19MS151 PH3201 ~~Class~~ Quiz

Rough

Q2 $|\psi_k\rangle = \begin{bmatrix} 1 \\ k \end{bmatrix}$, $|\phi_k\rangle = \begin{bmatrix} k \\ 1 \end{bmatrix}$

$$\rho_1 = \frac{1}{32}\left(|\psi_3\rangle\langle\psi_3| + |\phi_3\rangle\langle\phi_3| + 6|\psi_{-1}\rangle\langle\psi_{-1}|\right)$$

$$= \frac{1}{32}\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \end{bmatrix} + 6\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix}\right)$$

$$= \frac{1}{32}\left[\begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} + \begin{pmatrix} 9 & 3 \\ 3 & 1 \end{pmatrix} + \cancel{6}\begin{pmatrix} 6 & -6 \\ -6 & 6 \end{pmatrix}\right]$$

$$= \frac{1}{32}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}.$$

$$\rho_2 = \frac{1}{18}\left(|\psi_2\rangle\langle\psi_2| + |\phi_2\rangle\langle\phi_2| + 4|\psi_{-1}\rangle\langle\psi_{-1}|\right)$$

$$= \frac{1}{18}\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \end{pmatrix} + 4\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix}\right)$$

$$= \frac{1}{18}\left(\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} + \cancel{4}\begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}\right)$$

$$= \frac{1}{18}\begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\rho_3 = \frac{1}{2}\left(|\psi_{-1}\rangle\langle\psi_{-1}| + |\phi_1\rangle\langle\phi_1|\right)$$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \end{pmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right)$$

$$= \cancel{\frac{1}{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho_4 = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Q.3 $E_0(5) = 3\times 2\frac{\hbar\omega}{2} + 5\times 3\frac{\hbar\omega}{2} = \frac{21}{2}\hbar\omega$

$E_0(3) = 6\frac{\hbar\omega}{2} + \frac{5}{2}\hbar\omega = \frac{11}{2}\hbar\omega$.

$E_1(5) = \frac{4\times 5}{2}\hbar\omega + \frac{3\hbar\omega}{2} = \frac{23}{2}\hbar\omega$.

Ground will be

$\uparrow \; \uparrow \; \uparrow \longrightarrow$ 8 if spins flipped.

$\uparrow\downarrow$

$\uparrow\downarrow \; \uparrow \; \cancel{\uparrow} \longrightarrow$ 12 if spins flipped.

$\uparrow\downarrow$ $\underline{\underline{20}}$.

Q8

$\epsilon \propto |p|^2$, $PV = \frac{2}{3} U$

$\Rightarrow P = \frac{2U}{3 \cdot V}$

Q1

$\langle n_{fer} \rangle = \frac{1}{e^{\frac{(E - E_F)}{kT}} + 1}$

$\langle n_{Bos} \rangle = \frac{1}{e^{\frac{E - E_F}{kT}} - 1}$

$\mu < 0$ for Bosons

Q7 $P \sim \frac{e^{-E/kT}}{1+e^{-E/kT}} = \frac{e^{-\left(\frac{0.1\times 1.6\times 10^{-19}}{1.38\times 10^{-23}\times 6}\right)}}{e^{-\frac{0.1\times 1.6}{1.38\times 600\times 10^{-4}}}+1}$

≈ 0.13

$\Rightarrow \underline{0.1 < P < 0.2}$

Q5 $\rho = \sum_{k=0}^{n} |\psi_k\rangle\langle\psi_k| + |\phi_k\rangle\langle\phi_k|$

$= \sum_{k=0} \begin{pmatrix} 1 & k \\ k & k^2 \end{pmatrix} + \begin{pmatrix} k^2 & k \\ k & 1 \end{pmatrix}$

$= \sum \begin{pmatrix} k^2+1 & 2k \\ 2k & k^2+1 \end{pmatrix}$

$Tr(\rho^n) = 2\sum_{k=0}^{n} (\cancel{2}k^2+1)$

$= (n+1)\frac{(2n^2+n+6)}{3}$

Scanned with CamScanner

Q8

$$E = \hbar\omega \left[n_1 + n_2 + n_3 + \frac{3}{2} \right)$$

$$M = \sum_{1}^{3} n_i = \frac{E}{\hbar\omega} - \frac{\cancel{B}}{2}$$

$$\Omega = \frac{(M+3-1)!}{M!\,2!}$$

$$S = \ln(\Omega)\, k_B$$

$$\frac{1}{T} = \frac{\partial S}{\partial E}\Big|_N \approx \frac{k_B}{\hbar\omega} \ln\left(\frac{M+\cancel{B}-1}{M} \right) \quad \text{-- Stirling}$$

$$= \cancel{k} \frac{k_B}{\hbar\omega} \ln\left(\frac{\frac{E}{\hbar\omega} + \frac{3}{2} - 1}{\frac{E}{\hbar\omega} - \frac{3}{2}} \right)$$

$$\approx \frac{k_B}{\hbar\omega} \ln\left(\frac{\cancel{E} + \frac{7\hbar\omega}{2} + \frac{3}{2}\hbar\omega}{\frac{9\hbar\omega}{2} - \frac{3}{2}\hbar\omega} \right)$$

$$\frac{1}{T} \approx \frac{k_B}{\hbar\omega} \ln\left(\frac{5}{3}\right)$$

$$T > \frac{1}{\ln\left(\frac{5}{3}\right)}$$

Q4 $\frac{P}{kT} \sim \int 2\pi p\, dp \ln\left(1 \pm z e^{-\frac{\beta p^2}{2m}}\right)$.

2D.

$d^2p \sim 2\pi p\, dp$.

$$\frac{P}{kT} \propto \int x\, dx \sum_{l=1}^{\infty} -\left(\frac{\mp z e^{-x^2}}{l}\right)^l.$$

$$= \sum (-1)^{l+1} \frac{z^l}{l^2}$$

$\rightarrow$ 2 is the exponent.

in 1-D $\int 2\pi p\, dp \rightarrow \int dp$.

exponent $\frac{3}{2}$.