Assignment 1

1.) Calculate the number of oscillation modes of electromagnetic radiation filled in a cavity of volume V in the frequency range ν to $\nu + d\nu$. Using the Planck's distribution of black body radiation , derive the Stefan-Boltzmann law.

2.) (a) Compute the Fourier transform of the following potential:

$$V(x) = \begin{cases} 0 & -\infty \leq x \leq -a/2 \\ V_0 & -a/2 \leq x \leq a/2 \\ 0 & a/2 \leq x \leq \infty \end{cases} \tag{1}$$

(b) Discuss the limiting case when $a \to 0$, $V_0 \to \infty$, but $V_0 a = 1$, and interpret your results.

3.) (a) Compute the Fourier transform of the Gaussian distribution function $P_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$ in k space.
(b) Using the definition $\langle g(x) \rangle = \int_{-\infty}^{\infty} g(x) P_\sigma(x) dx$, calculate the standard deviation, $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$
(c) From the Fourier transform of $P_\sigma(x)$ obtain $\Delta k = \sqrt{\langle k^2 \rangle - \langle k \rangle^2}$.
(d)Verify that $\Delta x \Delta k \geq 1/2$.

4.) Consider the system of rigid rotor, which has a moment of inertia I and angular momentum L_z along the z-axis.
(a) Solve the following Schrödinger's equation:

$$\frac{\hat{L}_z^2}{2I}\psi = E\psi \quad \left(\text{where} \quad \hat{L}_z = \frac{\hbar}{i}\frac{\partial}{\partial\theta}\right) \tag{2}$$

with the boundary condition, $\psi(\theta + 2\pi) = \psi(\theta)$ and obtain the discrete energy levels.
(b) Also use the Bohr quantization rule $\int p dq = 2\pi n\hbar$, to calculate the allowed energies of the system.

5.) (a) Use Bohr quantization rule to find out the allowed energies for the potential $V(r) = V_0(r/a)^k$ in two dimensions.
(b) Check that for $k = 2$, you recover the same expression of energy as that of the harmonic oscillator.

6.) Using Heisenberg uncertainty relation, estimate the ground state energy of the 1D harmonic oscillator.