PH3103

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Q1 Prove C-R conditions in polar coordinates

$f = u + iv$ is analytic

if $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$\underbrace{\hspace{6em}}_{\text{Cauchy Riemann.}}$

$z = x + iy = r\cos\theta + i\, r\sin\theta$

$x = r\cos\theta,\ y = r\sin\theta.$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta = \frac{1}{r}(r)\left(\frac{\partial v}{\partial y}.\cos\theta - \frac{\partial v}{\partial x}\sin\theta\right)$$

$$= \frac{1}{r}\left(\frac{\partial v}{\partial \theta}\right)$$

$$\left[\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}(-r\sin\theta) + \frac{\partial v}{\partial y}(r\cos\theta)\right]$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\sin\theta = \frac{-1}{r}\left(\frac{\partial u}{\partial y}r\cos\theta - \frac{\partial u}{\partial x}r\sin\theta\right)$$

$$= \frac{-1}{r}\frac{\partial u}{\partial \theta}$$

$$\left[\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}(-r\sin\theta) + \frac{\partial u}{\partial y}(r\cos\theta)\right]$$

$\Rightarrow$ C-R conditions in polar coordinates

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$$