Adwart 19MS181
CT-2 Rough

1) $H = \frac{p^2+q^2}{2}$, $A = \frac{p+iq}{\sqrt{2}}$

$\{A, H\} = \frac{\partial A}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial H}{\partial q}$

$= \frac{i}{\sqrt{2}} p - \frac{1}{\sqrt{2}} q = \frac{ip-q}{\sqrt{2}}$

$= \frac{i(p+iq)}{\sqrt{2}} = iA.$

2) $q = \frac{\dot{q}^2 - q^2}{2} \Rightarrow H = \frac{p^2+q^2}{2}$

Harmonic oscillator.

$\Rightarrow \ddot{q} = -q \rightarrow q \propto e^{it}$ or e^{-it}

$p \propto ie^{it}$ or $-ie^{it}$

$\Rightarrow p+iq = (i+i)e^{it}$

$\Rightarrow \ln(p+iq) = \ln(2i) + it$

$\Rightarrow \ln(p+iq) - it = \ln(2i)$ is constant.

or if $q \propto e^{-it} \Rightarrow p = -ie^{it}$

$\Rightarrow p - iq = (-i-i)e^{-it} = \ln(-2i) - it$

$\Rightarrow \ln(p-iq) + it$ is also conserved

Q6 $\frac{\partial L}{\partial \dot q} = \frac{m}{2}(2\dot q \sin^2 \omega t + q\omega \sin(2\omega t))$

$\mathcal{H} = m\dot q^2 \sin^2 \omega t + \frac{m}{2} \dot q q \sin(2\omega t)$
$- \frac{m}{2}(\dot q^2 \sin^2 \omega t + \dot q q \sin(2\omega t) + q^2\omega^2)$

$= \frac{m}{2}\dot q^2 \sin^2 \omega t - \frac{m}{2} q \omega^2$

$p = \frac{m}{2}(2\dot q \sin \omega t + q\omega \sin 2\omega t)$

$\frac{p - \frac{mq}{2}\omega \sin(2\omega t)}{m \sin^2 \omega t} = \dot q$

$\Rightarrow H = \frac{m}{2}\frac{(p - \frac{q\omega m}{2}\sin(2\omega t))^2}{2m^2 \sin^4(\omega t)} - \frac{q^2\omega^2}{2}$

Q1 $T = \int \frac{ds}{dx} = \frac{1}{a}\int_0^a \frac{\sqrt{1+y'^2}}{x}\, dx$

Brachistochrone

Q4 it's a projectile motion thingy
so it will be parabola.

Q7 H is conserved $H = p_x^2 + \dots$

and also

$L_z = (r \times p)_z = x p_y - y p_x$.
$H = \frac{1}{2m}(p_x^2 + p_y^2 + x^2 + y^2 + 2(x p_y - y p_x))$
$= \frac{1}{2m}(p_x^2 + p_y^2 + x^2 + y^2) + \frac{L_z}{m}$ conserved.

L_z is conserved