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Midsem PH3101

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Rough.

$F = m\gamma x$

$\frac{dF}{dt} = m v_0 \dot{x}$

$L = \frac{1}{2} m \dot{x}^2 + m v_0 x$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x} + m v_0 = p_x$

$\Rightarrow H = p_x \cdot \left(\frac{p_x - m v_0}{m}\right) -$

$L = \frac{1}{2} m \left(\frac{p_x - m v_0}{m}\right)^2 + m v_0 \left(\frac{p_x - m v_0}{m}\right)$

$= \left(\frac{p_x - m v_0}{m}\right)\left(\frac{1}{2}(p_x + m v_0) \cancel{+ m v_0}\right)$

$= \frac{(p_x - m v_0)(p_x + m v_0)}{2m}$

$= \frac{p_x^2 - m^2 v_0^2}{2m}$

$\Rightarrow$ $\cancel{\frac{p_x^2}{} - mp}$

$\frac{p_x^2}{m} - p_x v_0 - \frac{p_x^2}{2m} + \frac{m v_0^2}{2}$

$\Rightarrow H = \frac{p_x^2}{2m} - p v_0 + \frac{m v_0^2}{2}$

Q5 $L = \frac{m}{2}(\dot{x}^2+\dot{y}^2) - mgy.$

$$H = \frac{m}{2}(\dot{x}^2+\dot{y}^2) + mgy$$

$$= \frac{p^2}{2m} + \frac{m}{2}(at)^2 + mg\frac{at^2}{2}$$

$$= \frac{p^2}{2m} + \frac{m}{2}a(a+g)t^2$$

Q7 $F(\vec{r}) = -\left(\alpha + \frac{\beta}{r}\right)\vec{r} = -(\alpha r+\beta)\hat{r}$

$F = -\nabla V$

$= V = \int \alpha r + \beta \, dr = \frac{\alpha r^2}{2} + \beta r$

No vel dependence

$\Rightarrow H = T + V$

$$= \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \left(\frac{\alpha}{2}r^2 + \beta r\right)$$

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Q6 $L = \frac{m}{2}\dot{x}^2 + m(\dot{y}^2 + \dot{z}^2) - \frac{k}{2}x^2 - \frac{k}{2}(y+z)^2$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x}, \quad \frac{\partial L}{\partial \dot{y}} = 2m\dot{y}, \quad \frac{\partial L}{\partial \dot{z}} = 2m\dot{z}$

$H = \cancel{m\dot{x}^2 + 2m\dot{y}} \left(p_x \frac{p_x}{m} + p_y \frac{p_y}{2m} + p_z \frac{p_z}{2m} \right.$

$= \sum_i p_i \dot{q}_i - L \quad - \frac{p_x^2}{2m} - \frac{p_y^2}{4m} - \frac{p_z^2}{4m}$

$+ \frac{k}{2}\dot{x}^2 + \frac{k}{2}(y+z)^2$

$\Rightarrow \frac{p_x^2}{2m} + \frac{p_y^2}{4m} + \frac{p_z^2}{4m} + \frac{k}{2}\dot{x}^2 + \frac{k}{2}(y+z)$

Q4 $f(x,y) = (x+y)^2$

$\frac{\partial f}{\partial y} = 2(x+y) = V$

$\frac{\partial g}{\partial V} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial V} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial V}$

$= \frac{\partial g}{\partial x}\cdot\frac{1}{2} + \frac{\partial g}{\partial y}\cdot\frac{1}{2} = y$

$f = (x+y)^2 \Rightarrow V = \frac{\partial f}{\partial y}$ -

$g(x,V) = y = \frac{\partial g}{\partial V}$

$f = x^2 + y^2 + 2xy, \quad \frac{\partial f}{\partial y} = 2x + 2y$

$y = \frac{V - 2x}{2}$,

$\frac{V-2x}{2} = \frac{\partial g}{\partial V} \Rightarrow dg = \frac{V\,dV}{2} - x\,dV$

$= \frac{V^2}{4} - xV$

$g = \left(\frac{V}{4} - x\right)V$

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Q2 $\frac{dJ}{dt} = m\ddot{r}\dot{r} - m r \dot{r} a^2$

$~~= m\dot{r}(\ddot{r} - m\ddot{r}) = 0~~$

$J = \frac{1}{2} m \dot{l}^2 - \frac{1}{2} m a^2 l^2$

$= \frac{1}{2} m \left(\frac{a^2 \omega^2}{4} (e^{i\omega t} - e^{-i\omega t})^2 - \frac{\omega^2 a^2}{4} (e^{i\omega t} + e^{-i\omega t})^2 \right)$

$= \frac{m \omega^2 a^2}{8} (-4) = \frac{-m\omega^2 a^2}{2}$

Now $L = \frac{1}{2} (m(\dot{r}^2 + r^2 a^2))$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = m\ddot{r} = \frac{\partial L}{\partial r} = m r a^2$

$\Rightarrow r = Ae^{i\omega t} + Be^{-i\omega t}$

$\dot{r} = 0 \Rightarrow r = a$

$A + B = a, \quad A = B = \frac{a}{2}$

$\Rightarrow r = \frac{a}{2}(e^{i\omega t} + e^{-i\omega t})$

$J = m\ddot{r}r - \frac{1}{2} m \dot{l}^2 - \frac{1}{2} m r^2 a^2 = \frac{1}{2} m \dot{r}^2 - \frac{1}{2} m l^2 a^2$