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19MS(5)
Midsem

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Q1

$$f(z) = \frac{-z^2 - z + 1}{-z^2 - z + 1} = 1$$

$f(z)$ is analytic ~~$\Rightarrow$ no poles~~ except $z = -\left(\frac{1 \pm \sqrt{5}}{2}\right)$ $\rightarrow$ $-\left(\frac{1-\sqrt{5}}{2}\right)$

$$f(z) = \frac{-z^2 - z + 1}{-z^2 - z + 1} = \frac{\left(z + \frac{1 \pm \sqrt{5}}{2}\right)\left(z + \frac{1-\sqrt{5}}{2}\right)}{\left(z + \frac{1+\sqrt{5}}{2}\right)\left(z + \frac{1-\sqrt{5}}{2}\right)}$$

$$\operatorname{Res}_{z = -\left(\frac{1+\sqrt{5}}{2}\right)} (f(z)) = \left(z + \frac{1+\sqrt{5}}{2}\right) f(z) \Big|_{z \to -\left(\frac{1+\sqrt{5}}{2}\right)} = \left(z + \frac{1+\sqrt{5}}{2}\right)$$

$$= 0$$

$$\operatorname{Res}_{z = -\left(\frac{1 \mp \sqrt{5}}{2}\right)} (f(z)) = \left(z + \frac{1-\sqrt{5}}{2}\right) f(z) = z + \frac{1-\sqrt{5}}{2} = 0$$

$f(z)$ isn't defined at $z = -\left(\frac{1+\sqrt{5}}{2}\right), -\left(\frac{1-\sqrt{5}}{2}\right)$

~~No singularities~~

but these aren't singularities

The residues at them are also 0.

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Q2 $f(z) = \dfrac{z^2+z-1}{-z^2-z+2}$

$= \dfrac{z^2+z-1}{(z-1)(z+2)}$

so this has poles at $z = 1, -2$.

$\int_C f(z)\, dz$ where $C: |z| < 1$

the pole $z = 1$ lies just outside the contour

$f(z)$ is analytic inside C.

$\Rightarrow \oint f(z)\, dz = 0$ by Cauchy-integral theorem

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Q4 $f(z) = \frac{1}{(z^2+z+1)^2} = \frac{1}{\left[\left(z-\left(\frac{1+i\sqrt{3}}{2}\right)\right)\left(z-\left(\frac{1-i\sqrt{3}}{2}\right)\right)\right]^2}$

$\left(\frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}\right) \Rightarrow$ poles

z_1 z_2

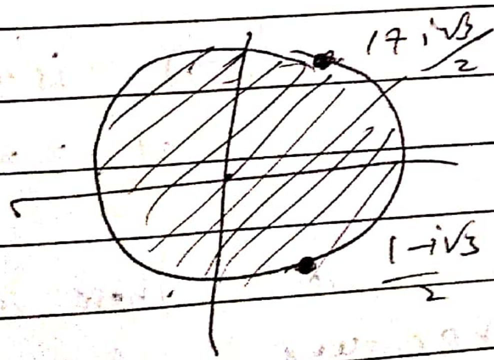
$|z_1| = |z_2| = 1$

$\int_C f(z)\,dz$ for $C: |z| < 1$

the poles z_1 & z_2 lie just outside the contour.

$f(z)$ is analytic in C.

$\Rightarrow \int_C f(z)\,dz = 0$



Q3 $f(z) = \dfrac{1}{z^2+z-1}$

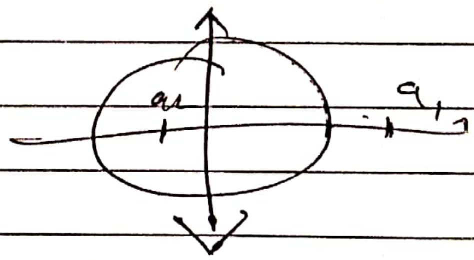
$$= \frac{1}{\left(z+\frac{1+\sqrt{5}}{2}\right)\left(z+\frac{1-\sqrt{5}}{2}\right)}$$

$a_1 = \dfrac{1+\sqrt{5}}{2}$, $a_2 = \dfrac{1-\sqrt{5}}{2}$

$$f(z) = \frac{1}{(a_1-a_2)}\left(\frac{1}{(z+a_2)} - \frac{1}{z+a_1}\right)$$

$a_1 = \dfrac{1+\sqrt{5}}{2} > 1.$

a_2 is inside.



for $0 < |z| < 1$

for $|z| < |a_2|$

$\dfrac{|z|}{|a_1|} < 1$, $\dfrac{|z|}{|a_2|} < 1$

$$f(z) = \frac{1}{(a_1-a_2)}\left(\frac{1}{a_2\left(1+\frac{z}{a_2}\right)} - \frac{1}{a_1\left(1+\frac{z}{a_1}\right)}\right)$$

$$= \frac{1}{(a_1-a_2)}\left(\frac{1}{a_2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{a_2}\right)^n - \frac{1}{a_1}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{a_1}\right)^n\right).$$

$$= \frac{1}{(a_1-a_2)}\left(\sum(-1)^n\left(\frac{1}{a_2^{n+1}} - \frac{1}{a_1^{n+1}}\right)z^n.\right)$$

for $|z| > |a_2|$

$\left|\frac{a_2}{z}\right| < 1$

$$\Rightarrow f(z) = \frac{1}{(a_1-a_2)}\left(\frac{1}{z\left(1+\frac{a_2}{z}\right)} - \frac{1}{a_1\left(1+\frac{z}{a_1}\right)}\right)$$

$$= \frac{1}{(a_1-a_2)}\left(\sum_{n=0}^{\infty}\frac{(-1)^n a_2^n}{z^{n+1}} - \sum\frac{(-1)^n z^n}{a_1^{n+1}}\right)$$

for $|z| = |a_2|$

laurent series does not converge.

$\left(\sum_{n=0}^{\infty}(-1)^n \text{ does not converge}\right)$

laurent series for $|z| = |a_2|$ doesn't exist

$$f(z) = \begin{cases} \frac{1}{(a_1-a_2)}\sum(-1)^n\left(\frac{1}{a_2^{n+1}} - \frac{1}{a_1^{n+1}}\right)z^n & 0 < |z| < |a_2| < 1 \\ \frac{1}{(a_1-a_2)}\left(\sum_{n=0}^{\infty}\frac{(-1)^n a_2^n}{z^{n+1}} - \sum_{n=0}^{\infty}\frac{(-1)^n z^n}{a_1^{n+1}}\right) & |a_2| < |z| < 1. \end{cases}$$