

Section-A

Attempt all questions:

(3 x 2 = 6 Marks)

Q.1. Evaluate  $\int_0^1 \int_0^y x y e^{-x^2} dx dy$ . (2)

Q.2. Change the order of the integration of the double integral

$$\int_0^2 \int_{\sqrt{4-x^2}}^{4-x} f(x, y) dy dx. \quad (2)$$

Q.3. Find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$ ,

where  $\vec{F} = \text{grad } (x^3 + y^3 + z^3 - 3xyz)$ . (2)

Section-B

Attempt all questions:

(3 x 3 = 9 Marks)

Q.1. (a) Transform the integral to Cartesian form and hence

evaluate:  $\int_0^\pi \int_0^a r^3 \sin \theta \cos \theta dr d\theta$ . (1.5)

(b) Evaluate:  $\iiint_R (x + y + z) dx dy dz$ ,

where  $R: 0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3$ . (1.5)

Q.2. (a) Find the area bounded between the parabolas  $y^2 = 4x$  and

$x^2 = 4y$ . (1.5)

(b) Find the mass of an octant of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

the density at any point being  $\rho = xyz$ . (1.5)

Q.3. Find a vector towards the outward normal to the surface

$x^2 + y^2 + z^2 = 11$ . Also find the directional derivative of the scalar point function  $f(xyz) = xyz$  at the point  $P(1, 1, 3)$  in the direction of this vector. (3)