# End-Sem Exam

December 03, 2021 @10:00 PM

Duration: 120 Minutes (+ 30 Minutes for other formalities)

- Login in to usual Google meeting and remain logged in during the exam.
- Keep you video on
- Answer all the questions.
- All questions carry equal marks.
- Answer questions explicitly, with all the necessary step.
- Upload the answer script as PDF file to welearn.
- You need not upload question paper.
- Do not use improper methods
- Have a good day.

**[1 of 6]** Find the Cauchy principal value of the integrals

1. $$\int_0^\infty \frac{1-\cos x}{x^2} dx \tag{1}$$
2. $$\int_{-\infty}^\infty \frac{1}{x^2-2x+2} dx \tag{2}$$

**[2 of 6]** Verify if the following function is harmonic and if it is harmonic find the harmonic conjugate function

$$U(x,y) = e^x \left(x \cos y - y \sin y\right) \tag{3}$$

**[3 of 6]** Find the inverse Fourier transform, $\mathcal{F}^{-1}\left(\tilde{g}\right)$, of the function given by,

$$\tilde{g}(f) = \frac{2}{1+f^2} \tag{4}$$

**[4 of 6]** For the differential equation in the following form

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right) y = 0 \tag{5}$$

1. find the property of singularity at point $x = 0$
2. find the property of singularity at point $x = \infty$
3. find the indicial-equation at $x = 0$
4. find the recurrence relation

**pts, . . . there is something more on the next page!**

**[5 of 6]** A function of two variables, $\phi(x, y)$ satisfies the Laplace equation in a rectangular region, *i.e.*

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0\,, \tag{6}$$

with the boundary condition

$$\begin{aligned} \phi(x = 0, y) &= 0\,, \\ \phi(x, y = 0) &= 0\,, \\ \phi(x = a, y) &= 0\,, \\ \phi(x, y = b) &= V\,. \end{aligned} \tag{7}$$

Find the solution for $\phi(x, y)$ in the region $0 \le x \le a$ and $0 \le y \le b$.

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**[6 of 6]** Express the following function in terms of the Fourier series in the interval $[-1, 1]$,

$$f(x) = \begin{cases} = 1 & -1 \le x \le -\frac{1}{2} \\ = 0 & -\frac{1}{2} < x < \frac{1}{2} \\ = 1 & \frac{1}{2} \le x \le 1 \end{cases} \tag{8}$$

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**That's all floks!**