General instructions: answer as much as you can. The person with the highest mark would be scaled to the total 30. Others would have same scaling factor. People caught in the *noble act of cheating* would be given equal *negative* credits.

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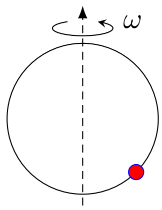
1. Consider a circular rigid wire in the vertical plane. A bead is constrained to slide along the wire under the action of gravity as shown in the figure. The circular wire is being rotated with a constant angular speed ω about the vertical axis passing through its center. [10]



   (a) Classify the constraint (using all four types, as taught in the class). [1]

   (b) Identify the generalized coordinates for the bead. [1]

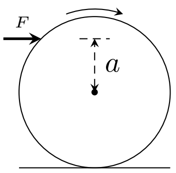
   (c) Find the Lagrangian and the equations of motion. [2+1]

   (d) Find the Hamiltonian. Is it conserved? [1+1]

   (e) Is the total energy conserved? [1]

   (f) The bead is released from the rest and is found to be not sliding along the wire. What is the angle of release? [2]

2. A cylinder is rolling without slipping on a horizontal plane under the action of a constant force (F) applied a distance above the axis of the cylinder. [7]



   (a) Identify the constraints. [1]

   (b) Using Lagrange's equations of the first kind, find the equations of motion. [2]

   (c) Find the directions and the magnitudes of the constraint forces. [2]

   (d) Is there a value of a for which the constraint force vanishes? Why does it vanish (discuss the reason behind the vanishing constraint force). [2]

   (e) For which value of a, the constraint force is maximum? [1]

3. Consider a transformation $(q,p) \to (Q,P)$, such that we have, [3]

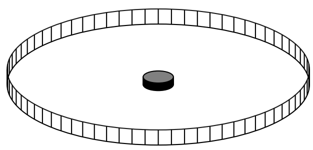
$$Q = q^m p^n; \quad P = q^k p^l$$

   where, the numbers in the powers are real.

   (a) What are the relations among k, l, m, n, that ensure the above transformation is canonical? [2]

   (b) Identify the transformation for $k = 0$. [1]

4. Consider a puck of mass m released to undergo bouncing motion along the diagonal of a horizontal tray with walls (see the figure below). Assume all motions to be frictionless. The tray is placed at the co-latitude λ. [7]



   (a) Find the equations of motion for all velocity components of the puck. [2]

(b) Show that the path of the puck rotates about the local vertical axis. [2]

(c) Find the rate of rotation of the path. [2]

(d) Is it the same as that of the a pendulum (Foucault)? [1]

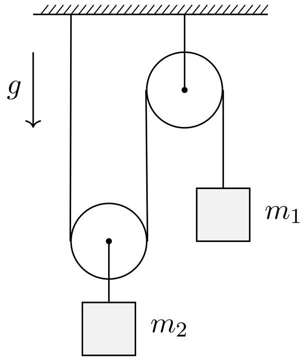
5. Consider the transformation [4]

$$Q = \sin^{-1}\left(\frac{q}{\sqrt{q^2 + \frac{p^2}{\alpha^2}}}\right), \quad P = \frac{1}{2}\left(\alpha q^2 + \frac{p^2}{\alpha}\right).$$

(a) Show that the above transformation is canonical. [2]

(b) Find the generating function of (the first kind, i.e. $F_1(q, Q)$) for this transformation. [2]

6. Two masses are connected by a massless thin inextensible string and are arranged to move vertically with two massless pulleys as shown in the figure. Both the mass moves only vertically. [5]



(a) Write down the constraints. [1]

(b) Identify the virtual displacements. [2]

(c) Use D'Alembert's principle to find the acceleration of the mass m_1. [2]

7. Suppose a function $f(q, p, t)$ and H are integrals of motion. [6]

(a) Show that $\frac{\partial f}{\partial t}$ is also an integral of motion. [2]

(b) Consider a free particle of mass m. Show that H is an integral of motion. [1]

(c) For this free particle show that [2]

$$f(q, p, t) = q - \frac{pt}{m}$$

is an integral of motion.

(d) Verify that $\frac{\partial f}{\partial t}$ for the above example is indeed an integral of motion. [1]