Adwait Naravane 19MS151

PH3104 - PS-4

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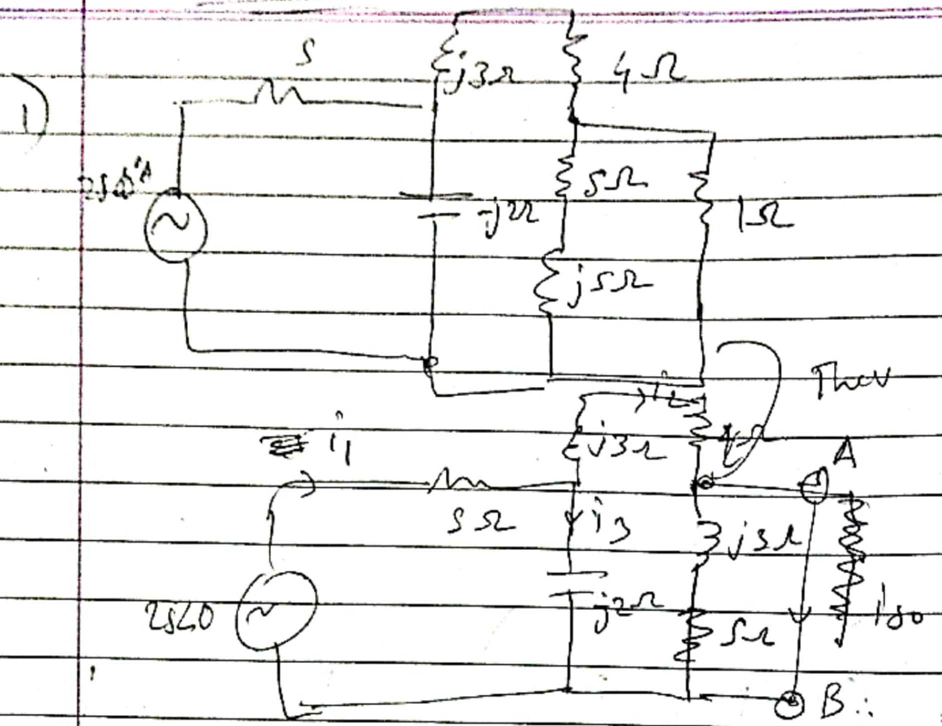
1)



$i_1 = i_2 + i_3,$

$i_2(3j+4+5+5j) = -2 i_3 j$

$i_3 = i_2(9j-8)$

$i_1 = i_2(1-4+\frac{9}{2}j)$

$= 3i_2(-1+\frac{3}{2}j)$

$V_{Th} = \frac{5}{8}(-3+7j)i_1$

$i_2 = \frac{i_1}{-3+j\frac{9}{2}} = \frac{4i_1(-3-9j/2)}{117}$

$V_{Th} = i_2(5+5j) = \frac{-30 i_1}{117}(1+3j)$

$V_A = V_B \Rightarrow i_2 = i_{50}$

$(4+3j) i_{50} = -i_3(2j).$

$i_1 = i_3 + i_{50} = i_{50}(-\frac{1}{2}+2j)$

$\Rightarrow i_{50} = \frac{i_1(-1/2-2j)}{(\frac{1}{4}+4)} = \frac{-4i_1}{17}(\frac{1}{2}+2j)$

$\Rightarrow R_{Th} = \frac{V_{Th}}{I_{50}} = \frac{30/117\,(1+3j)}{4/17\,(\frac{1}{2}+2j)}$

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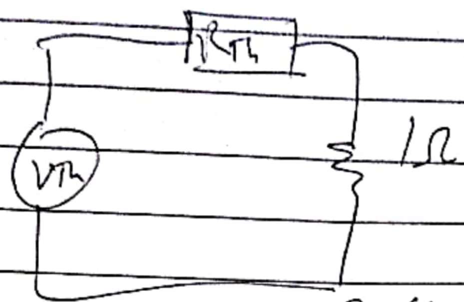
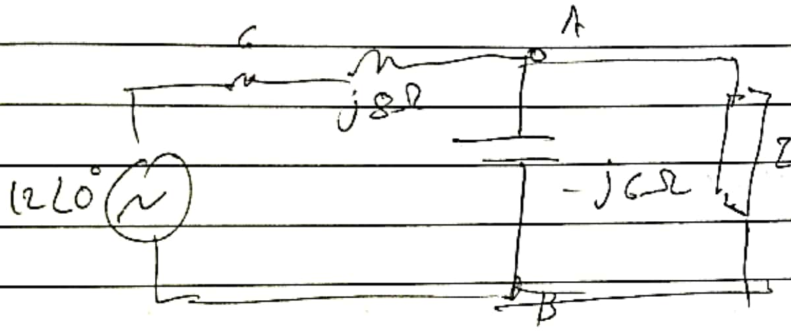
$R_{Th} = \frac{15}{117}(13-j)$



$\Rightarrow I = \frac{V_{Th}}{R_{Th}+1} = \frac{-30(1+3j)}{312-13j} = \frac{-30(1+3j)(312+13j)}{(312)^2+13^2}$

$Re(I) = \frac{-30\times(312-45)}{312^2+15^2} = 0.082A.$

2)



Max power = ?

$Z_{AB} = \frac{Z_L(-j6)}{Z_L-6j}$

$i = \frac{12}{6+8j-\frac{6Z_Lj}{Z_L-6j}}$

$i = \frac{12(Z_L-6j)}{(6Z+48)+j(2Z_L-36)}$

$i_0 = Re(i) = \frac{12\sqrt{Z_L^2+36}}{\sqrt{(6Z+48)^2+(2Z_L-36)^2}}$

$\therefore$ Current through Z_L

$\Rightarrow \frac{i\times -6j}{Z_L-6j} = |i_2| = \frac{6\times12}{\sqrt{(6Z_L+48)^2+(2Z_L-36)^2}}$

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Power $P(Z_L) = \frac{1}{2} i_2^2 Z_2 = \frac{36(12)^2 Z_L}{2((6Z_L+48)^2 + (2Z_L-36)^2)}$

$\frac{dP}{dZ} = 0$

$\Rightarrow \frac{1}{(6Z+48)^2+(2Z-36)^2} = \frac{2Z \times (6 \times (6Z+48) + 2(2Z-36))}{(6Z+48)^2 + (2Z-36)^2}$

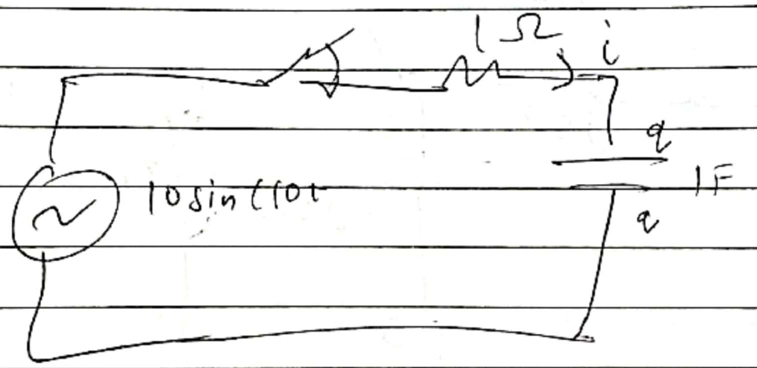
$\Rightarrow Z^2 = 90$

$\Rightarrow Z = 3\sqrt{10}$

Max power for $Z_L = 3\sqrt{10}\ \Omega$ impedence

Max power = 1.752 W

Q3



$\Rightarrow 1 \frac{dq}{dt} + \frac{q}{1} = 10\sin(10t)$

Laplace trans.

$\mathcal{L}\left(\frac{dq}{dt} + q\right) = \mathcal{L}\left(10\sin(10t)\right)$

$\Rightarrow s\mathcal{L}(q) - \underbrace{q(0)}_{0} + \mathcal{L}(q) = \frac{100}{s^2+100}$

$\Rightarrow \mathcal{L}(q) = \frac{100}{(s+1)(s^2+100)}$

$= \frac{100}{101}\left(\frac{1}{s^2+100} + \frac{1}{s+1} - \frac{s}{s^2+100}\right)$

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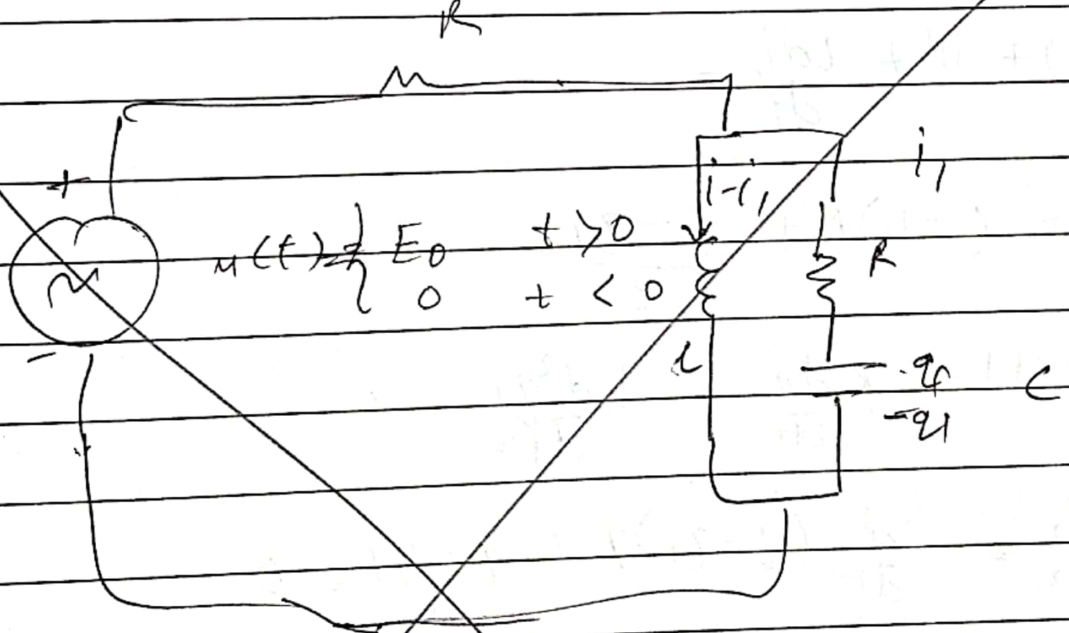
$$q = \mathcal{L}^{-1}\left(\frac{100}{101}\left(\frac{1}{s^2+100} + \frac{1}{s+1} - \frac{s}{s^2+100}\right)\right)$$

$$q(t) = \frac{100}{101}\left(\sin(10t) + 10e^{-t} - 10\cos(10t)\right)$$

$$i = \frac{dq}{dt} = \frac{10}{101}\left(10\cos(10t) - 10e^{-t} + 100\sin(10t)\right)$$

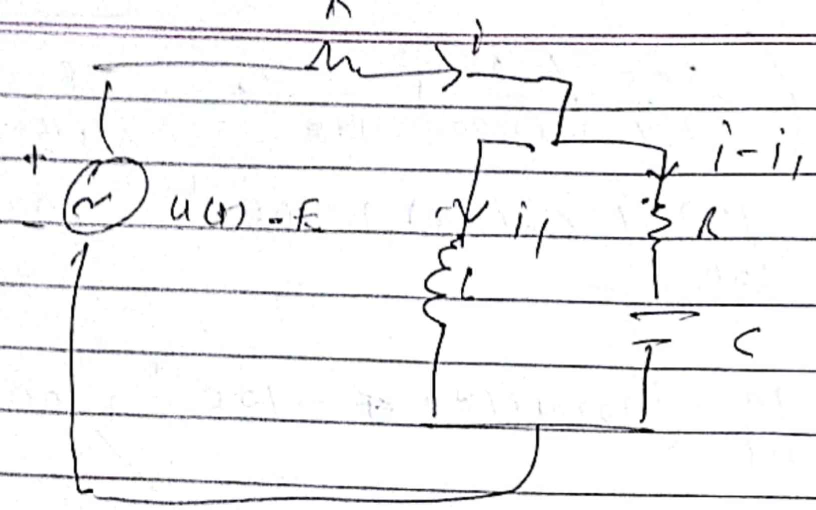


$$(i + i_1)R + \frac{q_1}{C} = u(t) - E_0$$

$$\frac{d}{dt}(i + i_1)R + \frac{i_1}{C} = 0$$

$$i_1 R + \frac{q_1}{} = L\frac{di_2}{} = L\frac{d(i - i_1)}{}$$

Q4



$$-u(t) + iR + L\frac{di_1}{dt} =$$

$$L\frac{di_1}{dt} = (i-i_1)R + \frac{q-q_1}{c}$$

$$\Rightarrow -u(t) + R\frac{dq}{dt} + L\frac{d^2q_1}{dt^2} = 0$$

Laplace

$$L\frac{d^2q_1}{dt^2} = \frac{d}{dt}(q-q_1)R + \frac{q-q_1}{c} = 0$$

$$-\frac{E_0}{s} + Rs\,\mathcal{L}(q) + Ls^2\,\mathcal{L}(q_1) = 0$$

$$Ls^2\,\mathcal{L}(q_1) = Rs\,\mathcal{L}(q-q_1) + \frac{1}{c}\mathcal{L}(q-q_1)$$

$$\left(2Rs + \frac{1}{c}\right)\mathcal{L}(q) - \left(Rs + \frac{1}{c}\right)\mathcal{L}(q_1) = \frac{E_0}{s}$$

2 - eq.

$$Rs\,\mathcal{L}(q) + Ls^2\,\mathcal{L}(q_1) = \frac{E_0}{s}$$

$$\left(Rs + \frac{1}{c}\right)\mathcal{L}(q) - \left(Rs + \frac{1}{c} + Ls^2\right)\mathcal{L}(q_1) = 0$$

solve

$$\mathcal{L}(q) = \frac{\left(Rs + \frac{1}{c} + Ls^2\right)E_0}{s(Rs+Ls^2)(Rs+\frac{1}{c})}$$

$$\mathcal{L}(q_1) = \frac{E_0}{s(Rs+Ls^2)(Rs+\frac{1}{c})}$$

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$$Q(s) = \frac{E_0 R}{(Rs+Ls^2)\left(Rs+\frac{1}{C}\right)} + \frac{E_0}{sC(Rs+Ls^2)\left(Rs+\frac{1}{C}\right)} + \frac{L}{(R+Ls)\left(Rs+\frac{1}{C}\right)}$$

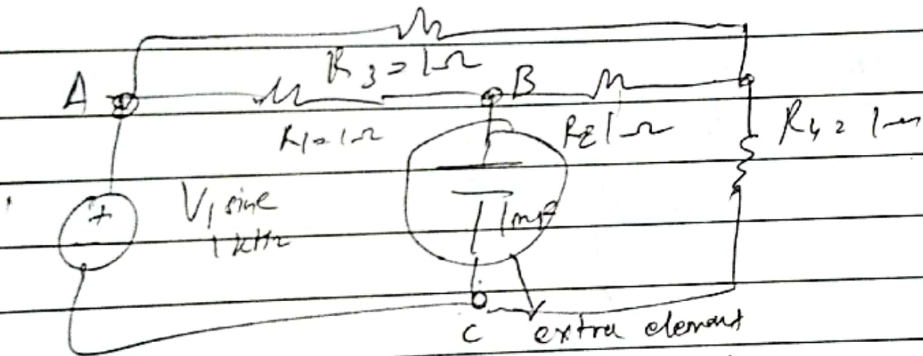
$$q = E_0 R\left(\frac{C}{R} - \frac{CR^2}{CR^2-L}e^{-t/CR} - \frac{CL^2}{L(CR^2-L)}e^{-Rt/L}\right)$$

$$+ \frac{E_0}{C}\left(\frac{Ct}{R} + \frac{C^2R^2 e^{-t/CR}}{CR^2-L} - \frac{CL^2 e^{-Rt/L}}{R^2(CR^2-L)} - \frac{C(CR^2+L)}{R^2}\right)$$

$$+ L\left(\frac{Ce^{-t/RC}}{CR^2-L} - \frac{Ce^{-Rt/L}}{CR^2-L}\right)$$

$$i = \frac{dq}{dt} = \frac{E_0}{R} + \frac{E_0 L}{R(CR^2-L)}\left(e^{-tR/L} - e^{-t/RC}\right)$$

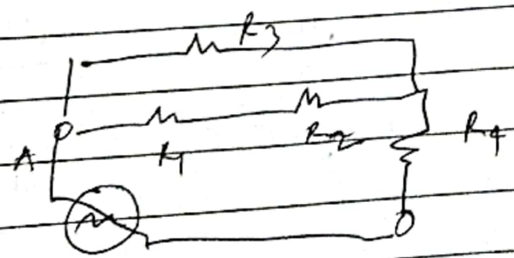
Q5



$$R_C = \frac{1}{\omega C_1}$$

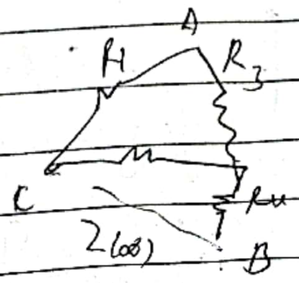
$$R_{in}(R_C) = R_{in}(\infty)\left(\frac{1+\frac{Z(0)}{R_C}}{1+\frac{Z(\infty)}{R_C}}\right)$$

$Z(R)$ is impedence from B–C



$$R_{in}(\infty) = \frac{1\times(1+1)}{3} + 1 = \frac{5}{3}\,\Omega.$$

$Z(\infty) =$



$= \frac{5}{3}\,\Omega$

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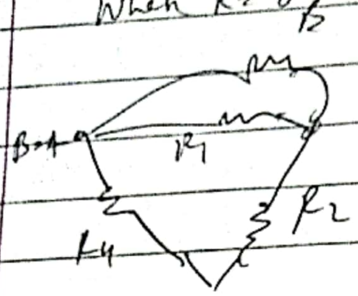
When $R = 0$



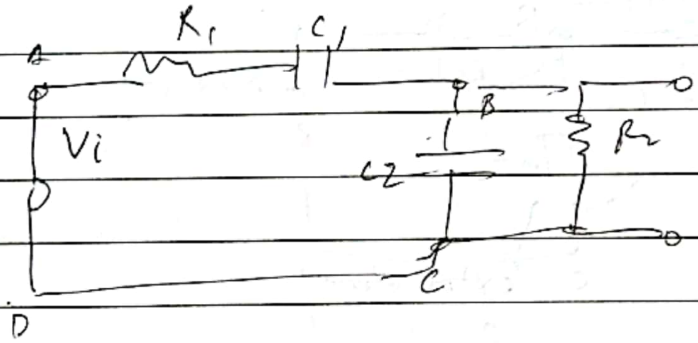
$$r(R=0) = \frac{1+\frac{1}{2}}{\frac{1}{2}+2}\ \Omega$$

$$= \frac{3}{5}\ \Omega$$

$$R_c = \frac{1}{\omega C} = \frac{1}{2\pi \times 10^{+3} \times 10^{-3}}$$

$$R_{in}(R_c) = \frac{5}{3}\frac{\left(1+\frac{3}{5}\times 2\pi\right)}{\left(1+\frac{5}{3}\times 2\pi\right)} = 0.69\ \Omega$$

Q6



AB impedence $Z_1 = R_1 + \frac{1}{j\omega C_1}$

BC im $Z_2 = \frac{R_2 \frac{j}{\omega C_2}}{R_2 + j/\omega C_2}$

$$\frac{V_o}{V_1} = \frac{1}{1+\frac{Z_1}{Z_2}} = \frac{R_1 + \frac{1}{j\omega C_1}}{\frac{R_2}{1+j\omega R_2 C_2}} = \frac{j\omega C_1 R_2}{(1-\omega^2 C_1 C_2 R_1 R_2) + j(\omega C_1 R_2 + \omega C_1 R_1 + \omega C_2 R_2)}$$

$$\left|\frac{V_o}{V_1}\right| = \frac{C_1 R_2}{\sqrt{\left(\frac{1}{\omega} - \omega C_1 C_2 R_1 R_2\right)^2 + (C_1 R_2 + C_1 R_1 + C_2 R_2)^2}}$$

$$< \frac{C_1 R_2}{C_1 R_2 + C_1 R_1 + C_2 R_2}$$

$\left|\cancel{\frac{V_0}{V_i}}\right|$

$$\Rightarrow \left(\frac{1}{\omega_0} - \omega_0 C_1 R_1 C_2 R_2\right)^2 = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{C_1 R_1 C_2 R_2}}$$

Phase

$$\Delta\phi = \tan^{-1}\left(\frac{1 - \omega^2 C_1 R_2 C_2 R_1}{\omega(C_1 R_2 + C_1 R_1 + C_2 R_1)}\right)$$

at resonance $\omega = \omega_0$;

$$\left|\frac{V_0}{V_1}\right| = \frac{1}{\sqrt{2}}.$$