Intermediate Quantum Mechanics(PH3102) Mid. Sem. Exam

time 1hr 30min

[*Read the questions carefully. Extra credit for clean and precise answers. Marks will be deducted (even no marks) for final answers without any logical steps or physical arguments. Part marking (even full marks) for correct logical steps even if final answer maybe wrong*]

1. Consider a particle in 1D potential well $V(x) = -\frac{V_0}{\sqrt{\pi}\sigma} e^{-x^2/\sigma^2}$

i)Estimate the ground state energy of the particle.

ii) Find out expectation value of $\hat{x}$ (position) and $\hat{p}$ (momentum) corresponding to an eigenstate of the Hamiltonian.

iii)Find out the asymptotic behavior (functional form) of the ground state wavefunction for $x \to \pm\infty$.

iv) Discuss the behavior of the bound state wavefunction and its derivative across $x = 0$, as $\sigma \to 0$. (2+1+2+3).

2. Show that the eigenvalues of Hamiltonian in 1D (for simplicity) are real. (3).

3. Consider a potential well:

$$\begin{aligned} V(x) &= \infty \quad for\ x < 0 \\ &= -V_0 \quad for \quad 0 \leq x < a \\ &= 0 \quad for \quad x \geq a \end{aligned}$$

i) Find out the eigenfunctions corresponding to scattering states.

ii) Derive the mathematical condition for finding the eigenvalues of the bound states.

iii) From graphical solution find out asymptotic value of energy eigenvalues for large depth of potential well $V_0 \gg \frac{\hbar^2}{ma^2}$. (3 + 4 + 2)

Mathematical formula: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$