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19MS151

Assigmnent – 2 Forced-Damped oscillator

(1)

$$\frac{d^2y}{dx^2} + \lambda \frac{dy}{dx} + \omega_0^2 y = F\cos\omega x.$$

try. ~~x(t)~~ $y(x) = A \cancel{e^{i(\omega t+\phi)}} \cos(\omega x + \phi)$.

$$\Rightarrow A\left(-\omega^2 \cos(\omega x+\phi) - \lambda\omega\sin(\omega x+\phi) + \omega_0^2 \cos(\omega x+\phi)\right)$$

$$= F\cos(\omega x \cancel{+\phi})$$

$$A = \frac{F}{\left(\lambda^2\omega^2 + (\omega_0^2-\omega^2)^2\right)^{1/2}}$$

$$\phi = \tan^{-1}\left(\frac{\lambda\omega}{\omega^2-\omega_0^2}\right) \leftarrow \cos(A+B) = \cos A\cos B - \sin A \sin B.$$

Now this is the true solution but it is just from a guess and might not have spanned the full set. So we will do it using Fourier transform.

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2) $\ddot{y} + \lambda \dot{y} + \omega_0^2 y = F\cos(\omega x)$.

Do Fourier transform :-

$$\hat{f}(\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\nu x} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\nu) e^{i\nu x} d\nu$$

$f'(x) \rightarrow i\nu \hat{f}(\nu)$. — (we had a small amount of Fourier transform in QM class).

$f''(x) \longrightarrow -\nu^2 \hat{f}(\nu)$.

$\Rightarrow$ y transforms to $\hat{y}$

$F\cos\omega x \longrightarrow \hat{F}$

$$\Rightarrow (-\nu^2 + i\lambda\nu + \omega_0^2)\hat{y} = \hat{F}.$$

$$\Rightarrow \hat{y} = \frac{\hat{F}}{(-\nu^2 + i\lambda\nu + \omega_0^2)}$$

Now we do Fourier trans on $F\cos\omega x$

$$\cos(\omega x) = \frac{1}{2}\left(e^{i\omega x} + e^{-i\omega x}\right).$$

$$F\cos\omega x \longrightarrow \frac{F}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i(\nu-\omega)x} dx + \int_{-\infty}^{\infty} e^{-i(\nu+\omega)x} dx.$$

Now. $\displaystyle\int_{-\infty}^{\infty} e^{i\nu x} dx = \int_{-\infty}^{\infty} e^{-i\nu x} dx = 2\pi\,\delta(x)$.

$$\Rightarrow F\cos\omega x \xrightarrow{\text{Fourier}} \sqrt{\frac{\pi}{2}}\, F\left(\delta(\nu-\omega) + \delta(\nu+\omega)\right)$$

Now,

$$y(x) = \frac{F}{2}\int_{-\infty}^{\infty} \frac{\delta(\nu-\omega)+\delta(\nu+\omega)}{(-\nu^2+i\lambda\nu+\omega_0^2)} e^{i\nu x}\, d\nu$$

$$= \frac{F}{2}\int_{-\infty}^{\infty} \frac{\delta(\nu-\omega)}{-\nu^2+i\lambda\nu+\omega_0^2} e^{i\nu x}\, d\nu + \int_{-\infty}^{\infty} \frac{\delta(\nu+\omega)}{-\nu^2+i\lambda\nu+\omega_0^2} e^{i\nu x}\, d\nu$$

$$y(x) = \frac{F}{2}\left(\frac{e^{i\omega x}}{\omega_0^2-\omega^2+i\lambda\omega} + \frac{e^{-i\omega x}}{\omega_0^2-\omega^2-i\lambda\omega}\right)$$

$$= \frac{F}{(\omega_0^2-\omega^2)^2+\lambda^2\omega^2}\left((\omega_0^2-\omega^2)\cos\omega x + \lambda\omega\sin(\omega x)\right)$$

$$= \frac{F}{\sqrt{(\omega_0^2-\omega^2)+\lambda^2\omega^2}}\cos(\omega x-\phi)$$

$$\phi = \tan^{-1}\left(\frac{\lambda\omega}{\omega_0^2-\omega^2}\right)$$

By using the whole

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

and giving us

$$\frac{\sin A}{\cos A} = \tan A = \frac{\lambda\omega}{\omega_0^2-\omega^2}$$

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