Assignment 2

1.) A particle in the infinite square well from $x = 0$ to $x = a$ has the following normalized initial wave function,

$$\psi(x,0) = A\,x\,(a-x)$$

(a) Compute the value of A and plot the wavefunction.
(b) Argue intuitively, which stationary state of the Hamiltonian it closely resembles to and calculate the probability of finding the particle in that state.
(c) Compute the expectation value of the energy.
(d) Compute Δx, Δp and verify the Heisenberg uncertainty relation.

2.) Consider the following potential with finite depth V_0 between $x = -a/2$ to $x = a/2$,

$$V(x) = \begin{cases} 0 & -\infty \leq x < -a/2 \\ -V_0 & -a/2 \leq x \leq a/2 \\ 0 & a/2 < x \leq \infty \end{cases} \tag{1}$$

(a) Consider the case when the width of the potential decreases and the depth increases, such that $aV_0 = \alpha$ (where $\alpha > 0$). Find out the ground state of this potential and show that it corresponds to bound state of an attractive δ function, $V(x) = -\alpha\delta(x)$.
(b) Assuming that a is now fixed, but the depth is decreasing, find out the critical value V_c, below which there will be no odd wavefunction.
(c) Now consider the case that $V(x) = +V_0$ from $x = -a/2$ to $x = a/2$ in Eq.1. Then, calculate the Transmission and reflection coefficients for the case when energy $E < V_0$.

3.) Consider the following double delta function potential

$$V(x) = -\alpha[\delta(x+a) + \delta(x-a)] \tag{2}$$

where, α and a are positive constant.
(a) Find out the bound states and their corresponding energies for $\alpha = \hbar^2/ma$.
(b) Roughly sketch the wavefunction of the bound states found in (a) and discuss their associated symmetries.

4.) A particle of mass m moving in a 1D potential,

$$V(x) = \begin{cases} \infty & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2 & x > 0 \end{cases} \tag{3}$$

Without explicitly solving, find out the allowed energy levels and wavefunctions of the system.

5.) If the n^{th} normalized eigenstate of the linear harmonic potential is given by $\psi_n = CH_n(y)e^{-y^2/2}$, where $H_n(y)$ is the n^{th} Hermite polynomial. The generating function for the Hermite polynomial is given as, $G(t,x) = e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n(y)t^n/n!$. Then
(a) Compute the width Δx of the n^{th} eigenstate.
(b) Compute the expectation value $\langle p^2 \rangle$ for the n^{th} eigenstate. (Hint: you may use the following recursion relation, $H_n^{''}(y) = 4nyH_{n-1}(y) - 2nH_n(y)$). Check if the Heisenberg uncertainty relation

remains valid for the n^{th} eigenstate.

6.) Using the energy eigenstates as basis of 1D harmonic oscillator, compute the matrix elements of position operator, $x_{mn} = \int \psi_m^\star \hat{x} \psi_n dx$, momentum operator, $p_{mn} = \int \psi_m^\star \hat{p} \psi_n dp$.