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19MS151
PH-1
CT-3
Rough

Q1 $\dot{q} = \frac{\partial H}{\partial p}$, $\dot{p} = -\frac{\partial H}{\partial q}$

$\dot{x} = \frac{p}{m}$, $\dot{p} = -(-ma) = ma$

$\ddot{x} = \frac{\dot{p}}{m} = a$

$\dot{x} = at + \alpha$, $x = \beta + \alpha t + \frac{at^2}{2}$

$\Rightarrow x = x_0 + \alpha t + \frac{at^2}{2}$

$\Rightarrow x = x_0 + \frac{p}{m}t + \frac{at^2}{2}$

Q5 $\frac{\partial Q}{\partial p} = -q^{\alpha}\sin(\beta p)\beta$, $\frac{\partial Q}{\partial q} = \alpha q^{\alpha-1}\cos(\beta p)$

$\frac{\partial P}{\partial p} = q^{\alpha}\beta\cos(\beta p)$, $\frac{\partial P}{\partial q} = \alpha q^{\alpha-1}\sin(\beta p)$

$\{Q, P\} = \alpha\beta q^{2\alpha-1} = 1 \Rightarrow \alpha = 1/2$ & $\beta = 2$

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Q5

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2+y^2)$$

$$\frac{\partial H}{\partial p_x} = \frac{p_x}{m}, \quad \frac{\partial H}{\partial p_y} = \frac{p_y}{m}$$

$$\frac{\partial H}{\partial x} = m\omega^2 x, \quad \frac{\partial H}{\partial y} = m\omega^2 y.$$

let $A = \frac{1}{4m\omega}\left(p_x^2 - p_y^2 + m^2\omega^2(y^2 - x^2)\right)$

$$\frac{\partial A}{\partial p_x} = \frac{p_x}{2m\omega}, \quad \frac{\partial A}{\partial p_y} = \frac{-p_y}{2m\omega}$$

$$\frac{\partial A}{\partial x} = -\cancel{2}x \cdot \frac{m^2\omega^2}{\cancel{4m\omega}} = -\frac{m\omega x}{2}, \quad \frac{\partial A}{\partial y} = \frac{m\omega y}{2}$$

$$\{A, H\} = \frac{\partial A}{\partial x}\frac{\partial H}{\partial p_x} + \frac{\partial A}{\partial y}\frac{\partial H}{\partial p_y} - \frac{\partial A}{\partial p_x}\frac{\partial H}{\partial x} - \frac{\partial A}{\partial p_y}\frac{\partial H}{\partial y}$$

$$= -\frac{\cancel{m}\omega x}{2}\cdot\frac{p_x}{\cancel{m}} + \frac{\cancel{m}\omega y}{2}\cdot\frac{p_y}{\cancel{m}} - \frac{p_x}{2\cancel{m}\cancel{\omega}}\cancel{m}\omega^{\cancel{2}}x + \frac{p_y}{2m\omega}\cdot m\omega^2 y$$

$$= -\frac{\omega x p_x}{2} - \frac{\omega x p_x}{2} + \frac{\omega y p_y}{2} + \frac{\omega y p_y}{2}$$

$$= -\omega x p_x + \omega y p_y$$

$$= \omega(y p_y - x p_x) \neq 0$$

-- integral of motion = constant of motion (without explicit time)

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Q3

$$H = \frac{p_x^2 + p_y^2 + \cancel{p_z^2}}{2m} - \frac{\alpha}{\sqrt{x^2+y^2+\cancel{z^2}}}$$

$$A = p_y + \frac{\beta x}{\sqrt{x^2+y^2+\cancel{z^2}}}$$

$$\frac{\partial H}{\partial x} = \frac{\cancel{+}1}{\cancel{2}}\alpha \frac{\cancel{2}x}{(x^2+y^2)^{3/2}}, \quad \frac{\partial H}{\partial y} = \frac{\alpha y}{(x^2+y^2)^{3/2}}$$

$$\frac{\partial H}{\partial p_x} = \frac{p_x}{m}, \quad \frac{\partial H}{\partial p_y} = \frac{p_y}{m}$$

$$\frac{\partial A}{\partial x} = \frac{\beta}{\sqrt{x^2+y^2}} - \frac{\beta x^2}{(x^2+y^2)^{3/2}} = \frac{\beta}{\sqrt{x^2+y^2}}\left(1 - \frac{x^2}{x^2+y^2}\right) = \frac{\beta y^2}{(x^2+y^2)^{3/2}}$$

$$\frac{\partial A}{\partial y} = -\frac{\beta x y}{(x^2+y^2)^{3/2}}, \quad \frac{\partial A}{\partial p_x} = 0, \quad \frac{\partial A}{\partial p_y} = 1$$

$$\{A, H\} = \frac{\partial A}{\partial x}\frac{\partial H}{\partial p_x} + \frac{\partial A}{\partial y}\frac{\partial H}{\partial p_y} - \frac{\partial A}{\partial p_x}\frac{\partial H}{\partial x} - \frac{\partial A}{\partial p_y}\frac{\partial H}{\partial y}$$

$$= \cancel{\frac{\beta}{\sqrt{x^2+y^2}}\left(1 - \frac{x^2}{x^2+y^2}\right)}$$

$$= \frac{\beta y^2}{(x^2+y^2)^{3/2}} \cdot \frac{p_x}{m} - \frac{\beta x y}{(x^2+y^2)^{3/2}} \cdot \frac{p_y}{m} - 0 - \frac{\alpha y}{(x^2+y^2)^{3/2}}$$

$$= \frac{\beta y}{\cancel{(x^2+y^2)^{3/2}}} \frac{(y p_x - x p_y)}{m} - \frac{\alpha y}{\cancel{(x^2+y^2)^{3/2}}} = 0$$

$$= \beta \cancel{y} (y p_x - x p_y) = m\alpha \cancel{y}$$

$$\Rightarrow \beta = \frac{m\alpha}{y p_x - x p_y}$$

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Q3 $-\frac{1}{2}pq$ & ~~cotQ are same~~

$\frac{1}{2}pq$ are not gen fun.

$\frac{\partial}{\partial q}(-\frac{1}{2}pq) = p = -\frac{1}{2}p$ & $\frac{\partial}{\partial q}(\frac{1}{2}pq) = p = \frac{1}{2}p$

cotQ is only gen.

Q7 $v = \exp(x) \Rightarrow x = \ln v = \frac{\partial g}{\partial v}$

$\Rightarrow \underline{g = v(\ln v - 1)}$