QFT

Adwait

Quantum Field Theory.

Noether's theorem.

For a field $\phi(x^\nu)$,

$$\phi(x^\nu) \to \phi(x^\nu + \lambda a^\nu) = \phi(x^\nu) + \lambda a^\nu \partial_\nu \phi(x^\nu) = \phi(x^\nu) + \lambda D\phi$$

For such a continuous transformation, will lead to a symmetry in the Langrangian.

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi} + \frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}\delta(\partial_\nu\phi)$$

With $\Pi^\nu = \frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}$,

$$\delta\mathcal{L} = (\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\nu\diamond^\nu)\delta\phi + \partial_\nu(\diamond^\nu\delta\phi) = \partial_\nu(\diamond^\nu\delta\phi)$$

With the transformation in the lagrangian being,

$$\partial\mathcal{L} = \partial_\nu(\diamond^\nu\mathcal{D}\phi)\delta\lambda = \mathcal{D}\phi\delta\lambda$$

Noether's theorem: $\partial_\nu(\Pi^\nu D\phi - W^\nu) = \partial_\nu J_N^\nu = 0 \implies Q = \int dx J_N^0$ is conserved.

where $\partial_\nu W^\nu = D\mathcal{L}$

Energy conservation

Energy conservation comes about when the Lagrangian is symmetric under time translation. Let's take a general translation in space-time.

$$x^\nu \to x^\nu + \delta x^\nu D\mathcal{L} = \partial(\dashv^\nu\mathcal{L}) \implies \mathcal{W}^\nu = \dashv^\nu\mathcal{L}$$

$$J_N^\nu = \Pi^\nu D\phi - W^\nu = a^\nu(\Pi^\nu \partial_\nu \phi - \delta_\nu^\mu \mathcal{L}) = \dashv^\nu \mathcal{T}^{\mu\nu}$$

The conserved charge being,

$$P^\alpha = \int d^3 x T^{0\alpha}$$