## Electromagnetic Waves

Lecture34: Electromagnetic Theory

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Electromagnetic wave at the interface between two dielectric media

We have so far discussed the propagation of electromagnetic wave in an isotropic, homogeneous, dielectric medium, such as in air or vacuum. In this lecture, we woulddiscuss what happens when a plane electromagnetic wave is incident at the interface between two dielectric media. For being specific, we will take one of the medium to be air or vacuum and the other to be a dielectric such as glass. We have come across this in school in connection with the reflection and transmission of light waves at such an interface. In this lecture, we would investigate this problem from the point of view of electromagnetic theory.
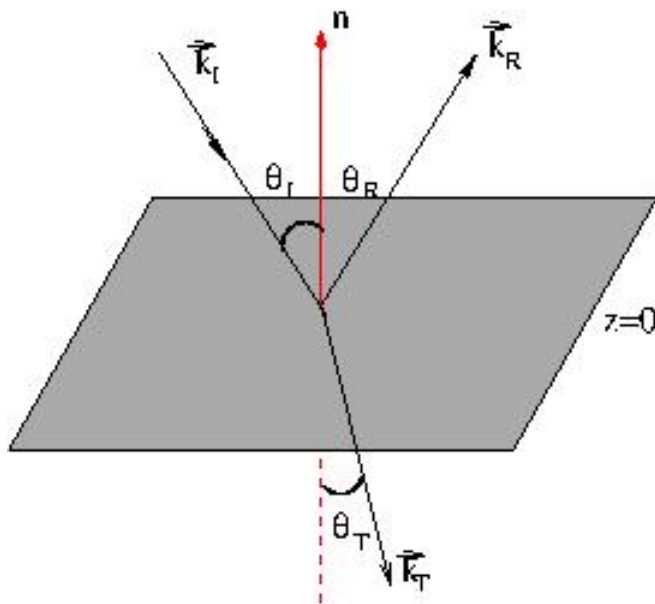
Let us choose the interface to be the xy plane (z=0). The angles of incidence, reflection and refraction are the angles made by the respective propagation vectors with the common normal at the interface.

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We have indicated the propation vectors in the apprpriate medium by capital letters I, R and T so as not to confuse with the notation for the position vector $\vec{r}$ and time t.

The principle that we use to establish the laws of reflection and refraction is the continuity of the tangential components of the electric field at the interface, as discussed extensively during the course of these lectures. Let us represent the component of the electric field parallel to the interface by the superscript ∥. We then have,

$$E_{0I}^{\parallel} \exp(i\, \overrightarrow{k_I} \cdot \vec{r} - i\omega t) + E_{0R}^{\parallel} \exp(i\overrightarrow{k_R} \cdot \vec{r} - i\omega t) = E_{0T} \exp(i\, \overrightarrow{k_T} \cdot \vec{r} - i\omega t)$$

This equation must remain valid at all points in the interface and at all times. That is obviously possible if the exponential factors is the same for all the three terms or if they differe at best by a constant phase factor. Considering, the incident and the reflection terms, we have,

$$\overrightarrow{k_I} \cdot \vec{r} = \overrightarrow{k_R} \cdot \vec{r} + \phi_1$$
$$\left(\overrightarrow{k_I} - \overrightarrow{k_R}\right) \cdot \vec{r} = \phi_1$$

We are familiar with the vector equation to a plane and we know that if the position vector of a plane is $\vec{r}$ and the normal to the plane is represented by $\hat{n}$, the equation to the plane is given by $\hat{n} \cdot \vec{r} =$ constant. Thus $\overrightarrow{k_I} - \overrightarrow{k_R}$ is normal to the interface plane. Since the interface is x-y plane, we take the plane containing the $\overrightarrow{k_I}$ and $\overrightarrow{k_R}$ in the x-z plane.

Since $\overrightarrow{k_I} - \overrightarrow{k_R}$ and the normal to the plane $\hat{n}$ is normal to the interface $\hat{n}$ are in the same plane, we have,

$$\left(\overrightarrow{k_I} - \overrightarrow{k_R}\right) \times \hat{n} = 0$$

Thus we have, $\left|\overrightarrow{k_I}\right| \sin\theta_I = \left|\overrightarrow{k_R}\right| \sin\theta_R$. Since $\overrightarrow{k_I}$ and $\overrightarrow{k_R}$ are in the same medium, though their directions are different, their magnitudes are the same $\left|\overrightarrow{k_I}\right| = \left|\overrightarrow{k_R}\right| = \frac{\omega}{c}$ and hence, we have,

$$\sin\theta_I = \sin\theta_R$$
$$\theta_I = \theta_R$$

which is the law of reflection.

We will now prove the **Snell's law**.

Let us look into the equation

$$\left(\overrightarrow{k_I} - \overrightarrow{k_T}\right) \cdot \vec{r} = \phi_2$$

In a fashion similar to the above, we can show that
$$\left(\overrightarrow{k_I} - \overrightarrow{k_T}\right) \times \hat{n} = 0$$

which gives $\left|\vec{k_I}\right| \sin \theta_I = \left|\vec{k_T}\right| \sin \theta_T$. Since $\vec{k_I}$ and $\vec{k_T}$ are in different media, we have, recognizing that as the wave goes from one medium to another, its frequency does not change,

$$\left|\vec{k_I}\right| = \frac{\omega}{c}, \qquad \left|\vec{k_T}\right| = \frac{\omega}{v_T}$$

where v_T is the velocity of the wave in the transmitted medium.
We, therefore, have,

$$\frac{\sin \theta_I}{\sin \theta_T} = \frac{\left|\vec{k_T}\right|}{\left|\vec{k_I}\right|} = \frac{c}{v} = \frac{n_T}{n_I} \equiv n$$

Here n is the refractive index of the second medium with respect to the incident medium.

Fresnel's Equations

What happens to the amplitudes of the wave on reflection and transmission?

Let us summarize the boundary conditions that we have derived in these lectures. We will assume that both the media are non-magnetic so that the permeability of both media are the same, viz., μ_0. The two media differ by their dielectric constant, the incident medium is taken to be air as above. We further assume that there are no free charges or currents on the surface so that both the normal and tangential components of the fields are continuous.
We will consider two cases, the first case where the electric fields are parallel to the incident plane. This case is known as p- polarization, p standing for "parallel". The second case is where the electric field is perpendicular to the incident plane. This is referred to as s- polarization, s standing for a German word "*senkrec*ht " meaning perpendicular.

p- polarization

In this case, since the magnetic fields are perpendicular to the plane of incidence, we take the directions of the H fields to come out of the plane of the page (the incident plane). Since the electric field, the magnetic field and the direction of propagation are mutually perpendicular being a right handed triad, we have indicated the directions of the electric fields accordingly. It may be noted that in this picture, for the case of normal incidence, the incident and the reflected electric fields are directed oppositely. (The assumption is not restrictive because, if it is not true, a negative sign should appear in the equations).
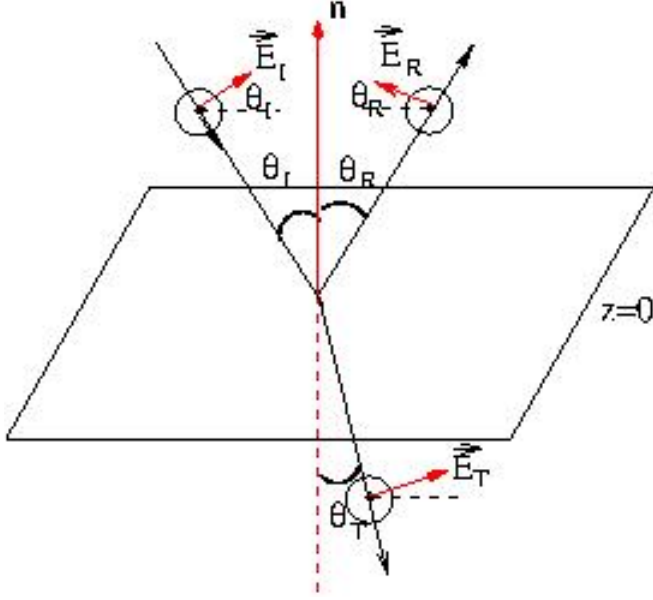
Taking the tangential components of the electric field (parallel to the interface), we have,

$$E_I \cos \theta_I - E_R \cos \theta_R = E_T \cos \theta_T \quad (1)$$
$$H_I = H_R = H_T$$

As our medium is linear, we have the following relationship between the electric and the magnetic field magnitudes,

$$H = \frac{B}{\mu} = \frac{E}{v\mu} = \frac{E\sqrt{\mu_0 \epsilon}}{\mu} = \sqrt{\frac{\epsilon}{\mu}} E$$

Thus the continuity of the tangential component of the magnetic field H can be expressed in terms of electric field amplitudes

$$\sqrt{\frac{\epsilon_I}{\mu_I}} (E_I + E_R) = \sqrt{\frac{\epsilon_T}{\mu_T}} E_T \quad (2)$$

Equations (1) and (2) can be simplified (we use $\theta_R = \theta_I$)

$$\frac{E_I + E_R}{E_I - E_R} = \sqrt{\frac{\mu_I \epsilon_T}{\mu_T \epsilon_I}} \frac{\cos \theta_I}{\cos \theta_T}$$

$$\sqrt{\frac{\mu_I \epsilon_T}{\mu_T \epsilon_I}} = \frac{\sqrt{\mu_T \epsilon_T}}{\sqrt{\mu_I \epsilon_I}} \frac{\mu_I}{\mu_T} = \frac{v_I}{v_T} \frac{\mu_I}{\mu_T} = \frac{n_T}{n_I} \frac{\mu_I}{\mu_T}$$

where, n_T and n_I are refractive indices of the transmitted medium and incident medium, respectively, with respect to free space.

Thus, we have,

$$\frac{E_I + E_R}{E_I - E_R} = \frac{n_T}{n_I}\frac{\mu_I}{\mu_T}\frac{\cos\theta_I}{\cos\theta_T}$$

which gives,

$$\frac{E_R}{E_I} = \frac{(n_T/\mu_T)\cos\theta_I - (n_I/\mu_I)\cos\theta_T}{(n_T/\mu_T)\cos\theta_I + (n_I/\mu_I)\cos\theta_T} \qquad (3)$$

Substituting (3) in (2),

$$\frac{E_T}{E_I} = \frac{2\,(n_I/\mu_I)\cos\theta_I}{(n_T/\mu_T)\cos\theta_I + (n_I/\mu_I)\cos\theta_T} \qquad (4)$$

Let us take $\mu_I = \mu_T$. The Fresnel's equations are now expressible in terms of refractive indices of the two media and the angles of incidence and transmission

$$\frac{E_R}{E_I} = \frac{n_T\cos\theta_I - n_I\cos\theta_T}{n_T\cos\theta_I + n_I\cos\theta_T} \equiv r_p$$

$$\frac{E_T}{E_I} = \frac{2\,n_I\cos\theta_I}{n_T\cos\theta_I + n_I\cos\theta_T} \equiv t_p$$

where, r_p and t_p are, respectively, the reflection and transmission coefficients for field amplitudes. (Caution : the phrases "reflection/transmission" coefficients are also used to denote fraction of transmitted intensities.)

Brewster's Angle

Using $\frac{n_T}{n_I} = \frac{\sin\theta_I}{\sin\theta_T}$, we can express , r_p as follows.

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$$\begin{aligned}
r_p &= \frac{\sin\theta_I\cos\theta_I - \sin\theta_T\cos\theta_T}{\sin\theta_I\cos\theta_I + \sin\theta_T\cos\theta_T}\\
&= \frac{\sin 2\theta_I - \sin 2\theta_T}{\sin 2\theta_I - \sin 2\theta_T}\\
&= \frac{2\sin(\theta_I - \theta_T)\cos(\theta_I + \theta_T)}{2\sin(\theta_I + \theta_T)\cos(\theta_I - \theta_T)}\\
&= \frac{\tan(\theta_I - \theta_T)}{\tan(\theta_I + \theta_T)}
\end{aligned}$$

If $\theta_I + \theta_T = \frac{\pi}{2}$, i.e. if the angle between the reflected ray and the transmitted ray is 90^0, the reflection coefficient for the parallel polarization becomes zero, because $\tan(\theta_I + \theta_T) \to \infty$. If we had started with an equal mixture of p polarized and s polarized waves (i.e. an unpolarized wave), the reflected ray will have no p component, so that it will be plane polarized. The angle of incidence at which this happens is called the **Brewster's angle**".

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For this angle, we have

$$\tan \theta_I = \cot \theta_T = \frac{\cos \theta_T}{\sin \theta_T} = \frac{\sin \theta_I}{\sin \theta_T} = n$$

The following figure (right) , the red curve shows the variation in the reflected amplitude with the angle of incidence for p-polarization. The blue curve is the corresponding variation for s – polarization to be discussed below.
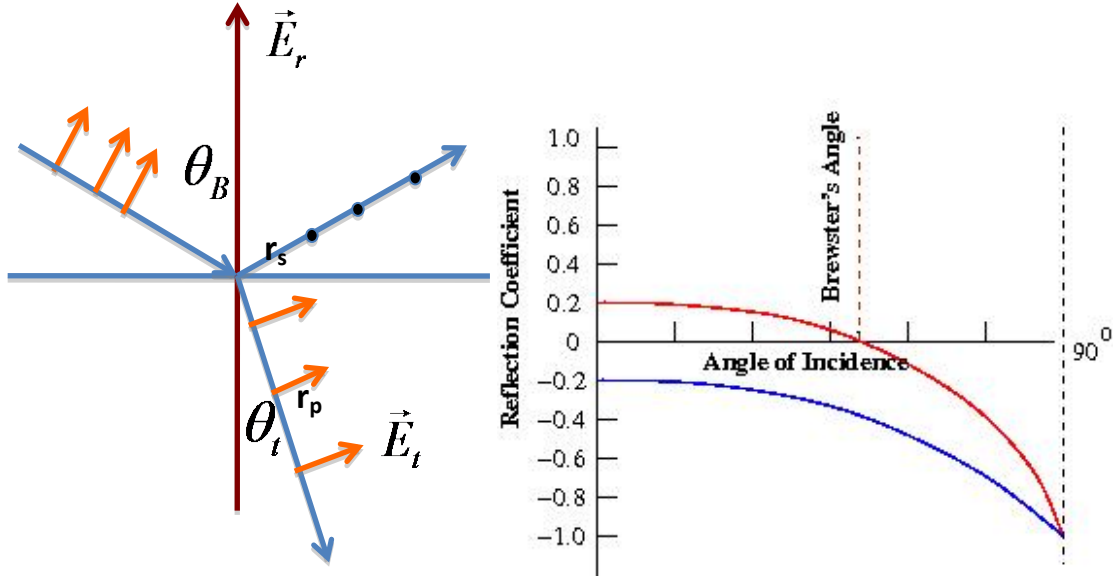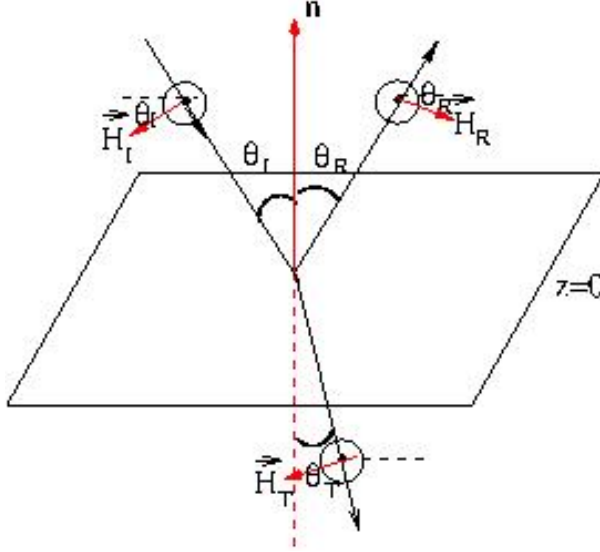

For normal incidence, $\theta_I = \theta_T = 0$, we have,

$$r_p = \frac{n_T - n_I}{n_T + n_I} = \frac{n - 1}{n + 1}$$

s –polarization

We next consider s polarization where the electric field is perpendicular to the incident plane. As we have taken the plane of incidence to be the plane of the paper, the electric field will be taken to come out of the plane of the paper.

The corresponding directions of the magnetic field is shown in the figure. The boundary conditions give

$$E_I + E_R = E_T$$
$$(H_I - H_R)\cos\theta_I = H_T\cos\theta_T$$

Substituting $H = \sqrt{\frac{\epsilon}{\mu}}E$, we get for the reflection and transmission coefficient for the amplitudes of the electric field

$$\frac{E_R}{E_I} = \frac{n_I\cos\theta_I - n_T\cos\theta_T}{n_I\cos\theta_I + n_T\cos\theta_T} \equiv r_s$$
$$\frac{E_T}{E_I} = \frac{2\,n_T\cos\theta_I}{n_I\cos\theta_I + n_T\cos\theta_T} \equiv t_s$$

Using Snell's law, we can simplify these expressions to get,

$$r_s = -\frac{\sin(\theta_I - \theta_T)}{\sin(\theta_I + \theta_T)}$$
$$t_s = \frac{2\sin\theta_T\cos\theta_I}{\sin(\theta_I + \theta_T)}$$

For normal incidence, $\theta_I = \theta_T = 0$, we have,

$$r_s = \frac{n_I - n_T}{n_T + n_I} = \frac{1-n}{1+n}$$

Notice that the expression differs from the expression for r_p for normal incidence, while both the results should have been the same. This is because of different conventions we took in fixing

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the directions in the two cases; in the p –polarization case, for normal incidence, the electric field directions are opposite for the incident and reflected rays while for the s-polarization, they have been taken to be along the same direction.

Total Internal Reflection and Evanescent Wave

Let us return back to the case of p polarization and consider the case where in the incident medium has a higher refractive index than the transmitted medium. In this case, we can write the amplitude reflection coefficient as

$$r_p = \frac{n_T \cos \theta_I - n_I \cos \theta_T}{n_T \cos \theta_I + n_I \cos \theta_T}$$

$$= \frac{n\cos \theta_I - \sqrt{1 - \sin^2 \theta_T}}{n\cos \theta_I + \sqrt{1 - \sin^2 \theta_T}}$$

where we have used the refractive index of the second medium as $n = \frac{n_T}{n} < 1$. Substituting Snell's law into the above, we have, $\sin \theta_T = \sin \theta_I / n$, we can write the above as

$$r_p = \frac{n^2 \cos \theta_I - \sqrt{n^2 - \sin^2 \theta_I}}{n^2 \cos \theta_I + \sqrt{n^2 - \sin^2 \theta_I}}$$

The quantity under the square root could become negative for some values of θ_I since n <1 . we therefore write,

$$r_p = \frac{n^2 \cos \theta_I - i\sqrt{\sin^2 \theta_I - n^2}}{n^2 \cos \theta_I + i\sqrt{\sin^2 \theta_I - n^2}}$$

In a very similar way, we can show that the reflection coefficient for s polarization can be expressed as follows:

$$r_s = \frac{n_I \cos \theta_I - n_T \cos \theta_T}{n_I \cos \theta_I + n_T \cos \theta_T}$$

$$= \frac{\cos \theta_I - i\sqrt{\sin^2 \theta_I - n^2}}{\cos \theta_I + i\sqrt{\sin^2 \theta_I - n^2}}$$

It may be seen that for $\sin \theta_I > n$, the magnitudes of both r_p and r_s are both equal to unity because for both these, the numerator and the denominator are complex conjugate of each other. Thus it implies that when electromagnetic wave is incident at an angle of incidence greater than a "critical angle" defined by

$$\sin \theta_c = n$$

where n here represents the refractive index of the (rarer) transmitted medium with respect to the(denser) incident medium , the wave is totally reflected. (In text books on optics, the critical angle is defined by the relation $\sin \theta_c = 1/n$, because the refractive index there is conventionally defined as that of the denser medium with respect to the rarer one).

In case of total internal reflection, is there a wave in the transmitted medium? The answer is yes, it does as the following analysis shows.

Let us take the incident plane to be xz plane and the interface to be the xy plane so that the normal to the plane is along the z direction. The transmitted wave can be written as

$$\vec{E}_T = \vec{E}_{T0} \exp(i\vec{k}_T \cdot \vec{r} - i\omega t)$$

The space part of the wave can be expressed as

$$i\vec{k}_T \cdot \vec{r} = i(k_T x \sin \theta_T + k_T z \cos \theta_T)$$

Writing this in terms of angle of incidence θ_I

$$i\vec{k}_T \cdot \vec{r} = i\left(\frac{k_T}{n} x \sin \theta_I + k_T z \sqrt{1 - \frac{\sin^2 \theta_I}{n^2}}\right)$$

For angles greater than the critical angle the quantity within the square root is negative and we rewrite it as

$$i\vec{k}_T \cdot \vec{r} = i\left(\frac{k_T}{n} x \sin \theta_I + i k_T z \sqrt{\frac{\sin^2 \theta_I}{n^2} - 1}\right)$$
$$= i\beta x - \alpha z$$

where, we define "propagation vector " β as

$$\beta = \frac{k_T}{n} \sin \theta_I$$

and the "attenuation factor" α as

$$\alpha = k_T \sqrt{\frac{\sin^2 \theta_I}{n^2} - 1}$$
$$= \frac{\omega n_T}{c} \sqrt{\frac{\sin^2 \theta_I}{n^2} - 1} = \frac{\omega}{c} \sqrt{n_I^2 \sin^2 \theta_I - n_T^2}$$

With this, the wave in the transmitted medium becomes,

$$\vec{E}_T = \vec{E}_{T0} e^{-\alpha z} \exp(i\beta x - i\omega t)$$

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which shows that the wave in the second medium propagates along the interface. It penetrates into the medium but its amplitude attenuates exponentially. This is known as "**evanescent wave**".

The amplitude variation with angle of incidence is shown in the following figure.
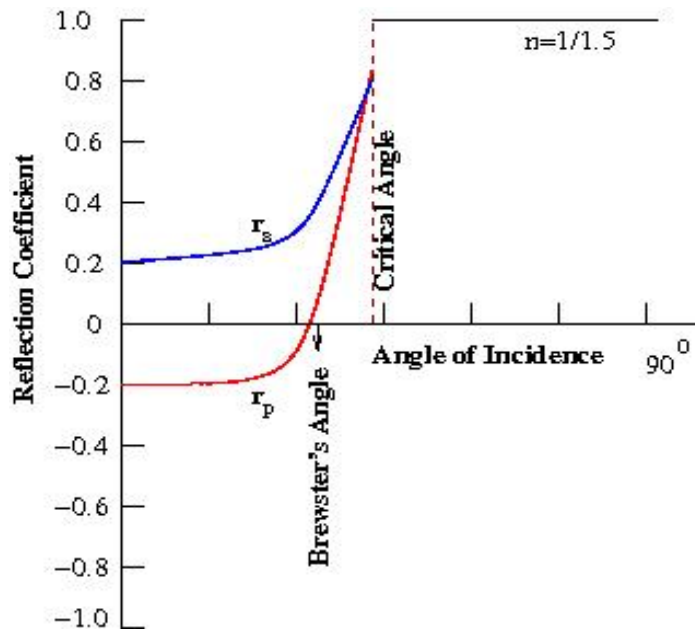
What is the propagation vector?

Recall that the magnitude of the propagation vector is defined as $\frac{2\pi}{\lambda}$, where the "wavelength" λ is the distance between two successive crests or troughs of the wave measured along the direction of propagation. However, consider, for instance, a water wave which moves towards the shore. Along the shore, one would be more inclined to conclude that the wavelength is as measured by the distance between successive crests or troughs along the shore. This is the wavelength with which the attenuating surface wave propagates in the second medium.

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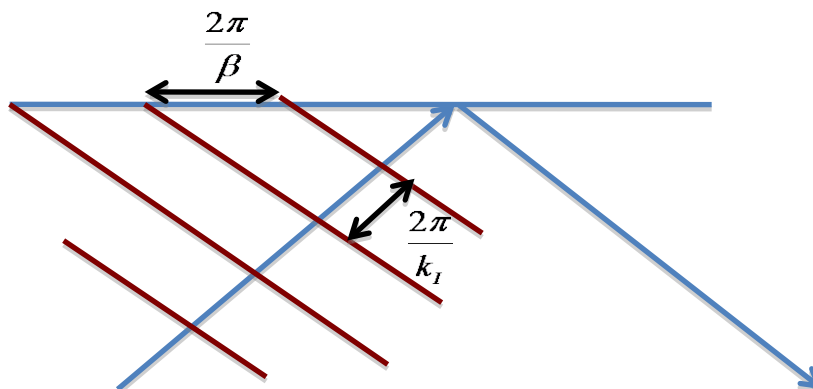
No transfer of energy into the transmitted medium:

Though there is a wave in the transmitted medium, one can show that on an average, there is no transfer of energy into the medium from the incident medium.

Consider, p polarization, for which the transmitted electric field, being parallel to the incident (xz) plane is along the x direction.

$$\vec{E}_T = E_{T0}e^{-\alpha z}\exp(i\beta x - i\omega t)\hat{\imath}$$

From this we obtain the H- field using Faraday's law, Since the H field is taken perpendicular to the electric field , it would be in the y-z plane . Taking the components of $\nabla \times \vec{E}$, we have,

$$-\frac{\partial B_y}{\partial t} = i\mu_0\omega H_y = \left(\nabla \times \vec{E}\right)_y = \frac{\partial E_x}{\partial z} = -\alpha E_{T0}e^{-\alpha z}\exp(i\beta x - i\omega t)$$

and

$$-\frac{\partial B_z}{\partial t} = i\mu_0\omega H_z = \left(\nabla \times \vec{E}\right)_z = \frac{\partial E_y}{\partial x} = +i\beta E_{T0}e^{-\alpha z}\exp(i\beta x - i\omega t)$$

so that,

$$\vec{H} = \frac{E_{T0}}{\mu_0\omega}e^{-\alpha z}\left(i\alpha\hat{\jmath} + \beta\hat{k}\right)\exp(i\beta x - i\omega t)$$

Thus the average energy transfer Poynting vector. The complex Poynting vector can be written as

$$\vec{S} = \frac{1}{2}\vec{E} \times \vec{H}^*$$

so that the average power transferred into the second medium is

$$\begin{aligned}
\langle\vec{S}\rangle &= \frac{1}{2}\mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) \\
&= \frac{E_{T0}^2}{2\mu_0\omega}e^{-2\alpha z}\mathrm{Re}\left(-i\alpha\hat{k} - \beta\hat{\jmath}\right) \\
&= -\frac{E_{T0}^2}{2\mu_0\omega}e^{-2\alpha z}\beta\hat{\jmath}
\end{aligned}$$

Since the normal to the surface is along the z direction, the average energy transferred to the second medium is zero.

Tutorial Assignment

1. A plane electromagnetic wave described by its magnetic field is given by the expression
$$\vec{H} = H_0 \sin(kz - \omega t)\hat{y}$$
Determine the corresponding electric field and the time average Poynting vector.
If it is incident normally on a perfect conductor and is totally reflected   what would be the pressure exerted on the surface? Determine the surface current generated at the interface.

2. A circularly polarized electromagnetic wave is given by
$$\vec{E} = E_0 \sin(kz - \omega t)\hat{x} + E_0 \cos(kz - \omega t)\hat{y}$$
Show that the average value of the Poynting vector for the wave is equal to the sum of the Poynting vectors of its components.

3.

Solutions to Tutorial Assignments

1. The wave is propagating along the + z direction (before reflection). The electric field is given by Maxwell- Ampere law,

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Since $\vec{H}$ is along $\hat{y}$ direction, and its y-component depends on z only, the curl is given by

$$-\hat{x} \frac{\partial H_y}{\partial z} = -\hat{x} H_0 k \cos(kz - \omega t) = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

This gives,

$$\vec{E} = \frac{H_0 k}{\omega \epsilon_0} \sin(kz - \omega t)\hat{x} = \frac{H_0}{c\epsilon_0} \sin(kz - \omega t)\hat{x}$$

The Poynting vector is given by

$$\vec{S} = \vec{E} \times \vec{H} = \frac{H_0^2}{c\epsilon_0} \sin^2(kz - \omega t)\hat{z}$$

The time average Poynting vector is $\langle \vec{S} \rangle = \frac{H_0^2}{2c\epsilon_0}\hat{z}$.

Since the wave is totally reflected, the change in momentum is twice the initial momentum carried. Thus the pressure is given by

$$P = \frac{2|\langle S \rangle|}{c} = \frac{H_0^2}{c^2 \epsilon_0}$$

At the metallic interface (z=0), the tangential component of the electric field is zero. Since the wave is totally reflected, the reflected wave must have oppositely directed electric field, i.e. in $-\hat{x}$ direction. The direction of propagation having been reversed, the magnetic the field is given by

$$\vec{H}_r = -H_0 \sin(kz + \omega t)\hat{y}$$

At the interface the total magnetic field is $-2H_0 \sin(\omega t)\hat{y}$.

The surface current can be determined by taking an Amperian loop perpendicular to the interface, Taking the direction of the loop parallel to the magnetic field, the line integral is seen to be $2H_0 \sin(\omega t)l = Kl$, where K is the surface current density. The direction of the surface current is along the $\hat{x}$ direction. Thus

$$\vec{K} = 2 H_0 \sin(\omega t)\hat{i}$$

2. The electric field is given by

$$\vec{E} = E_0 \sin(kz - \omega t)\hat{x} + E_0 \cos(kz - \omega t)\hat{y}$$

As the wave is propagating in the z direction, the corresponding magnetic field is given by

$$\vec{H} = -\frac{1}{\mu_0 c} E_0 \cos(kz - \omega t)\hat{x} + \frac{1}{\mu_0 c} E_0 \sin(kz - \omega t)\hat{y}$$

Poynting vector is

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0 c} E_0^2 \hat{z}$$

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The individual components have average Poynting vectors given by

$$\langle S_1 \rangle = \frac{1}{\mu_0 c} E_0^2 \langle \sin^2(kz - \omega t) \rangle = \frac{1}{2\mu_0 c} E_0^2$$

$$\langle S_2 \rangle = \frac{1}{\mu_0 c} E_0^2 \langle \cos^2(kz - \omega t) \rangle = \frac{1}{2\mu_0 c} E_0^2$$

Thus $\langle S \rangle = \langle S_1 \rangle + \langle S_2 \rangle$.

Electromagnetic Waves

Lecture34: Electromagnetic Theory

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Self Assessment Questions

1. The electric field of a plane electromagnetic wave propagating in free space is described by
$$\vec{E} = E_0 \sin(kx - \omega t)\hat{y}$$
Determine the corresponding magnetic field and the time average Poynting vector for the wave.

2. Show that an s –polarized wave cannot be totally transmitted to another medium.

3. An electromagnetic wave   given by
$$\vec{E} = E_0 \sin(kz - \omega t)\hat{x}$$
is incident on the surface of a perfect conductor and is totally reflected. The incident and the reflected waves combine and form a pattern. What is the average Poynting vector?

Solutions to Self Assessment Questions

1. The wave is propagating in +x direction so that the propagation vector is $\vec{k} = k\hat{x}$. Using Faraday's law, we have, $\vec{k} \times \vec{E} = \omega\vec{B}$ which gives magnetic field directed along the z direction,

$$\vec{B} = \frac{E_0}{c}\sin(kx - \omega t)\hat{z}$$

   The Poynting vector is given by

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \frac{E_0^2}{c\mu_0}\sin^2(kx - \omega t)\,\hat{x}$$

   The time average of the Poynting vector is $\langle\vec{S}\rangle = \frac{E_0^2}{2c\mu_0}$.

2. Considering Fresnel equations for s polarization,

$$r_s = \frac{n_I\cos\theta_I - n_T\cos\theta_T}{n_I\cos\theta_I + n_T\cos\theta_T}$$

For total transmission $r_s = 0$, so that $n_I\cos\theta_I = n_T\cos\theta_T$. From Snell's law, we have, $n_I\sin\theta_I = n_T\sin\theta_T$. Squaring and adding, we get $n_I^2 = n_T^2$, which is not correct.

3. The incident wave is

$$\vec{E_i} = E_0\sin(kz - \omega t)\hat{x}$$

   Since the tangential component of the electric field must vanish at the interface (z=0), the reflected wave us given by

$$\vec{E_r} = E_0\sin(kz + \omega t)\hat{x}$$

   The corresponding magnetic fields are given by

$$\vec{H_i} = \frac{E_0}{c\mu_0}\sin(kz - \omega t)\hat{y}$$

$$\vec{H_r} = -\frac{E_0}{c\mu_0}\sin(kz + \omega t)\hat{y}$$

   The individual Poynting vectors can be calculated and on adding it will turn out that the average Poynting vector is zero. The waves of the type obtained by superposition of the two waves have the structure,

$$\vec{E} = \vec{E_i} + \vec{E_r} = 2E_0\sin kz\cos\omega t$$

   where the space and time parts are separated. These are known as "standing waves" and they do not transport energy.