QFT

Adwait

Quantum Field Theory

Canonical Quantisation.

For Quantising fields using second quantisation, we follow the algorithm.

step 1:

Get the Lagrangian Density $\mathcal{L}$

step 2:

Get the momentum and Hamiltonian density

$$\Pi^\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \mathcal{H} = \Pi^0 \partial_0 \phi - \mathcal{L}$$

step 3:

$$[\hat{\phi}(x^\nu), \hat{\Pi}^0(y^\nu)] = i\delta^3(x^\nu - y^\nu)$$

Step 4:

Expand the field in plane waves and get other operators. Use Wick ordering to prevent infinities.

Free Scalar field:

step I:

$$\mathcal{L} = \frac{1}{2}((\partial_\nu \phi)^2 - m^2 \phi^2)$$

Step II:

$$\Pi^\nu = \partial^\nu \phi$$

$$\mathcal{H} = \frac{1}{2}(\dot{\phi}^2 + (\nabla\phi)^2 + m^2\phi^2)$$

Step III:

Do the above

Step IV:

$$\hat{\phi}(x^\nu) = \int \frac{d^4p}{(2\pi)^4}(\hat{a}(k_\nu)e^{ik_\nu x^\nu} + \hat{a}^\dagger(k_\nu)e^{-ik_\nu x^\nu}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}(\hat{a}_p e^{ip.x} + \hat{a}_p^\dagger e^{-ip.x})$$

Where $E_p = \sqrt{p^2 + m^2}$

$$\mathcal{H} = \hat{T}\mathcal{H} = \int d^3p E_p \hat{a}_p^\dagger \hat{a}_p$$

Complex Scalar Field.

Step I:

$$\mathcal{L} = \partial^\nu\psi\partial_\nu\psi - m^2\psi^\dagger\psi$$

Step II:

$$\Pi^0_\psi = \partial^0\psi,\ \Pi^0_{\psi^\dagger} = \partial^0\psi$$

$$\mathcal{H} = \partial_0\psi^\dagger\partial_0\psi + \nabla\psi^\dagger.\nabla\psi + m^2\psi^\dagger\psi$$

Step III:

$$[\psi, \Pi^0_\psi] = [\psi^\dagger, \Pi^0_{\psi^\dagger}] = i\delta^3(x-y)$$

Step IV:

$$\hat{\psi}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2E_p}}(\hat{a}_p e^{-ip.x} + \hat{b}_p^\dagger e^{ip.x})$$

$$\hat{\psi}^\dagger(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2E_p}}(\hat{a}_p^\dagger e^{ip.x} + \hat{b}_p e^{-ip.x})$$

Where a is for matter, b for anti-matter.

There is a symmetry in a phase transition.

$$\psi \to \psi e^{i\alpha},\ \psi^\dagger \to \psi^\dagger e^{-i\alpha} \text{ does not change the Klein-Gordon Equation.}$$

The conserved charge is, $Q = N_{\text{anti-matter}} - N_{matter}$ which is the difference between particles and anti-particles.