Assignment Vid[7]

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1 Fields near the intersection of two conducting planes:

The electric potential near the intersection of two conducting plates follows the 2D laplace equation, $\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$ with boundary conditions $\phi(0) = \phi(\theta_0) = \text{constant} = 0$.

In our case, we are only restricted to $0 \leq \theta \leq theta_0$ and the potential must obey the boundary conditions, the solution becomes.

$$\phi(r,\theta) = \sum a_m r^{m\pi/\theta_0} \sin(m\pi\theta/\theta_0)$$

For close to the origin, $\phi(r,\theta) \approx r^{\pi/\theta_0} \sin(\pi\theta/\theta_0)$

The conjugate of ϕ, say ψ acts similar to the electric field, if $\phi = r^{\pi/\theta_0} \sin(\pi\theta/\theta_0)$ then $\psi = r^{\pi/\theta_0} \cos(\pi\theta/\theta_0)$.
This ψ is the same as the angular electric field like $E_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} \propto r^{\pi/\theta_0 - 1} \cos(\pi\theta/\theta_0)$

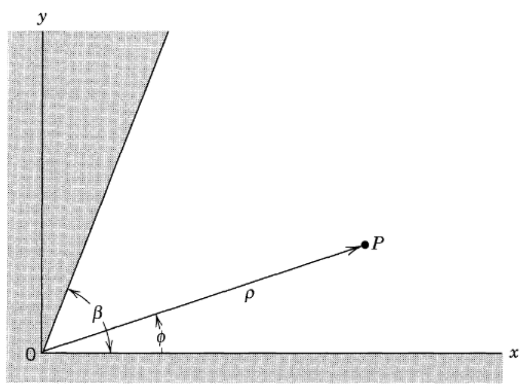

Figure 1: Two conducting plates in 2D with angle θ_0

The physical significance here is that the equipotential surfaces run parallel to the plates while the field is perpendicular to them. The potentials become more densely packed near the intersection between the plates or in case of $\theta_0 \geq \pi$ ”edges”. We can understand this by finding out that the charge density is greater near such edges or corners is.

$$\sigma(r) = \varepsilon_0 E_\phi \propto r^{\pi/\theta_0 - 1}$$

This shows us that for greater θ_0, the charge density will fall down before rising again for $\theta_0 \geq \pi$ where the charge accumulates at the edge does increasing the field and therefore the number of potential lines.

Below we have the ϕ and ψ curves for $\theta_0 = \pi/2, \pi, 3\pi/2, 2\pi$ for perpendicular plates,a flat plate, a perpendicular ”wedge” and a half plate.

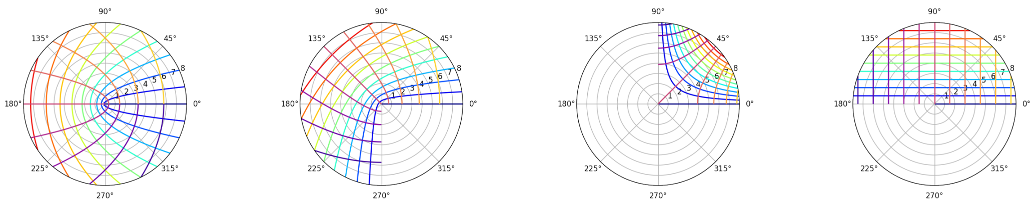

Figure 2: $\theta_0 = 2\pi, 3\pi/2, \pi/2, \pi$

The one for $\theta_0 = \pi$ is our usual plate, with perpendicular electric fields moving till infinity and the flat lines being equipotential surfaces.