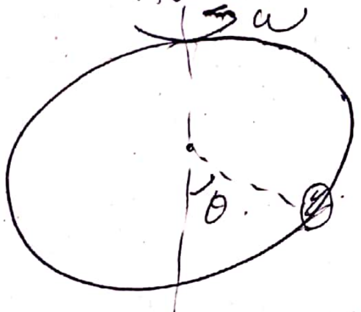
Endsem 19MS151 PH3101

Adwait Naravane

Q1



(a) $|r| = R$ is constant is holonomic constraint
from below we can derive eq. of motion
and $\ddot{\theta} + \frac{g}{R}\sin\theta - \omega^2 \sin\theta\cos\theta = 0$ has not
explicit time dependence, so ~~scle~~ schleronomic constraint.

(b) The angle θ is the single generalised coordinate for system.

$T = \frac{1}{2} m \dot{v}\dot{v}$ $\dot{r} = 0$, $r = R$.

$\dot{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\,\dot{\phi}\,\hat{\phi}$

ϕ is the one going into the plane

$\dot{\phi} = \omega$.

$T = \frac{m}{2}(R^2\dot{\theta}^2 + R^2\omega^2\sin^2\theta)$.

$V = -mgR\cos\theta$. is potential energy.

(c) $L = T - V = \frac{mR^2}{2}(\dot{\theta}^2 + \omega^2\sin^2\theta) + mgR\cos\theta$.

$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{mR^2}{2}.2\dot{\theta} = mR^2\dot{\theta}$..

(d) $H = \dot{\theta}p_\theta - L = \dot{\theta}mR^2\dot{\theta} - \frac{mR^2}{2}(\dot{\theta}^2 + \omega^2\sin^2\theta) - mgR\cos\theta$

$\Rightarrow H = \frac{mR^2}{2}(\dot{\theta}^2 - \omega^2\sin^2\theta) - mgR\cos\theta$.

$= \frac{mR^2}{2}\left(\frac{p_\theta^2}{(mR^2)^2} - \omega^2\sin^2\theta\right) - mgR\cos\theta$

$H = \frac{p_\theta^2}{2mR^2} - \frac{mR^2\omega^2}{2}\sin^2\theta - mgR\cos\theta$

(e) $\frac{dH}{dt} = -\frac{\partial L}{\partial t} = 0$ .. Hammy is conserved.
~~Energy is conserved, no time dependence~~

But Energy is not

$E = T + U$

$= \frac{1}{2}m(R^2\dot{\theta}^2 + R^2\sin^2\theta\,\omega^2) + mgR\cos\theta$

$\frac{\partial E}{\partial t} = 0$

But $\{E, H\} \neq 0$. as E & H are not same.

E is not conserved.

(f) $\frac{\partial L}{\partial \theta} = F_\theta = 0 \Rightarrow mR^2\omega^2 \sin\theta\cos\theta - mgR\sin\theta$

$\Rightarrow \cos\theta = \frac{g}{R\omega^2}$

$\theta = \cos^{-1}\left(\frac{g}{R\omega^2}\right)$ angle at which bead doesn't move (it released from rest with no initial $\dot{\theta}$).

(c) again eq of motion.

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$

$mR^2\ddot{\theta} - (mR^2\omega^2\sin\theta\cos\theta - mgR\sin\theta)$

has no explicit time dependence

Scanned with CamScanner

Q3. $(q, p) \longrightarrow (Q, P)$

(a) $Q = q^m p^n, \quad P = q^k p^l$.

for ~~transformat~~ canonical transformation

for two functions F & G.

$\{F, G\}_{p,q} = \{F, G\}_{P,Q}$.

take P & Q as the functions itself.

$\{Q, P\}_{p,q} = \{Q, P\}_{P,Q} = 1 \ldots$ (derivative with itself)

$$\{Q, P\}_{p,q} = \frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q}.$$

$$= m q^{m-1} p^n \cdot q^k l p^{l-1} - n q^m p^{n-1} \cdot k q^{k-1} p^l.$$

$$= ml \cdot p^{n+l-1} q^{k+m-1} - nk \, q^{m+k-1} p^{n+l-1}$$

$$= 1$$

$\Rightarrow n + l - 1 = 0, \quad k + m - 1 = 0.$

$\Rightarrow n + l = 1, \quad k + m = 1$

$ml - nk = 1.$

$n = 1 - l, \quad m = 1 - k.$

$(1-k)l - (1-l)k = 1$

$l - kl - k + kl = 1$

$l - k = 1 \quad \Rightarrow \quad n + 1 + k = 1 \Rightarrow n + k = 0$

$n + l = k + m = l - k = 1$

$n = -k$,

relations would be $n = -k$, $n + l = k + m = l - k = 1$, $m + l = 2$ and so on.

knowing one gives all.

Scanned with CamScanner

(b) $k = 0$

$\Rightarrow n = 0$

$\Rightarrow m = l = 1$

$\Rightarrow Q = q, \quad P = p$

so the transformation in this case is identity.

Scanned with CamScanner

Q5 (a)

$$Q = \sin^{-1}\left(\frac{q}{\sqrt{q^2+\frac{p^2}{\alpha^2}}}\right), \quad P = \frac{1}{2}\left(\alpha q^2 + \frac{p^2}{\alpha}\right)$$

$\{Q, P\}_{p,q} = 1$ as discussed in Q3

$$\frac{\partial Q}{\partial q} = \frac{1}{\sqrt{1-\frac{q^2}{q^2+\frac{p^2}{\alpha^2}}}}\left(\frac{1}{\sqrt{q^2+\frac{p^2}{\alpha^2}}} - \frac{q^2}{\left(q^2+\frac{p^2}{\alpha^2}\right)^{3/2}}\right)$$

$$\frac{\partial P}{\partial p} = \frac{p}{\alpha}$$

$$\frac{\partial Q}{\partial p} = \frac{1}{\sqrt{1-\frac{q^2}{q^2+\frac{p^2}{\alpha^2}}}}\left(\frac{-pq}{\alpha^2\left(q^2+\frac{p^2}{\alpha^2}\right)^{3/2}}\right)$$

$$\frac{\partial P}{\partial q} = \alpha q$$

$$\{Q, P\} = \frac{1}{\sqrt{1-\frac{q^2}{q^2+\frac{p^2}{\alpha^2}}}}\left(\frac{p}{\alpha\sqrt{q^2+\frac{p^2}{\alpha^2}}} - \frac{pq^2 + pq^2}{\alpha\left(q^2+\frac{p^2}{\alpha^2}\right)^{3/2}}\right)$$

$$= \frac{p}{\alpha\sqrt{q^2+\frac{p^2}{\alpha^2}}\sqrt{1-\frac{q^2}{\frac{p^2}{\alpha^2}+q^2}}}$$

$$= \frac{\sqrt{q^2+\frac{p^2}{\alpha^2}}}{\sqrt{q^2+\frac{p^2}{\alpha^2}}} = 1$$

$\Rightarrow$ ~~Eq~~ Transformation is canonical.

Scanned with CamScanner

(Q5) (b).

$$Q = \sin^{-1}\left(\frac{q}{\sqrt{q^2 + \frac{p^2}{\alpha^2}}}\right)$$

$$P = \frac{1}{2}\left(\alpha q^2 + \frac{p^2}{\alpha}\right)$$

$$= \frac{\alpha}{2}\left(q^2 + \frac{p^2}{\alpha^2}\right)$$

$$\left(\frac{2P}{\alpha}\right)^{1/2} = \sqrt{q^2 + \frac{p^2}{\alpha^2}}$$

$$\cancel{\sin Q =} \sqrt{q^2 + \frac{p^2}{\alpha^2}} = \frac{q}{\sin Q}$$

$$P = \frac{\alpha}{2} \cdot \frac{q^2}{\sin^2 Q}$$

$$\frac{p^2}{\alpha^2} = \frac{q^2}{\sin^2 Q} - q^2$$

$$p^2 = \alpha^2 q^2 (\operatorname{cosec}^2 Q - 1)$$

$$= \alpha^2 q^2 (\cot^2 Q)$$

$$p = \alpha q \cot Q$$ .. transformations

$$P = \frac{\alpha}{2} \frac{q^2}{\sin^2 Q}$$

Generation function $F(q, Q)$.

$$P = -\frac{\partial F}{\partial Q}, \quad p = \frac{\partial F}{\partial q}$$

$$F = -\int \frac{\alpha}{2} \frac{q^2}{\sin^2 Q}\, dQ + \ldots$$

$$= -\frac{\alpha}{2} q^2 \int \operatorname{cosec}^2 Q\, dQ$$

$$= \frac{\alpha}{2} q^2 \cot Q.$$

$$F = \int \alpha q \cot Q\, dq$$

$$= \alpha \cot Q \cdot \frac{q^2}{2}$$

Both sides match.

$F = \frac{\alpha}{2} q^2 \cot Q$ is generating function

7) (a) for $f(p,q,t)$ to be integral of motion

$$\{f(p,q,t), H\} + \frac{\partial f}{\partial t} = 0$$

$$\frac{\partial f}{\partial t} = -\{f(p,q,t), H\} = \{H, f(p,q,t)\}$$

$$\{\frac{\partial f}{\partial t}, H\} = \{\{H, f(p,q,t)\}, H\}$$

$$= \{H, \{f, H\}\} + \{\cancel{f}, \cancel{\{H, H\}}\}$$

$$\Rightarrow \{\{H, f\}, H\} = \cancel{\{H, f\}} \{H, \{f, H\}\}$$

$$\Rightarrow \{\{H, f\}, H\} = 0$$

$$\Rightarrow \{\frac{\partial f}{\partial t}, H\} = 0$$

$\Rightarrow \frac{\partial f}{\partial t}$ is also an integral of motion.

(b) free particle $\Rightarrow$ $H = \frac{p^2}{2m}$

$$\frac{dH}{dt} = \{H, H\} + \frac{\partial H}{\partial t} = 0 + 0 = 0$$

H is integral of motion.

(c) $f = q - \frac{pt}{m}$

$$\frac{df}{dt} = \{H, f\} + \frac{\partial f}{\partial t}$$

$$= \{\frac{p^2}{2m}, q - \frac{pt}{m}\} - \frac{p}{m}$$

$$= (0 + \frac{p}{m}) - \frac{p}{m} = 0$$

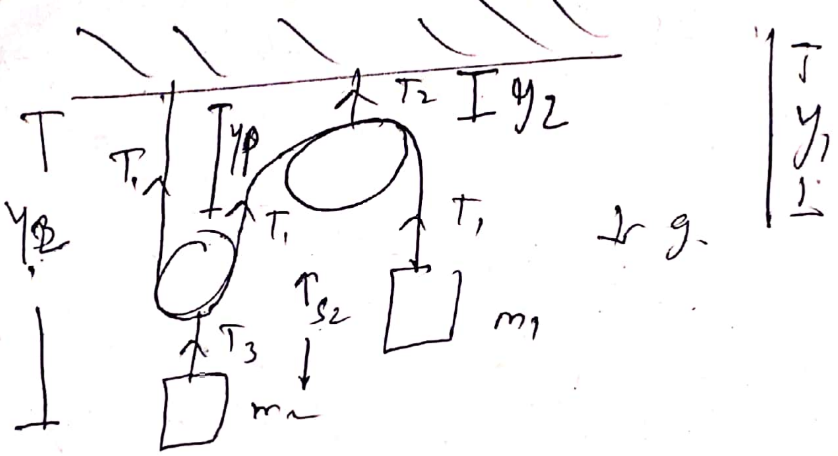
f is an integral of motion

(d) $\frac{\partial f}{\partial t} = -\frac{p}{m}$

$$\{\frac{p^2}{2m}, \frac{p}{m}\} = 0 - 0 = 0$$

$\frac{\partial f}{\partial t}$ is an integral of motion.

Q6.



for $m_1 \Rightarrow \quad m_1 g - T_1 = m_1 a_1$

for $m_2 \Rightarrow \quad m_2 g - T_3 = m_2 a_2$.

$2T_1 = T_2$

$T_3 = 2T_1$

Stretching the mass m_1 by l, gives.

(a)

01-06-2018 10:48:19
PAGE NO. 3

$l = y_B + \pi R + (y_B - y_2) + \pi R + (y_1 - y_2)$.

$\frac{d^2 l}{dt^2} = a_p + a_1 \Rightarrow 2a_p + a_1 = 0$.

$\Rightarrow a_1 = -2a_p$.

S_2 is constant. $\quad S_2 = y_2 - y_p$

$\frac{d^2 S_2}{dt^2} = a_2 - a_p \Rightarrow a_2 = a_p$.

$\Rightarrow a_1 = -2a_2$

(b). $l = y_p + \pi R + (y_p - y_2) + \pi R + (y_1 - y_2)$

$dy_2 = dy_p$

$dy_1 = -2dy_2$

$dl = 2dy_p + dy_1 = 0$

$2dy_2 + dy_1 = 0$

$dy_1 = -2dy_2 \quad$ virt displacement

p kandivli(RK0304 VS043375
AZAR (BHB) RK0304 VS043375
RANCH (MDS) RK0304 VS043375
p kandivli(RK0304 VS043375
NEW MUMBAI) RK0304 VS043375
MAHINDRA BA RK0304 VS043375
MAHINDRA BA RK0304 VS043375
(E) (PRL) RK0304 VS043375
AZAR (BHB) RK0304 VS043375
E BRANCH (S RK0304 VS043375
E BRANCH(SE RK0304 VS043375
MAHINDRA BA RK0304 VS043375

Scanned with CamScanner

(c) $a_1 = -2a_2$

$m_1 g - T = m_1 a_1$

$m_2 g - 2T = m_2 a_2$

$2m_1 g - 2T = 2m_1 a_1$

$m_2 g - 2T = m_2 a_2$

$(2m_1 - m_2) g = 2m_1 a_1 - m_2 a_2$

$= -4m_1 a_2 - m_2 a_2$

$= -a_2 (4m_1 + m_2)$.

$\Rightarrow \quad a_2 = \dfrac{(m_2 - 2m_1) g}{4m_1 + m_2}$

$a_1 = \dfrac{(4m_1 - 2m_2) g}{4m_1 + m_2}$

Scanned with CamScanner