University Roll No. | | | | | | | | | | |

G.L.A. UNIVERSITY, MATHURA

B.TECH. I YEAR I SEM. FIRST MID – TERM EXAMINATION, SEPT. 2011

M.M.: 40 **MATHEMATICS - I (AHM – 101)** **Time : 1 Hr. 30 Min.**

Instructions :

1. This question paper has **three** parts – A, B and C.

2. Attempt **ALL** parts of this question paper.

3. Marks are indicated against each part.

PART A

Attempt ALL questions of this Part. **(8 x 2 = 16 Marks)**

Q1. If $y = (x + a)^n$ where 'a' is a constant then find the n^{th} derivative of y.

Q2. Write down the n^{th} derivative of $\log_e x$.

Q3. If $y = \exp.(\tan^{-1}x)$ then find A in $(1 + x^2)\, y_2 = Ay_1$.

Q4. If $u = x^2 + y^2$ then find $\partial^2 u / \partial x\, \partial y$.

Q5. If $u = (x^{1/4} + y^{1/4}) / (x^{1/5} + y^{1/5})$ then find $x\, \partial u/\partial x + y \partial u/\partial y$.

Q6. If $f = x + 4y$ where $x = 2s + t$, $y = s + 2t$ then find $\partial f / \partial t$.

Q.7. If $x = r \cos \theta$, $y = r \sin \theta$ then find $\partial(x, y) / \partial(r, \theta)$

Q.8. If u (x,y) is a homogeneous function in x and y of degree n then find

$$x^2 (\partial^2 u / \partial x^2) + 2xy\, (\partial^2 u / \partial x\, \partial y) + y^2 (\partial^2 u / \partial y^2)$$

P.T.O.

PART B

Attempt any THREE of the following: (3 x 5 = 15 Marl

Q9. If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)\, y_2 - x\, y_1 + m^2 y = 0$

Q10. If $u = \log_e [(x^4 + y^4)/(x + y)]$, Use Euler's theorem to prove that

$$x\, \partial u/\partial x + y\, \partial u/\partial y = 3.$$

Q11. If $x^x y^y z^z = \text{constant}$ show that $\partial^2 z/\partial x\, \partial y = -(x \log_e ex)^{-1}$
when $x = y = z$

Q12. If $x = u(1 - v)$, $y = uv$ Prove that $JJ' = 1$

where $J = \partial(x, y)/\partial(u, v)$ and $J' = \partial(u, v)/\partial(x, y)$

Q13. Evaluate $\int_0^1 [(x^\alpha - 1)/\log x]\,dx, \alpha \geq 0$ using differentiation under the integral sign.

PART C

Attempt any ONE of the following questions : (1 x 9 = 9 Mark

Q14. If $y = [x + \sqrt{(1 + x^2)}]^m$, prove that

$$(1 + x^2)\, y_{n+2} + (2n + 1)\, x\, y_{n+1} + (n^2 - m^2)\, y_n = 0.$$

Also find $(y_n)_0$.

Q15. If $V = f(e^{y-z}, e^{z-x}, e^{x-y})$ find the value of $V_x + V_y + V_z$.

Q16. Given $u = x^n f_1(y/x) + y^{-n} f_2(x/y)$ then prove that

$$x^2 (\partial^2 u/\partial x^2) + 2xy\, (\partial^2 u/\partial x\, \partial y) + y^2 (\partial^2 u/\partial y^2) + x\, \partial u/\partial x + y\, \partial u/\partial y = n^2 u$$