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Angular Momentum

$\vec{L} = r \times p$

Cartesian :-

$L_x = \cancel{x}\, y p_z - z p_y, \quad L_y = z p_x - x p_z,$
$L_z = x p_y - y p_x$

$[x_i, p_j] = i\hbar \delta_{ij}$

$L_i^\dagger = L_i$

Commutation relations of Angular momentum :-

$[\hat{L}_i, \hat{x}_j] = i\hbar \varepsilon_{ijk} \hat{x}_k, \quad [\hat{L}_i, \hat{p}_j] = i\hbar \varepsilon_{ijk} \hat{p}_k$

Some identities :-

$a.b = b.a + [a_i, b_i]$

$\Rightarrow r.p = p.r + 3i\hbar$

$(a \times b)_i = -(b \times a)_i + \varepsilon_{ijk}[a_j, b_k].$

$\Rightarrow r \times p = -p \times r$

if ~~vector~~ operator 'u' follows $[\hat{L}_i, \hat{u}_j] = i\hbar \varepsilon_{ijk} \hat{u}_k$
then it is vector under rotations.

$[L_i, L_j] = i\hbar \varepsilon_{ijk} L_k$

L: orbital, S: spin, J: total.

$$[\hat{J}_i, \hat{J}_j] = i\hbar \varepsilon_{ijk} \hat{J}_k$$

and
$$[\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z$$
$$[\hat{J}_y, \hat{J}_z] = i\hbar \hat{J}_x$$
$$[\hat{J}_z, \hat{J}_x] = i\hbar J_y$$

and $[\hat{J}_i, \hat{J}^2] = 0$

Define ladder operators :—

$$J_+ = J_x + iJ_y, \quad J_- = J_x - iJ_y$$

$$J_+ J_- = J_x^2 + J_y^2 - i[J_x, J_y] = \hat{J}_x^2 + \hat{J}_y^2 + \hbar \hat{J}_z$$
$$J_- J_+ = \hat{J}_x^2 + \hat{J}_y^2 - \hbar \hat{J}_z$$
$$[J_+, J_-] = 2\hbar J_z$$

$$\Rightarrow \hat{J}^2 = J_+ J_- + J_z^2 - \hbar J_z$$

$$\Rightarrow [J_\pm, \hat{J}^2] = 0$$
and $[\hat{J}_z, \hat{J}_\pm] = \pm\hbar \hat{J}_\pm$

as J^2 and J_z commute, they have simultaneous eigenstates.

$$J^2 |\alpha, \beta\rangle = \alpha |\alpha, \beta\rangle$$
$$J_z |\alpha, \beta\rangle = \beta |\alpha, \beta\rangle$$

$$J_z (J_+ |\alpha, \beta\rangle) = J_+ J_z + \hbar J_+ |\alpha, \beta\rangle$$
$$= (J_+ \beta + \hbar J_+) |\alpha, \beta\rangle$$
$$= (\beta + \hbar) J_+ |\alpha, \beta\rangle$$

and $J^2 J_+ |\alpha, \beta\rangle = \alpha J_+ |\alpha, \beta\rangle$.

in simpler ways,

$$J_z J_+ |\lambda, m\rangle = (J_+ J_z + \hbar J_+)|\lambda, m\rangle$$
$$= (m+1)\hbar . J_+ |\lambda, m\rangle$$
$$J_z J_- |\lambda, m\rangle = (m-1)\hbar J_- |\lambda, m\rangle.$$

$$J^2 J_\pm |\lambda, m\rangle = \lambda \hbar^2 J_\pm |\lambda, m\rangle.$$

Now square of J_z^2 will not exceed J^2.

$\Rightarrow \quad m^2 \leq \lambda$

$$\langle\lambda, m| J_x^2 + J_y^2 |\lambda, m\rangle \geq 0.$$

let $\max\{m\} = j.$

$$J_+ |\lambda, j\rangle = 0.$$
$$J_- J_+ = J^2 - J_z^2 - \hbar J_z$$
$$J_- J_+ |\lambda, j\rangle = (\lambda - j^2 - j)\hbar^2 |\lambda, j\rangle.$$

and for $\min\{m\} = j'$.

$$J_+ J_- |\lambda, j'\rangle = \lambda - j'^2 - j'$$

$$\Rightarrow \lambda = j'^2 - j'$$

$$\Rightarrow J^2(|j, m\rangle) = \hbar^2 j(j+1)|j, m\rangle.$$
$$J_z(|j, m\rangle) = \hbar m |j, m\rangle$$

Ladder operators :-

$J_+ |j,m\rangle = c_+ \hbar |j,m+1\rangle.$

$J_- |j,m\rangle = c_- \hbar |j,m-1\rangle.$

$\langle j,m | J_- J_+ | j,m\rangle = c_+^* c_+ \hbar^2 \langle j,m+1 | j,m+1\rangle.$

$\langle j,m | J^2 - J_z^2 - \hbar J_z | j,m\rangle = [j(j+1) - m(m+1)] \hbar^2 \langle j,m | j,m\rangle$

$= c_+^* c_+ \hbar^2 \langle j,m+1 | j,m+1\rangle$

$\Rightarrow J_+ |j,m\rangle = \sqrt{j(j+1) - m(m+1)}\, \hbar |j,m+1\rangle$

$J_- |j,m\rangle = \sqrt{j(j+1) - m(m-1)}\, \hbar |j,m-1\rangle$

$j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots\ldots$

$m = j, j-1, j-2, \ldots, -j$

for orbital $J \equiv L$ case

$l = 0, 1, 2, \ldots$

$m = j, j-1, \ldots, -j.$

Spherical Harmonics :-

$$L^2 = -\hbar^2 \left(\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta} \cdot \frac{\partial^2}{\partial\phi^2} \right)$$

$$L_z \to -i\hbar \left(\frac{\partial}{\partial\phi} \right)$$

the state $|l,m\rangle$ in $|\theta,\phi\rangle$

is $\langle \theta,\phi | l,m\rangle = Y_{lm}(\theta,\phi)$

$L^2 |l,m\rangle = \hbar^2 l(l+1) |l,m\rangle.$

$L_z |l,m\rangle = \hbar m |l,m\rangle.$

$$\Rightarrow \hat{L}^2 \langle \theta,\phi | l,m\rangle = \hbar^2 \, l(l+1) \langle \theta,\phi | l,m\rangle.$$
$$\hat{L}_z \langle \theta,\phi | l,m\rangle = \hbar m \langle \theta\phi | l,m\rangle.$$

$$Y_{lm}(\theta,\phi) = P_{lm}(\theta)\, e^{im\phi}$$

So for radial equation :-

$$\hat{H} = \frac{\hat{P}^2}{2m} + V(r) = -\frac{\hbar}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{2mr^2} L^2\psi + V(r).$$

$$\psi_{Elm}(r,\theta,\phi) = f_{Elm}(r)\, Y_{lm}(\theta,\phi)$$

$$-\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}\left(rf\right) + \frac{\hbar^2 \, l(l+1)}{2mr^2} f_{Elm} + V(r) f_{Elm} = E f_{Elm}$$

$$u = r f_{El}.$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dr^2}u + \underbrace{\left(V(r) + \frac{\hbar^2 \, l(l+1)}{2mr^2}\right)}_{V_{eff}} u = Eu.$$

$$L_{\pm} = \hbar e^{\pm i\phi}\left(i\cot\theta \cdot \frac{\partial}{\partial\phi} \pm \frac{\partial}{\partial\theta}\right)$$

$$L_x = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$L_y = \frac{\hbar}{i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$L_\pm = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \pm iz\frac{\partial}{\partial x} \mp ix\frac{\partial}{\partial z}\right)$$

$Y_{11}(x, y, z)$

$$L_+ Y_{11} = 0$$

$$\frac{\hbar}{i}\left(y\frac{\partial}{\partial z}Y_{11} - z\frac{\partial}{\partial y}Y_{11} + iz\frac{\partial}{\partial x}Y_{11} - ix\frac{\partial}{\partial z}Y_{11}\right) = 0$$

$$\frac{\hbar}{i}\left((y - ix)\frac{\partial}{\partial z}Y_{11} - z\frac{\partial}{\partial y}Y_{11} + iz\frac{\partial}{\partial x}Y_{11}\right) = 0$$

⋮

$$Y_1^1 = -\left(\frac{3}{4\pi}\right)^{1/2} \cdot \frac{x + iy}{\sqrt{2}\,r}, \quad Y_1^{-1} = \sqrt{\frac{3}{4\pi}} \cdot \frac{x - iy}{\sqrt{2}\,r}.$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cdot \frac{z}{r}.$$

$$\psi = N(x + y + 2z)e^{-ar}$$

$$\cancel{Y_1^1 + Y_1^{-1} + 2Y_1^0 = \sqrt{\frac{3}{4\pi}}\left(\frac{x + iy + x - iy}{\sqrt{2}\,r} + \frac{2z}{r}\right)}$$

$$\cancel{= \sqrt{\frac{3}{4\pi}}\left(\frac{\sqrt{2}\,x + 2z}{r}\right)}$$

$$z = Y_1^0 \cdot r\sqrt{\frac{4\pi}{3}}, \quad x = \frac{(Y_1^1 + Y_1^{-1})}{\sqrt{2}}\, r\sqrt{\frac{4\pi}{3}}$$

$$y = \frac{(Y_1^1 - Y_1^{-1})}{\sqrt{2}}\, r\sqrt{\frac{4\pi}{3}}$$

$\Rightarrow \quad P(l=0) = \frac{2}{3}, \quad P(l=\pm 1) = \frac{1}{6}$

12.5.14

$$\Psi = A z e^{-r^2/a^2}$$

$$= A Y_1^0 r \sqrt{\frac{4\pi}{3}} e^{-r^2/a^2}$$

Rotation around x axis

$$x \to x, \quad y \to y\cos\theta - z\sin\theta_x$$

$$L_x^{l=1} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R(\theta_x) = e^{-\frac{i}{\hbar}\theta_x L_x} = \frac{1}{2}(\cos\theta_x - 1)\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$-\frac{i}{\sqrt{2}}(\sin\theta_x)\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$|1,0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |1,1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |1,-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$e^{-\frac{i}{\hbar}\theta_x L_x}|1,0\rangle = -\frac{i}{\sqrt{2}}\sin\theta\,|1,1\rangle + \cos\theta_x\,|1,0\rangle - \frac{i}{\sqrt{2}}\sin\theta\,|1,-1\rangle$$

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Free particle

$V(r) = 0$.

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}u = Eu.$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}, \quad \rho = kr.$$

$$-\frac{\partial^2 u}{\partial \rho^2} + \frac{l(l+1)}{\rho^2}u = u$$

$u = r\, j_l(kr)$. ... Spherical Bessel functions.

$$\psi(r,\theta\,\phi) = j_l(kr)\, Y_{lm}(\theta,\phi)$$

$$r\, j_l(r) \sim \frac{x^{l+1}}{(2l+1)!} \quad x \to 0, \quad r\, j_l(r) \sim \sin\left(r - \frac{l\pi}{2}\right) \; r \to \infty$$

For $V \neq 0$ but dissappears at $r \to \infty$.

$$u \approx \sin\left(kr - \frac{l\pi}{2} + \delta_l(E)\right) \quad r \to \infty$$

$\delta(E)$ is the phase shift which can be used to extract information from the potential.

Spherical infinit well :—

$u = r\, j_l(kr)$.

$j_l(ka) = 0$.

Here Energy is quantised :—

$J_0(ka) = \frac{\sin(ka)}{ka} = 0$, $ka = n\pi$, $\Rightarrow$ ~~E_{00}~~ $E_{n0} = \frac{\hbar^2 n^2 \pi^2}{2\mu a^2}$

Addition of Angular momentum

Feynman - Hellmann theorem :-

$H(\lambda)$ be Hamiltonian, $\psi_n(\lambda)$ eigenstate, $E_n(\lambda)$

then.

$$\frac{dE_n(\lambda)}{d\lambda} = \langle \psi_n(\lambda) | \frac{\partial H(\lambda)}{\partial t} | \psi_n(\lambda) \rangle$$

$$H(\lambda) | \psi(\lambda) \rangle = E(\lambda) | \psi(\lambda) \rangle$$

$$\Rightarrow E(\lambda) = \langle \psi(\lambda) | H(\lambda) | \psi(\lambda) \rangle$$

$$\frac{dE_n(\lambda)}{d\lambda} = \langle \psi_n(\lambda) | \frac{dH}{d\lambda} | \psi_n(\lambda) \rangle + \left(\frac{d}{d\lambda} \langle \psi(\lambda) | \right) H | \psi_n(\lambda) \rangle$$

$$+ \langle \psi(\lambda) | H(\lambda) \frac{d}{d\lambda} (| \psi_n(\lambda) \rangle).$$

$$= \langle \psi | \frac{\partial H}{d\lambda} | \psi \rangle + E_n \left(\frac{d}{d\lambda} \langle \psi | \psi \rangle \right.$$

$$+ \langle \psi | \frac{d}{d\lambda} | \psi \rangle$$

$$= \langle \psi | \frac{dH}{d\lambda} | \psi \rangle + \cancel{E_n \frac{d}{d\lambda} (1)}$$

$$H(\lambda) = H_0 + \lambda H_1 = H_0 + \Delta H.$$

$$\frac{dE_n}{d\lambda} = \langle \psi_n | H_1 | \psi_n \rangle$$

$$\Rightarrow E_n(\lambda) = E_n(0) + \lambda \frac{dE_n}{d\lambda} + O(\lambda^2)$$

$$= E_n(0) + \lambda \langle \psi_n^0 | H_1 | \psi_n^0 \rangle$$

Ang momentum

$J_i^1 \Rightarrow [J_i^1, J_j^1] = i\hbar \epsilon_{ijk} J_k^1$ on V
$J_i^2 \Rightarrow$ - - - - on W.

$J_i \in \mathcal{L}(V \otimes W)$ on $V \otimes W$.
$\Rightarrow J_i = J_i^1 \otimes \mathbb{1} + \mathbb{1} \otimes J_i^2$

$[J_i, J_j] = [J_i^1 \otimes \mathbb{1} + \mathbb{1} \otimes J_i^2, J_j^1 \otimes \mathbb{1} + \mathbb{1} \otimes J_j^2]$

comm do not

$= [J_i^1 \otimes \mathbb{1}, J_j^1 \otimes \mathbb{1}] + [\mathbb{1} \otimes J_i^1, \mathbb{1} \otimes J_j^2]$

$= [J_i^1, J_j^1] \otimes \mathbb{1} + \mathbb{1} \otimes [J_i^2, J_j^2]$

$= i\hbar \epsilon_{ijk} (J_k^1 \otimes \mathbb{1} + \mathbb{1} \otimes J_k^2)$

$[J_i, J_j] = i\hbar \epsilon_{ijk} J_k$

Spin Orbit Coupling :-

$\Delta H = -\mu \cdot B = \frac{e}{m} S \cdot B = \frac{ge}{2m} S \cdot B.$

Continued - - - -

$S_i^2 |s_1, m_1\rangle \otimes |s_2, m_2\rangle = \hbar^2 s_i(s_i+1) |s_1, m_1\rangle |s_2, m_2\rangle$
$S_{iz} |s_1, m_1\rangle |s_2, m_2\rangle = \hbar m_i |s_1, m_1\rangle |s_2, m_2\rangle.$
$i \in \{1, 2\}.$

For operator $S = S_1 + S_2 = S_1 \otimes 1 + 1 \otimes S_2$

$S_z |++\rangle = (S_{1z} + S_{2z}) |++\rangle = S_{1z} |+\rangle . |+\rangle + |+\rangle S_{2z} |+\rangle$
$= \left(\frac{\hbar}{2} + \frac{\hbar}{2}\right) |++\rangle$

$S_z |+-\rangle = \underbrace{\left(\frac{\hbar}{2} - \frac{\hbar}{2}\right)}_{0} |+-\rangle, \quad S_z |-+\rangle = 0$

$S_z |--\rangle = -\hbar |--\rangle$

$$[S_z] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

$$S^2 = (S_1 + S_2)(S_1 + S_2) = S_1^2 + S_2^2 + 2S_1 \cdot S_2$$

$$S^2 |++\rangle =$$

$$S_1 \cdot S_2 = (S_{1x}\hat{x} + S_{1y}\hat{y} + S_{1z}\hat{z})(\cdots)$$

$$= S_{1x}S_{2x} + S_{1y}S_{2y} + S_{1z}S_{2z}$$

$$= \left(\frac{S_{1+} + S_{1-}}{2}\right)\left(\frac{S_{2+} + S_{2-}}{2}\right) + \left(\frac{S_{1+} - S_{1-}}{2i}\right)\left(\frac{S_{2+} - S_{2-}}{2i}\right)$$

$$+ S_{1z}S_{2z}$$

$$2 S_1 S_2 = \frac{1}{2}\left(S_+^1 S_-^2 + S_-^1 S_+^2\right) + S_z^1 S_z^2$$

$$\Rightarrow S^2 = S_1^2 + S_2^2 + 2S_z^1 S_z^2 + (S_+^1 S_-^2 + S_-^1 S_+^2)$$

$$[S^2] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

~~Spin orbit~~ :— Spin – spin :—

For proton – electron spin coupling.

take z-basis $|\pm z, \pm z\rangle = |\pm, \pm\rangle$.

$$H = \frac{2}{\hbar} A\, S_1 \cdot S_2$$

$$= \frac{2}{\hbar} A \cdot \frac{\hbar}{2} \sigma_1 \cdot \sigma_2$$

$\sigma_1 = \sigma_{1x}\hat{x} + \sigma_{1y}\hat{y} + \sigma_{1z}\hat{z}$ ... Pauli matrices

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$$= S_1 \cdot S_2 = \frac{\hbar}{2}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)$$

$$= \frac{\hbar}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= H = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & -A & 2A & 0 \\ 0 & 2A & -A & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With eigenbasis

$|++\rangle, \frac{1}{\sqrt{2}}\left(|+-\rangle + |-+\rangle\right), |--\rangle$

for Energy $= \frac{A}{2}$

and $\frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$ Energy $= -\frac{3A}{2}$

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Spin-Orbit

$$\Delta H \propto S \cdot L$$

$$\vec{J} = L + S = L \otimes \mathbb{1} + \mathbb{1} \otimes S$$

$$J^2 = L^2 + S^2 + 2 L \cdot S.$$

For Hydrogen $\{H, L_z, L^2, S_z, S^2\}$ are complete set of commuting operators.

$$L \cdot S = \frac{1}{2}(J^2 - S^2 - L^2)$$

So $l = 1$,

$|1,1\rangle \otimes |\tfrac{1}{2}, \tfrac{1}{2}\rangle$, $|1,0\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$, $|1,1\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$,

$|1,0\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$, $|1,-1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$

$|1,-1\rangle \otimes |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$.

$$|j = \tfrac{3}{2}, \tfrac{3}{2}\rangle = |1,1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle.$$

$$|\tfrac{3}{2}, -\tfrac{3}{2}\rangle = |1,-1\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$|\tfrac{3}{2}, \tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\, |1,0\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{3}} |1,1\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$|\tfrac{3}{2}, -\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\, |1,0\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{3}} |1,-1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$|\tfrac{1}{2}, \tfrac{1}{2}\rangle = -\tfrac{1}{\sqrt{3}} |1,0\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\, |1,1\rangle \otimes |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$|\tfrac{1}{2}, -\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}} |1,0\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\, |1,-1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle.$$

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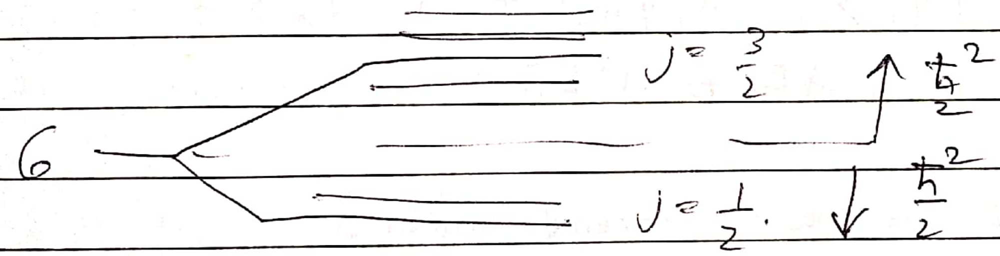
$$\langle L\cdot S\rangle = \frac{1}{2}\left(\langle J^2\rangle - \langle L^2\rangle - \langle S^2\rangle\right)$$

$$= \frac{\hbar}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$$

$$l = 1, \quad s = \frac{1}{2},$$

$$= \frac{\hbar}{2}\left(j(j+1) - \frac{11}{4}\right)$$ 6 states split into 4 with $j = \frac{3}{2}$, and 2 with $j = \frac{1}{2}$.



$$\underline{1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}}$$

General :—

$$J_1 \otimes J_2 = (J_1 + J_2) \oplus (J_1 + J_2 - 1) \oplus \cdots |j_1 - j_2|$$