Multiparticle states & Tensor products :—

Distinguishable only :— Tensor product

Particle 1 :— Vector space : V, operators $\mathcal{L}(V)$
Particle 2 :— Vector space : W, operators $\mathcal{L}(W)$

if $v \in V$, $w \in W$, $v \otimes w = V \otimes W$

$(\alpha v) \otimes w = v \otimes (\alpha w)$. $\alpha \in \mathbb{C}$

$(v_1 + v_2) \otimes w = v_1 \otimes w + v_2 \otimes w$.

$v \otimes (w_1 + w_2) = v \otimes w_1 + v \otimes w_2$

if $(e_1, e_2, \dots, e_n)$ is a basis of V.
and $(f_1, f_2, \dots, f_n)$ a basis for W.

$e_i \otimes f_j$ a basis for $V \otimes W$.

Operators on $V \otimes W$.

$T \in \mathcal{L}(V)$, $S \in \mathcal{L}(W)$

$T \otimes S \in \mathcal{L}(V \otimes W)$

$T \otimes S \, (v \otimes w) = (Tv) \otimes (Sw)$.

$T_1 \in \mathcal{L}(V) \rightarrow T_1 \otimes \mathbb{1} \in \mathcal{L}(V \otimes W)$.

$S_1 \in \mathcal{L}(W) \rightarrow \mathbb{1} \otimes S_1 \in \mathcal{L}(V \otimes W)$.

$[T_1 \otimes \mathbb{1}, \mathbb{1} \otimes S_1] = 0$

$H_T = H_1 \otimes \mathbb{1} + \mathbb{1} \otimes H_2$

$$|\psi\rangle = \alpha_1 |+\rangle_1 \otimes |+\rangle_2 + \alpha_2 |+\rangle_1 \otimes |-\rangle_2 + \alpha_3 |-\rangle_1 \otimes |+\rangle_2 + \alpha_4 |-\rangle_1 \otimes |-\rangle_2$$

$$(S_z^1 \otimes \mathbb{1})|\psi\rangle = \alpha_1 \hat{S}_z^1 |+\rangle \otimes |+\rangle + \alpha_2 \hat{S}_z^1 |+\rangle \otimes |-\rangle + \alpha_3 \hat{S}_z^1 |-\rangle \otimes |+\rangle + \alpha_4 \hat{S}_z^1 |-\rangle \otimes |-\rangle$$

$$= \frac{\hbar}{2}\left(\alpha_1 |+\rangle \otimes |+\rangle + \alpha_2 |+\rangle \otimes |-\rangle - \alpha_3 |-\rangle \otimes |+\rangle - \alpha_4 |-\rangle \otimes |-\rangle \right)$$

" $\mathbb{1} \otimes S_z^2$ "

$$S_z^{(total)} |\psi\rangle = \hbar \left(\alpha_1 |+\rangle \otimes |+\rangle - \alpha_4 |-\rangle \otimes |-\rangle \right)$$

$$\Rightarrow S_z^1 |+\rangle \otimes |+\rangle = \left(\frac{\hbar}{2} + \frac{\hbar}{2}\right) |+\rangle \otimes |+\rangle = \hbar |+\rangle \otimes |+\rangle$$

$$S_z^T |+\rangle \otimes |-\rangle = 0$$

$$S_z^T |-\rangle \otimes |+\rangle = 0$$

$$S_z^T |-\rangle \otimes |-\rangle = -\hbar |-\rangle \otimes |-\rangle$$

$$\langle v \otimes w, v \otimes w \rangle = \langle v, v \rangle \langle w, w \rangle$$

Entangled states :-

$V \otimes W$ has elements $\sum \alpha_{ij} \, v_i \otimes w_j$

if ψ in $V \otimes W$ can be written as $V_* \otimes W_*$ for $V_* \in V$, $W_* \in W \Rightarrow \psi$ is not entangled

for other ψ, they are entangled.

For example; in case of spin-1/2 with basis $\{e_1, e_2\}$ for V, $\{f_1, f_2\}$ for W.

$$\psi_A = a_{11} \, e_1 \otimes f_1 + a_{12} \, e_1 \otimes f_2 + a_{21} \, e_2 \otimes f_1 + a_{22} \, e_2 \otimes f_2$$

$$\Rightarrow A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

ψ_A is not entangled if $\psi_A = (a_1 e_1 + a_2 e_2) \otimes (b_1 f_1 + b_2 f_2)$

$\Rightarrow a_{11} = a_1 b_1$, $a_{22} = a_2 b_2$, $a_{12} = a_1 b_2$, $a_{21} = a_2 b_1$.

$\Rightarrow a_{11} a_{22} - a_{12} a_{21} = a_1 b_1 a_2 b_2 - a_1 b_2 a_2 b_1 = 0 \rightarrow \det(A) = 0$

Similarly we can prove that $\det(A) = 0 \rightarrow$ non-entangled.

For example -

$$|\psi_A\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_1 \otimes |+\rangle_2 - |-\rangle_1 \otimes |-\rangle_2 \right)$$

$$\Rightarrow A = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} \end{pmatrix}$$

$\Rightarrow$ State is entangled.

PAGE NO.:
DATE / /

for spin-1/2.

$|\phi_0\rangle = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle + |-\rangle|-\rangle)$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$|\phi_1\rangle = (\mathbb{1} \otimes \sigma_1)|\phi_0\rangle$

$= \frac{1}{\sqrt{2}}(|+\rangle|-\rangle + |-\rangle|+\rangle)$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$|\phi_2\rangle = (\mathbb{1} \otimes \sigma_2)|\phi_0\rangle = \frac{i}{\sqrt{2}}(|+\rangle|-\rangle - |-\rangle|+\rangle)$

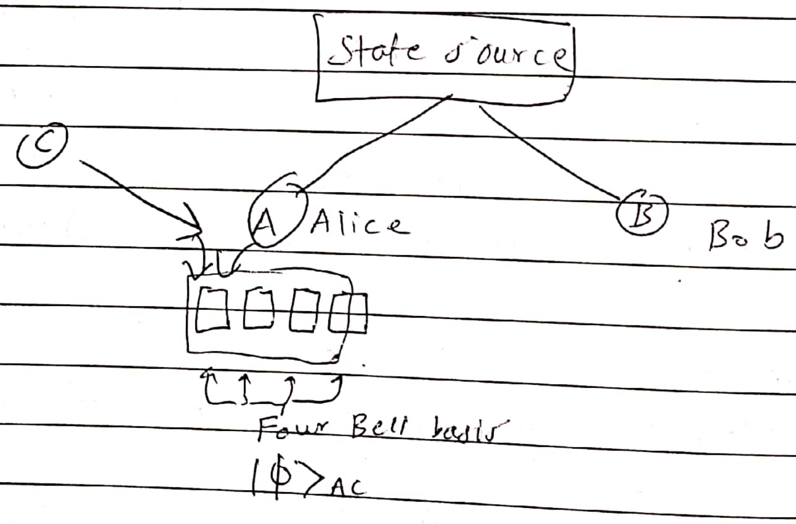
$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle - |-\rangle|-\rangle)$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

They are orthonormal.

These are called "Bell basis states" and are maximally entangled.

Quantum Teleportation:—

Alice has $|\psi\rangle_c = \alpha|+\rangle_c + \beta|-\rangle_c$



$|\psi_T\rangle = |\phi_0\rangle_{AB} \otimes (\alpha|+\rangle_c + \beta|-\rangle_c)$

say $|\phi_0\rangle_{AB} = \frac{1}{\sqrt{2}}(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B)$

$|\phi_0\rangle_{AB} \otimes |\alpha|+\rangle_c + \beta|-\rangle_c)$

$= \frac{1}{\sqrt{2}}(\alpha|+\rangle_A|+\rangle_c|+\rangle_B + \beta|+\rangle_A|-\rangle_c|+\rangle_B$
$+ \alpha|-\rangle_A|+\rangle_c|-\rangle_B + \beta|-\rangle_A|-\rangle_c|-\rangle_B)$

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$$= \frac{1}{2}\left\{\frac{1}{2}|\Phi_0\rangle_{AC} \otimes |\psi\rangle_B + \frac{1}{2}|\phi_1\rangle_{AC} \otimes \sigma_1 |\psi\rangle_B \right.$$

$$\left. + \frac{1}{2}|\phi_2\rangle_{AC} \otimes \sigma_2 |\psi_B\rangle + \frac{1}{2}|\phi_3\rangle_{AC} \otimes \sigma_3 |\psi\rangle_B \right.$$

if light - 0 shines, $|\psi\rangle_B$ is resulted on '0' slit by B.

results in C.

Original C is destroyed.

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EPR & Bell's inequalities :-

example of such entangled state

$$\eta_0 \rightarrow \mu^+ + \mu^-$$

meson ~~photons~~ muons

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2\right)$$

~~So~~

for $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|a;+\rangle_1 |a;-\rangle_2 - |a;-\rangle_1 |a;+\rangle_2\right)$

$$P(a,b) = |{}_1\langle a;+|{}_2\langle b;+|\psi\rangle|^2$$

$$= \frac{1}{2}\cos^2\left(\frac{\pi}{2} - \frac{\theta_{ab}}{2}\right) = \frac{1}{2}\sin^2\left(\frac{\theta_{ab}}{2}\right)$$

$\frac{1}{4}$ particles in $(z+, x+)$, $(-z, -x)$

$\frac{1}{4}$ particles in $(z, -x)$, $(-z, x)$

" " $(-z, x)$, $(z, -x)$

" " $(-z, -x)$, (z, x)

3-directions :— a, b, c

N_1	(a, b, c)	$(-a, -b, -c)$
N_2	$(a, b, c-)$	$(-a, -b, c)$
N_3	$(a, b-, c)$	$(-a, b, -c)$
N_4	$(a, b-, c-)$	$(-a, b, c)$
N_5	$(a-, b, c)$	$(a, -b, -c)$
N_6	$(a-, b, c-)$	$(a, -b, c)$
N_7	$(a-, b-, c)$	$(a, b, -c)$
N_8	$(a-, b-, c-)$	(a, b, c)

$P(a,b) = \frac{N_3 + N_4}{N}$, $P(a,c) = \frac{N_2 + N_4}{N}$, $P(c,b) = \frac{N_3 + N_7}{N}$

$\Rightarrow P(a,b) \leq P(a,c) + P(c,b)$

Bell's inequality

$\Rightarrow \sin^2\left(\frac{\theta_{ab}}{2}\right) \leq \sin^2\left(\frac{\theta_{ac}}{2}\right) + \sin^2\left(\frac{\theta_{cb}}{2}\right)$.

Let $\theta_{ab} = 2\theta$, $\theta_{ac} = \theta_{cb} = \theta$.

$\Rightarrow \frac{1}{2}\sin^2\theta \leq \sin^2(\frac{1}{2}\theta)$

contradictory.