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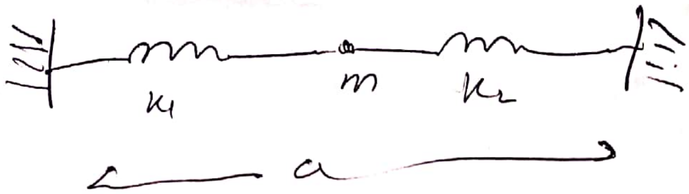
19MS151

CT – 4

PH3101

Q3




(a) Lagrangian
(i) $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}k_1(x - b_1)^2 - \frac{1}{2}k_2(a - x - b_2)^2$

$b_1$ & $b_2 = 0$ are equilibrium distances of particle ~~from~~.

$\Rightarrow L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - \frac{1}{2}k_2(a - x)^2$

$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x} \Rightarrow \dot{x} = \frac{p_x}{m}$.

Hamiltonian $\Rightarrow$ $H = \dot{x}p_x - L$

$= \frac{p_x^2}{m} - \frac{p_x^2}{2m} + \frac{k_1 x^2}{2} + \frac{k_2(a-x)^2}{2}$

$= \frac{p_x^2}{2m} + \frac{k_1 x^2}{2} + \frac{k_2(a-x)^2}{2}$

(ii) Hamiltonian does not explicitly dependent on time, so energy is conserved.

(b) $Q = q - b\sin\omega t$, $b = \frac{k_2 a}{k_1 + k_2}$

take $q = Q + b\sin\omega t$, $\dot{q} = \dot{Q} + b\omega\cos\omega t$.

replace $x$ with $q$ in above

$L = \frac{m}{2}(\dot{Q} + b\omega\cos\omega t)^2 - \frac{k_1}{2}(Q + b\sin\omega t)^2 - \frac{k_2}{2}(a - Q - b\sin\omega t)^2$

$P = \frac{\partial L}{\partial \dot{Q}} = m(\dot{Q} + b\omega\cos\omega t)$

$\Rightarrow \dot{Q} = \frac{P}{m} - b\omega\cos\omega t$

$H = \dot{Q}P - L = \frac{P^2}{m} - b\omega\cos\omega t - \frac{m}{2}\left(\frac{P}{m} - b\omega\cos\omega t + b\omega\cos\omega t\right)^2$
$- \frac{k_1}{2}(Q + b\sin\omega t)^2 - \frac{k_2}{2}(a - Q - b\sin\omega t)^2$

Q3 continued.

$$H = \frac{p^2}{2m} - b\omega p \cos\omega t - \frac{k_1}{2}(Q + b\sin\omega t)^2 - \frac{k_2}{2}(a - Q - b\sin\omega t)^2.$$

New Hammy is not conserved due to new time dependent coordinate system

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# Q4

Corriolis Force

$$F_C = -2m\vec{\omega}\times\vec{v}$$

let $\lambda$ be altitude.

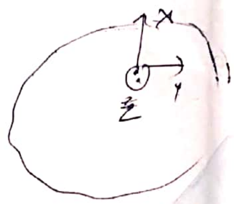
$$\vec{\omega} = \omega\cos\lambda\,\hat{x} + \omega\sin\lambda\,\hat{z}.$$

($x, y, z$ coordinate system is fixed to earth)

(a) for $\vec{v} = v_z\hat{z}$

$$\vec{F_c} = -2m\omega\times v = -2m\omega v_z(\hat{x}\cos\lambda + \hat{z}\sin\lambda)\times\hat{z}$$

$$= 2m\omega\cos\lambda\, v_z\,\hat{y}.$$




$V_z = V_{oz} - gt$ -- from the eq of motion.

$Z(t) = z_0 + V_{oz}t - \frac{gt^2}{2}$ for vertical motion.

y-direction motion.

$$m\ddot{y} = 2m\omega\cos\lambda\,(V_{oz} - gt)$$

$$\Rightarrow \dot{y} = 2m\omega\cos\lambda\,(V_{oz}t - gt^2/2) + V_{oy}.$$

$$\Rightarrow y = y_0 + V_{oy}t + 2m\omega\cos\lambda\left(\frac{V_{oz}t^2}{2} - \frac{gt^3}{6}\right).$$

It will reach max height in z-direction at $t = \frac{V_{oz}}{g}$, $z_0 = 0$ as base.

$$Z_m = V_{oy}\cdot\frac{V_{oy}}{g} - \frac{V_o^2}{2g} = \frac{V_{oz}^2}{2g}. \text{ -- Max height}$$

time of flight $\rightarrow$ $t = \frac{2V_{oz}}{g}$.

$$\Rightarrow \Delta y = 2m\omega\cos\lambda\left(\frac{V_{oz}}{2}\left(\frac{2V_{oz}}{g}\right)^2 - \frac{g}{6}\left(\frac{2V_{oz}}{g}\right)^3\right)$$

Deflection

$$= \frac{4}{3}\omega\cos(\lambda)\cdot\frac{V_{oz}^3}{g^2}.$$

$$= \frac{8\sqrt{2}}{3}\cdot\cos\lambda\cdot Z_m\sqrt{\frac{\omega^2 Z_m}{g}}$$

(b) Free fall from z-max,
time for free fall $\Rightarrow$

$$t = \sqrt{\frac{2z_m}{g}}$$

& deflection would be

$$\Delta y = -\frac{1}{3}\omega\cos\lambda\, g t^{3/2} - \frac{1}{3}\omega\cos\lambda\, g\left(\frac{2z_m}{g}\right)^{3/2}$$

$$\Rightarrow \Delta y = -\frac{2}{3}\sqrt{2}\cos\lambda\, z_m\sqrt{\frac{\omega^2 z_m}{g}}$$

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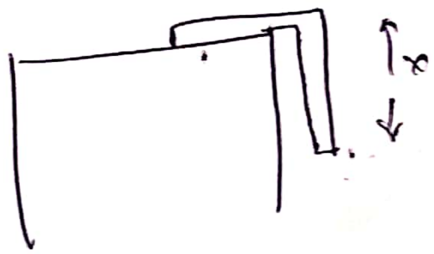
(c) Free falling particle has 4 times less deflection than the one which is thrown upwards.

Q1




(a) Total mass of rope be $M$.
& linear density $\mu = L/M$.

let generalised coordinate be length of rope hanging. '$x$'.

$T = \frac{1}{2} M \dot{x}^2 = \frac{1}{2} \mu L \dot{x}^2$.

Both forces, normal and weight are vertical.
if we move the rope slighty (virtual displacement). the forces displacement is perpendicular to due to and the work would be 0. normal force.
Work due to gravity -

$\Rightarrow \delta W = \mu g x \, \delta x = Q \, \delta x$

$Q = \mu g x$ .. generalised force

.. using this $\left( \frac{d}{dt} \frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x} = Q \right)$

$\Rightarrow$ Eq of motion. $\Rightarrow \mu g x = \mu L \ddot{x}$

$\rightarrow \ddot{x} - \frac{g}{L} x = 0$. exponential solution or hyperbolic cos or sin.

$\Rightarrow x = A \cosh(\omega t) + B \sinh(\omega t)$ $\quad \omega = \sqrt{\frac{g}{L}}$.

$x(0) = l$ at start, $\dot{x}(0) = 0$ at beginning.

$\Rightarrow x(t) = l \cosh\left(t \sqrt{\frac{g}{l}}\right)$

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(b) Here reaction force has both vertical & horizontal component.

At equilibrium, the rope on table has weight. $P$.

reaction force. $R$.

and rope weight (hanging) (tension). $= T$.

$P + R + T = 0$.

$R$ on horizontal $\Rightarrow R_h$

vertical comp $\Rightarrow R_v$.

$$R_h = T, \quad R_v = P = \underline{\mu (L - l) g.}$$

$\Rightarrow T = \mu l g.$

$\Rightarrow$ condition for static friction.

$R_h \leq f R_v \Rightarrow l \leq f(L - l)$

$\Rightarrow$ critical length $l_0 \cancel{\geq} l$

$$\Rightarrow \quad \underline{l_0 = \frac{f}{1+f} L.}$$

This time virtual work would have added horizontal reaction.

$\delta W = (\mu g x - R_h) \delta x$

$R_h = f R_v$ for dynamical

$\Rightarrow R_h = f \mu g (L - x)$

$\Rightarrow Q = \mu g ((1+f)x - fL).$

Eq of motion. $\mu L \ddot{x} = R$

$$\Rightarrow \ddot{x} = \frac{g}{L}((1+f)x - fL) \Rightarrow \ddot{x} = \frac{g}{L}(1+f)(x - l_0)$$

$$\Rightarrow \underline{x(t) - l_0} = \overline{(l - l_0) \cosh \left( t \sqrt{\frac{g(1+f)}{L}} \right)} \quad \underline{\text{for } l > l_0}$$

as last time