PH3202 19MS151

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Q1 $\phi = \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{c}\right)$

$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$ ~ poisson's eq.

$\frac{\partial^2}{\partial x^2}\phi + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} =$

$= \left(-\frac{\pi^2}{a^2}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{c}\right)$

$+ \left(-\frac{\pi^2}{b^2}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{c}\right)$

$+ \left(-\frac{\pi^2}{c^2}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{c}\right)$

$= -\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{c}\right)\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$

$= -\frac{\rho}{\varepsilon_0}$

$\Rightarrow \underline{\rho(x,y,z) = \pi^2\varepsilon_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{c}\right)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)}$

Q2 $\frac{d^2y}{dx^2} - k^2 y = f(x).$

$f(x) = \delta(x).$

$y = G(x).$ -- green's function

$\frac{d^2G}{dx^2} - k^2 y = \delta(x)$

$G(x) \to 0, \; x \to \infty$

Fourier transform.

$-p^2 \hat{G}(p) - k^2 \hat{G}(p) = 1$

$\hat{G}(p) = -\frac{1}{p^2+k^2}\left(\frac{1}{\sqrt{2\pi}}\right)$

We did Fourier transform for differential eq last sem

take $f(x) = e^{-k|x|}$

$\int_{-\infty}^{\infty} e^{-k|x|} e^{ipx}\,dx = \int_{-\infty}^{0} e^{kx+ipx}\,dx + \int_{0}^{\infty} e^{-kx+ipx}\,dx$

$= \left.\frac{e^{(k+ip)x}}{k+ip}\right|_{-\infty}^{0} - \left(\frac{e^{-(k-ip)x}}{k-ip}\right)_0^{\infty}$

$= \frac{1}{k+ip} + \frac{1}{k-ip} = \frac{2k}{k^2+m^2}.$

$\Rightarrow e^{-k|x|} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2k}{k^2+p^2} e^{-ipx}\,dp.$

$\Rightarrow G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{G(p)}{\sqrt{2\pi}} e^{-ipx}\,dp.$

$= \frac{\cancel{\pi}\, e^{-k|x|}}{k}$

~~or~~ $\cancel{G(x,x') = \frac{e^{-k|x-x'|}}{k}}$ is Green's function

$$\boxed{G(x,x') = \frac{e^{-k|x-x'|}}{k}}$$