_{x0} \cos k_x x \sin k_y y \cos k_z z$$

$$H_z = \frac{1}{-i\omega\mu} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = -\frac{1}{i\omega\mu} (E_{y0} k_x - E_{x0} k_y) \cos k_x x \cos k_y y \sin k_z z$$

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Tutorial Assignment

1. A rectangular waveguide has dimensions 8 cm x 4 cm. Determine all the modes that can propagate when the operating frequency is (a) 1 GHz, (b) 3 GHz and (c) 8 GHz.
2. Two signals, one of frequency 10 GHz and the other of 12 GHz are simultaneously launched in an air filled rectangular waveguide of dimension 2 cm x 1 cm. Find the time interval between the arrival of the signals at a distance of 10 m from the common place of their launch.
3. What should be the third dimension of a cavity having a length of 1 cm x 1 cm which can operate in a TE₁₀₃ mode at 24 GHz?

Solutions to Tutorial Assignments

1. The cutoff frequency for (m,n) mode is $\nu_c = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 1.5 \times 10^{10} \sqrt{\frac{m^2}{64} + \frac{n^2}{16}}$. Some of the lowest cutoffs are as under (in GHz) :

m	n	Cutoff frequency (GHz)
0	1	3.75
0	2	7.5
0	3	11.25
1	0	1.875
1	1	4.192
1	2	7.731
1	3	11.405
2	0	3.75
2	1	5.30
2	2	8.39
3	0	5.625
3	1	6.76
3	2	9.375
4	0	7.5
4	1	8.38
5	0	9.375

At 1 GHz, no propagation takes place. At 3 GHz only TE₁₀ mode propagates (recall no TM mode is possible when either of the indices is zero.) At 8 GHz, we have TE₀₁, TE₀₂, TE₁₀, TE₁₁, TE₁₂, TE₂₀, TE₂₁, TE₃₀, TE₃₁, TE₄₀, TM₁₁, TM₁₂, TM₂₁ and TM₃₁, i.e. a total of 14 modes propagating.

2. The cutoff frequency is given by

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 7.5 \text{ GHz}$$

The propagation of the wave is given by the group velocity $v_g = c \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$, which is 1.98×10^8 and 2.07×10^8 m/s respectively for $f=10$ GHz and 12 GHz respectively. Thus the difference in speed is 9×10^6 m/s, resulting in a time difference of approximately 10^{-5} s in travelling 10m.

3. The operating frequency is given by

$$\omega = \frac{1}{\sqrt{\mu\epsilon}} \left[\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{d}\right)^2 \right]^{\frac{1}{2}}$$

Substituting $l = 1, m = 0, n = 3$, we get $d = 2.4$ cm.

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Self Assessment Questions

1. A rectangular, air filled waveguide has dimension 2 cm x 1 cm. For what range of frequencies, there is a “single mode” operation in the guide?
2. The cutoff frequency for a TE₁₀ mode in an air filled waveguide is 1.875 GHz. What would be the cutoff frequency of this mode if the guide were to be filled with a dielectric of permittivity $9\epsilon_0$?
3. In an air filled waveguide of dimension 2 cm x 2 cm, the x component of the electric field is given by

$$E_x = -8 \sin\left(\frac{2\pi y}{a}\right) \sin(\omega t - 100z)$$

Identify the propagating mode, determine the frequency of operation and find H_z and E_z .

Solutions to Self Assessment Questions

1. The cutoff frequency for (m,n) mode is $\nu_c = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 1.5 \times 10^{10} \sqrt{\frac{m^2}{4} + n^2}$.
Substituting values, the cutoff for (1,0) mode is 7.5 GHz and for (2,0) is 15 GHz. The frequency for (0,1) is also 15 GHz. All other modes have higher cut off. Thus in order that only one mode propagates, the operating frequency should be in the range $7.5 < \nu < 15$ GHz. In this range, only TE₁₀ mode operates. Recall that there is no TM mode with either of the indices being zero.
2. The cutoff frequency is proportional to $\frac{1}{\sqrt{\mu\epsilon}}$. Thus the frequency would be reduced by a factor of 3 making it 625 MHz.
3. Since one of the standing wave factor is missing, one of the mode indices is zero. Thus it is a TE mode. The electric field of TE_{mn} mode is given by (with a=b)

$$E_x = \frac{i\omega\mu}{k^2} C k_y \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right) \sin(\beta z - \omega t)$$

Thus the mode is TE₀₂ mode. From the relation $k_x^2 + k_y^2 + \beta^2 = \frac{\omega^2}{c^2}$. Given $k_x = 0, k_y = \frac{2\pi}{0.02} = 100\pi, \beta = 100$,

$$\frac{\omega^2}{c^2} = (100\pi)^2 + (100)^2$$

the operating frequency is 15.74 GHz.

Since $k_x = 0, E_y = 0$. Further, by definition for TE mode $E_z = 0$.

Here we have $k^2 = k_x^2 + k_y^2 = k_y^2$. The ratio

$$\left| \frac{H_z}{E_x} \right| = \frac{k^2}{\omega \mu k_y} = \frac{k_y}{\omega \mu} = 0.0025$$

so that

$$H_z = 0.020 \cos\left(\frac{2\pi}{a}y\right) \cos(\omega t - 100z) \text{ A/m}$$