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Assignment 4 PH3102

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(1) $L_z = -i\hbar [x\partial_y - y\partial_x]$.

L_x, L_y are cyclic permutations.

$[L_z, r^2]\psi = -i\hbar [x\partial_y - \partial_x, x^2+y^2+z^2].\psi$

$= -i\hbar ((x^2+y^2+z^2)(x\partial_y - y\partial_x)\psi + (x\partial_y - y\partial_x)\psi$
$- (x^2+y^2+z^2)(x\partial_y - y\partial_x 2\psi]$.

$= 0$

$\Rightarrow \underline{[L_i, r^2] = 0}$ x, y, z are similar when it comes to r.

$[L_z, r.p] = [xp_y - yp_x, xp_x + yp_y + zp_z] = [xp_y, yp_y] - [yp_x, xp_x] - [yp_y, yp_y]$

$= [xp_y, xp_x] + [xp_y, yp_y] - [yp_x, xp_x]$

$= i\hbar [xp_y - xp_y + yp_x - yp_x] = 0$

& cyclic permutations $\Rightarrow \underline{[L_i, r.p] = 0}$

(2) (a) For $l=1$, choose basis $\{|1,0\rangle, |1,1\rangle, |1,-1\rangle\}$ $|l,m\rangle$.

$L_z|l,m\rangle = \hbar m|l,m\rangle$.

$\Rightarrow [L_z] = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$L_\pm |l,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\ |l, m\pm 1\rangle$.

so it's matrix form is

$[L_+] = \sqrt{2}\hbar \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $[L_-] = \sqrt{2}\hbar \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$L_x = \frac{L_+ + L_-}{2} = \frac{\hbar}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $L_y = \frac{L_+ - L_-}{2i}$

$= \frac{-i\hbar}{\sqrt{2}} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

For $l=2$, we can do similar derivation

Basis $= \{ |2,2\rangle, |2,1\rangle, |2,0\rangle, |2,-1\rangle, |2,-2\rangle \}$.

$$L_z = \hbar \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$$L_+ = \hbar \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}, \quad L_- = \hbar \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L_x = \frac{\hbar}{2} \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}, \quad L_y = \frac{-i\hbar}{2} \begin{bmatrix} 0 & -2 & 0 & 0 & 0 \\ 2 & 0 & -\sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & -\sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & -2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

(b) Simultaneous eigenstates of L^2 & L_z are coordinate basis vectors.

$$L_z^2 = \hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H = -L_x + \frac{L_z^2}{2I} = \begin{bmatrix} \hbar^2/2I & -\hbar/\sqrt{2} & 0 \\ -\frac{\hbar}{\sqrt{2}} & 0 & -\hbar/\sqrt{2} \\ 0 & -\hbar/\sqrt{2} & \hbar^2/2I \end{bmatrix}$$

3) (a) $$\frac{xy + yz + xz}{r^2} = \frac{\sin^2\theta \sin 2\phi}{2} + \sin\theta\cos\theta(\sin\phi + \cos\phi)$$

compare to spherical harmonics that look similar

$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2\theta\, e^{\pm 2i\phi}, \quad Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta\, e^{\pm i\phi}$$

$$\sin^2\theta \sin 2\phi = \sqrt{\frac{32\pi}{15}} \left(\frac{Y_2^{2} - Y_2^{-2}}{2i} \right)$$

$$\sin\theta\cos\theta\cos\phi = \sqrt{\frac{8\pi}{15}} \left(\frac{Y_2^{-1} - Y_2^{1}}{2} \right), \quad \sin\theta\cos\theta\sin\phi = \sqrt{\frac{8\pi}{15}} \left(\frac{Y_2^{1} + Y_2^{-1}}{2} \right)$$

$$\frac{xy + yz + xz}{r^2} = \frac{1}{4i}\sqrt{\frac{32\pi}{15}} \left(Y_2^{2} - Y_2^{-2} \right) + \frac{1}{2}\sqrt{\frac{8\pi}{15}} \left(1 - \frac{1}{i}\right) Y_2^{-1}$$
$$- \frac{1}{2}\sqrt{\frac{8\pi}{15}} \left(1 + \frac{1}{i}\right) Y_2^{1}.$$

$$\psi(x,y,z) = c(xy + yz + zx) e^{-\alpha r^2}.$$

all spherical harmonics are orthonormal.

$$\Rightarrow \langle \psi | \psi \rangle = c^2 \int_0^\infty r^6 \times \left(\frac{2\pi}{15} + \frac{2\pi}{15} + \frac{4\pi}{15} + \frac{4\pi}{15} \right) \times e^{-2\alpha r^2} dr$$

$$= \frac{12\pi}{15} c^2 \int_0^\infty dr\, r^6\, e^{-2\alpha r^2}$$

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$\langle\psi|\psi\rangle = 1 \Rightarrow c^2 = \dfrac{32\sqrt{2}\,\alpha^{7/2}}{3\pi^{3/2}}$

$\Rightarrow c = \sqrt{\dfrac{32\sqrt{2}\,\alpha^{7/2}}{3\pi^{3/2}}}$.

(c) Ang. momentum eq.

$\sin\theta \dfrac{\partial}{\partial\theta}\left(\sin\theta \dfrac{\partial Y_l^m}{\partial\theta}\right) + \dfrac{\partial^2 Y_l^m}{\partial\phi^2} = -l(l+1)\sin^2\theta\, Y_l^m.$

$L^2 = -\hbar^2\left(\dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial}{\partial\theta}\right) + \dfrac{1}{\sin^2\theta}\dfrac{\partial^2}{\partial\phi^2}\right)$

$\Rightarrow L^2 Y_l^m = \hbar^2 l(l+1) Y_l^m$.

$\underline{\underline{L^2\psi = 6\hbar^2\psi}}$ from above. as our $\underline{\underline{Y_l^m = Y_2^m}}$.

(d) $P(m=2) = |\langle Y_2^2|\psi\rangle|^2 = \dfrac{2\pi}{15} c^2 \int r^6 e^{-2\alpha r^2} dr = \dfrac{1}{6}$

$P(m=-2) = |\langle Y_2^{-2}|\psi\rangle|^2 = \dfrac{1}{6}$

$P(m=0) = 0$ ... (not Y_2^0).

$P(m=1) = P(m=-1) = \dfrac{1}{3}$.

$L_n = L\cdot\hat{n} = \sin\theta\cos\phi\, L_x + \sin\theta\sin\phi\, L_y + \cos\phi\, L_z.$

(f) L_+ L_- L_z basis.

in

$L_n = \dfrac{\sin\theta\, e^{-i\phi}}{2} L_+ + \dfrac{\sin\theta\, e^{i\phi}}{2} L_- + \cos\phi\, L_z.$

as $|l,m\rangle$ are orthonormal.

$\langle l,m|L_n|l,m\rangle = \hbar m\cos\phi.$

$L_n^2 = \dfrac{\sin^2\theta\, e^{-2i\phi}}{4} L_+^2 + \dfrac{\sin^2\theta\, e^{2i\phi}}{2} L_-^2 + \cos^2\phi\, L_z^2$

$+ \dfrac{\sin^2\theta}{4}(L_+L_- + L_-L_+) + \dfrac{\sin\theta\cos\phi\, e^{-i\phi}}{2}(L_+L_z + L_zL_+)$

$+ \dfrac{\sin\theta\cos\phi\, e^{-i\phi}}{2}(L_-L_z + L_zL_-].$

$\Rightarrow \langle l,m|L_n^2|l,m\rangle = \dfrac{\sin^2\theta}{4}\langle l,m|L_+L_- + L_-L_+|l,m\rangle$
$+ \cos^2\phi\,\langle l,m|L_z^2|l,m\rangle$

$= \dfrac{\sin^2\theta}{4}\hbar^2\left(l(l+1) - m(m-1) + l(l+1) - m(m+1)\right)$
$+ \hbar^2 m^2\cos^2\phi.$

$= \hbar^2\left(\dfrac{(2l(l+1) - m^2)}{4}\sin^2\theta + m^2\cos^2\phi\right)$

(5) (a) $\vec{L} = r \times p = x_i p_j \varepsilon_{ijk} = L_k$.

$L^2 = \sum (x_i p_j \varepsilon_{ijk} \hat{e}_k)(x_a p_b \varepsilon_{abc} \hat{e}_c)$

$= \sum x_i p_j x_a p_b \varepsilon_{kij} \varepsilon_{kab}$

$e_k \cdot e_c = \delta_{kc}$

using identity.

$= \sum x_i p_j x_a p_b \{\delta_{ia} \delta_{jb} - \delta_{ib} \delta_{ja}\}$.

$= \sum x_i x_i p_j p_j + \sum x_i (i\hbar - x_j p_j) p_i$

$= \sum x_i^2 \cdot \sum p_j^2 + i\hbar \sum x_i p_i - \sum x_i x_j p_i p_j$.

$= x^2 p^2 - (x \cdot p)^2 + i\hbar\, x \cdot p$

(b) $L^2 = x^2 p^2 - (x \cdot p)^2 + i\hbar\, x \cdot p$

$= \cancel{x} r p^2 + \hbar^2 r \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \hbar^2 r \frac{\partial}{\partial r}$,

$\left(\hat{p} = -i\hbar \frac{\partial}{\partial r}\right)$

$= r^2 p^2 + \hbar^2 r^2 \frac{\partial^2}{\partial r^2} + 2\hbar r \frac{\partial}{\partial r}$.

$L^2 |\alpha\rangle = r^2 p^2 |\alpha\rangle + \hbar^2 r^2 \frac{\partial^2}{\partial r^2} |\alpha\rangle + 2\hbar^2 r \frac{\partial |\alpha\rangle}{\partial r}$.

$\frac{1}{r^2} \langle x' | L^2 | \alpha\rangle = \langle x' | p^2 | \alpha\rangle + \hbar^2 \frac{\partial^2}{\partial r^2} \langle x' | \alpha\rangle$
$+ 2\hbar^2 \frac{1}{r} \frac{\partial}{\partial r} \langle x' | \alpha\rangle$.

$\frac{1}{2m} \langle x' | p^2 | \alpha\rangle = -\frac{\hbar^2}{2m} \cdot \left(\frac{\partial^2}{\partial r^2} \langle x' | \alpha\rangle + \frac{2}{r}\frac{\partial}{\partial r} \langle x' | \alpha\rangle - \frac{1}{\hbar^2 r^2} \langle x' | L^2 | \alpha\rangle\right)$.

⑥ ⓐ $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$, $[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k$.

$$\sigma_i\sigma_j = \frac{1}{2}\left([\sigma_i,\sigma_j] + \{\sigma_i,\sigma_j\}\right) = \delta_{ij} + i\varepsilon_{ijk}\sigma_k.$$

ⓑ ~~$a\cdot\sigma$~~ $a\cdot\sigma = \sum a_i\sigma_j\delta_{ij} = \underline{\sum a_i\sigma_i}$

$$(a\cdot\sigma)(b\cdot\sigma) = \sum a_i\sigma_i \sum b_j\sigma_j$$

$$= \sum_{ij} a_i b_j \sigma_i\sigma_j$$

$$= \sum_{ij} a_i b_j \left(\delta_{ij} + i\varepsilon_{ijk}\sigma_k\right).$$

$$\Rightarrow (a\cdot\sigma)(b\cdot\sigma) = a\cdot b + i(a\times b)\cdot\sigma.$$

⑦ ⓐ dimensions of $M_2(\mathbb{C})$ is 4 with field $\mathbb{C}$. $M_2(\mathbb{C})$ is 2×2 complex matrices vector space.

take basis $= \{\mathbb{1}, \sigma_1, \sigma_2, \sigma_3\}$.

to prove it is a basis, we show it is linearly independent.

$\mathbb{1}, \sigma_1, \sigma_2, \sigma_3 \in M_2(\mathbb{C})$.

$$c_0\mathbb{1} + c_1\sigma_1 + c_2\sigma_2 + c_3\sigma_3 = 0$$

$$\Rightarrow \begin{pmatrix} c_0 + c_2 & c_1 - ic_2 \\ c_1 + ic_2 & c_0 - c_3 \end{pmatrix} = 0$$

$$c_1 - ic_2 = c_1 + ic_2 = 0$$

$$\Rightarrow c_1 = c_2 = 0.$$

$$c_0 + c_2 = 0 = c_0 - c_3 = 0$$

$$\Rightarrow c_0 = c_3 = 0$$

$\Rightarrow \{\mathbb{1}, \sigma_1, \sigma_2, \sigma_3\}$ are linearly independent.

any $\rho \in M_2(\mathbb{C})$ can be written as $\rho = A\cdot\boldsymbol{\sigma}$

where $\boldsymbol{\sigma} = (\mathbb{1}, \sigma_i)$.

(2) $\rho = A + \sum B_i \sigma_i$, $\rho^\dagger = A^* + \sum B_j^* \sigma_j$

$\rho\rho^\dagger$ $\qquad (\sigma_j^\dagger = \sigma_j)$

$= (A + \vec{B}\cdot\sigma)(A^* + B^*\cdot\sigma)$

$= |A|^2 + \sum(A_i B_i^* + A_i^* B_i)\sigma_i + \sum B_i B_j^* \sigma_i \sigma_j)$

$= |A|^2 + |B|^2 + \sum(AB^* + A^*B)\sigma_j$

if ρ were unitary. $\rho\rho^\dagger = 1\!\!1$.

the $|A|^2 + |B|^2 = 1\!\!1$

& $AB_i^* + A^*B_i = 0$.

if ρ were Hermitian

$\rho = \rho^\dagger$

$\Rightarrow (A - A^*) + \sum(B_i - B_i^*)\sigma_i = 0$.

$A = A^*$, $B_i = B_i^*$.

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(8) (a). $[S_z, S_\pm] = \pm\hbar S_\pm$.

S_z follows ~~similar~~ same structure as L_z } as both are SU(2) generators)

so this is obvious from $[L_z, L_\pm] = \pm\hbar L_\pm$.

$[S_z, S_+] = \hbar^2[S - a^\dagger a, \sqrt{2S - a^\dagger a}\, a]$

$= -\hbar^2[n, \sqrt{2s-n}\; a]$

$= -\hbar^2\{\sqrt{2s-n}\,[n, a] + [n, \sqrt{2s-n}]\,a\}$.

$= -\hbar^2 \sqrt{2s-n}\,[n, a]$.

$= \hbar\sqrt{2s-n}\; a = \hbar S_+$.

Similarly for $[S_z, S_-]$.

$[S_+, S_-] = 2\hbar S_z$

$[S_+, S_-] = [\sqrt{2s-n}\; a, a^\dagger\sqrt{2s-n}]\hbar^2$

$= \hbar^2(\sqrt{2s-n}\,(a^\dagger[a, \sqrt{2s-n}] + [a, a^\dagger]\sqrt{2s-n})$

$+ a^\dagger[\sqrt{2s-n}, \sqrt{2s-n}] + [\sqrt{2s-n}, a^\dagger]\sqrt{2s-n})a\}$

$= \hbar^2((2s-n)(n+1) - a^\dagger a(2s-n+1))$.

$= \hbar^2(2s-n) = 2\hbar S_z$.

$S_x = \frac{S_+ + S_-}{2}$, $S_y = \frac{S_+ - S_-}{2i}$

$[S_z, S_\pm] = \pm\hbar S_\pm$

similarly from above we can show

$[S_z, S_x] = i\hbar S_y$, $[S_z, S_y] = -i\hbar S_x$

$[S_+, S_-] = 2\hbar S_z$, $[S_x, S_y] = i\hbar S_z$

$[S_i, S_j] = i\hbar \epsilon_{ijk} S_k$

(b) $S^2 = S_x^2 + S_y^2 + S_z^2$

$S_-S_+ = \cancel{S_x - S_+}\ (S_x - iS_y)(S_x + iS_y)$

$= S_x^2 + S_y^2 + i(S_xS_y - S_yS_x)$

$= S^2 - S_z^2 - \hbar S_z$

$S^2 = S_z^2 + \hbar S_z + S_-S_+$

and $S_-S_+ = \hbar^2 a^\dagger(2s-n)a = \hbar^2 n(2s-n+1)$

$\Rightarrow S^2 = \hbar^2(s-n)^2 + \hbar^2(s-n) + \hbar^2(n)(2s-n+1)$

$= \hbar^2 s(s+1)$

(9) (a) $I = \int_0^\infty \frac{e^{-xt/(1-t)}}{(1-t)^{1+\beta}} \frac{e^{-xs/(1-s)}}{(1-s)^{1+\beta}} e^{-x} x^\beta \, dx$

$= \int_0^\infty \frac{e^{-\frac{1-ts}{(1-t)(1-s)}x} \, x^\beta}{(1-t)^{1+\beta}(1-s)^{1+\beta}} \, dx$

let $y = \frac{(1-ts)x}{(1-s)(1-t)}$, $dy = \frac{1-ts}{(1-s)(1-t)} dx$.

$\Rightarrow \int_0^\infty \frac{e^{-y} y^\beta \, dy}{(1-ts)^{1+\beta}} = \frac{1}{(1-ts)^{1+\beta}} \int_0^\infty e^{-y} y^\beta \, dy$

$$= \frac{\Gamma(\beta+1)}{(1-ts)^{\beta+1}} = \frac{\beta!}{(1-ts)^{\beta+1}}$$

$$\left(\frac{1}{1-ts}\right)^{\beta+1} = \sum_{i=0}^{\infty} \frac{(i+\beta)!}{i!\,\beta!} (ts)^i$$

$$I = \sum_{i=0}^{\infty} \frac{(i+\beta)!}{i!} (ts)^i$$

$$\frac{e^{-xt/1-t}}{(1-t)^{1+\beta}} = \sum_{\alpha=0}^{\infty} L_\alpha^\beta(x) t^\alpha$$

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$$I = \sum_{\alpha=0}^{\infty} \sum_{\alpha'=0}^{\infty} \int_0^\infty \left(L_\alpha^\beta(x) \cdot L_{\alpha'}^\beta(x)\, e^{-x} x^\beta dx\right) t^\alpha s^{\alpha'}$$

$$\Rightarrow \int e^{-x} x^\beta L_\alpha^\beta(x) \cdot L_{\alpha'}^\beta(x)\, dx = \frac{(\alpha+\beta)!}{\alpha!} \delta_{\alpha\alpha'}$$

(b) $$\int_0^\infty r^2 (R_{nl}\, r)^2 dr = \frac{c^2}{k^2} \int r (kr)^{2l+1} e^{-kr} \left(L_{n-l-1}^{2l+1}(kr)\right)^2 d(kr)$$

$$(2(n-l-1)+2l+1+1)\, L_{n-l-1}^{2l+1}(kr).$$

$$r\, L_{n-l-1}^{2l+1}(kr) = \frac{1}{k}\Big(2(n-l-1)+2l+1+1)\, L_{n-l-1}^{2l+1}(kr) - (n+l)\, L_{n-l-2}^{2l+1}(kr) - (n-l) L_{n-l}^{2l+1}(kr)\Big)$$

$$\frac{c^2}{k^2} \int^\infty (kr)^{2l+1} e^{-kr} \frac{2n}{k} \left(L_{n-l-1}^{2l+1}(kr)\right)^2 d(kr).$$

$$= \frac{2nc^2}{k^3} \frac{(n+l)!}{(n-l-1)!}$$

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$$\Rightarrow c = \left(\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n(n+l)!}\right)^{1/2}$$

$$\langle r \rangle = \int_0^\infty r \cdot (R_{nl}(r))^2 r\, dr = \frac{c^2}{k^3} \int (kr)^{2l+1} \frac{e^{-kr}}{k} \big(-(4n^2)(n-l)(n+l+1) + (n+l)(n-l-1)\big) \left(L_{n-l-1}^{2l+1}\right)^2 d(kr).$$

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$$= \frac{k^3 (n-l-1)!}{2n(n+l)!\,k^n} (6n^2 - 2l^2 - 2l) \cdot \frac{(n+l)!}{(n-l-1)!}$$

$$= \frac{1}{2n}(3n^2 - l(l+1)) = \frac{a_0}{2}(3n^2 - l(l+1))$$

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tion have already been cancelled
n carried out for the day

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(10) (a) ~~Fy~~ Feynman - Hellmann Lemma

$$\frac{\partial E(\lambda)}{\partial \lambda} = \frac{\partial}{\partial \lambda} \langle \psi_n(\lambda) | H(\lambda) | \psi_n(\lambda) \rangle$$

$$= \langle \frac{\partial \psi_n}{\partial \lambda} | H | \psi_n \rangle + \langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_n \rangle + \langle \psi_n | H | \frac{\partial \psi_n}{\partial \lambda} \rangle$$

$$= E_n \langle \frac{\partial \psi_n}{\partial \lambda} | \psi_n \rangle + \langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_n \rangle + E_n \langle \psi_n | \frac{\partial \psi_n}{\partial \lambda} \rangle$$

$$= E_n \left(\langle \frac{\partial \psi_n}{\partial \lambda} | \psi_n \rangle + \langle \psi_n | \frac{\partial \psi_n}{\partial \lambda} \rangle \right) + \langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_n \rangle$$

$$= E_n \underbrace{\frac{\partial}{\partial \lambda} (\langle \psi_n | \psi_n \rangle)}_{0} + \langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_n \rangle$$

$$= \left\langle \frac{\partial H(\lambda)}{\partial \lambda} \right\rangle$$

$$\Rightarrow \frac{\partial E(\lambda)}{\partial \lambda} = \left\langle \frac{dH(\lambda)}{d\lambda} \right\rangle$$

(B) $E_n = \frac{-Z^2 m e^4 \hbar}{32\pi^2 \epsilon_0^2 \hbar^2 n^2} = \underline{\frac{-Z^2 \hbar^2}{2 m a_0^2 n^2}}$

$H = \frac{p^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 r}$ Take $\lambda = e$:

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$\frac{dE}{de} = \frac{-2Z^2\hbar^2}{m e a_0^2 n^2}$, $\langle \frac{\partial H}{\partial e} \rangle = \langle -\frac{2Ze^2}{4\pi\epsilon_0 r} \rangle$

$= -\frac{2Ze}{4\pi\epsilon_0} \langle \frac{1}{r} \rangle$

$\Rightarrow \langle \frac{1}{r} \rangle = \frac{Z}{n^2 a_0}$

take $\lambda = m$, $\frac{\partial E}{\partial m} = \frac{-Z e^2}{32\pi\epsilon^2 \hbar^2 n^2} = \frac{-Z^2\hbar^2}{2m^2 a_0^2 n^2}$

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$\langle \frac{\partial H}{\partial m} \rangle = -\frac{1}{2m^2} \langle p^2 \rangle$, $\Rightarrow \langle p^2 \rangle = \frac{Z^2 m^2 c^2 \alpha^2}{n^2}$

$\boxed{\alpha: \text{fine struct const} = \frac{\hbar}{m c a_0}}$

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(c) $\cancel{\langle E \rangle}$ $\langle T \rangle = \frac{\langle p^2 \rangle}{2m}$

$= \frac{Z^2 m c^2}{2n^2} \frac{\hbar^2}{m^2 c^2 a_0^2} = \frac{Z^2 \hbar^2}{2n^2 m a_0^2}$

$= -\frac{1}{2} \frac{\hbar^2}{m} \frac{Z m e^2}{4\pi\varepsilon_0 \hbar^2} \frac{Z}{n^2 a_0} = -\frac{1}{2} \langle V \rangle.$

Virial theorem.