PH3103

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Vid [4] Assignment. 19MS151

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Q1 $f(z) = \bar{z}$

$\lim_{z \to 0} f(z) = 0$, $f(z)$ is cont at $z = 0$

$\lim_{\substack{x \to 0 \\ y = 0}} \bar{z} = \lim_{x \to 0} x = 0$

$\lim_{\substack{y \to 0 \\ x = 0}} \bar{z} = \lim_{y \to 0} (-iy) = 0$

$\therefore \lim_{z \to 0} \bar{z}$ exists and is equal to 0.

$\lim_{\substack{x \to 0 \\ y \to 0}} (x - iy) = 0$

$f(z)$ is cont ~~at $z=0$~~ if $\lim_{z \to 0} f(z) = f(0)$.

$f(z) = \bar{z}$, $\bar{z} = x - iy$.

$z = 0 \Rightarrow x = 0, y = 0$

$\Rightarrow \bar{z} = x - iy = 0$

$\therefore \lim_{z \to 0} \bar{z} = \bar{z}(0)$

$\therefore f(z)$ is continuous at $z = 0$.

Q2 Polynomials in $\mathbb{C}$ are continuous.
Sum of 2 cont functions is continuous

a polynomial in $\mathbb{C}$ is of form.

$$p(z) = \sum_{k=0}^{n} a_k z^k$$

We just prove that z^k is

induction :-

$k=0 \Rightarrow z^k = 1$ which is continuous everywhere

$k=1 \Rightarrow z$.

$$\lim_{\substack{x \to x_0 \\ y = y_0}} z = \lim_{x \to x_0} x + iy_0 = x_0 + iy_0$$

$$\lim_{\substack{x = x_0 \\ y \to y_0}} z = \lim_{\substack{x = x_0 \\ y \to y_0}} x_0 + iy = x_0 + iy_0$$

$z_0 = x_0 + iy_0$

$\Rightarrow z$ is continous on $\mathbb{C}$.

Assume for $k = k \in \mathbb{N}$, z^k is continous

i.e $\lim_{z \to z_0} z^k = z_0^k$.

$$\lim_{z \to z_0} z^{k+1} = \left(\lim_{z \to z_0} z\right)\left(\lim_{z \to z_0} z^k\right) = z_0 z_0^k = z_0^{k+1}$$

$\Rightarrow z^{k+1}$ is continuous.

$\Rightarrow z^k$ ~~for~~ $\forall k \in \mathbb{N}$ is continuous

$\Rightarrow p(z) = \sum a_k z^k$ is continuous

or $\quad z^k \Rightarrow a_k z^k$ is continuous.

$$\lim_{z \to z_0} \sum a_k z^k = \sum a_k z_0^k = p(z_0)$$