Printed Pages: 03 **University Roll No....................**

Second Mid Term Examination, 2016 – 2017

B. Tech. I Semester

AHM – 1101: Engineering Mathematics - I

Time: 1 ½ Hrs. **Max. Marks: 20**

Note - Attempt ALL Sections.

SECTION A **(1 x 5 = 5 marks)**

Note: ALL questions of this section are **COMPULSORY.**

Q.1. Examine the linear dependence of the following set of vectors:

$$X_1 = [3,\ 1,\ 1],\ X_2 = [2,\ 0,\ 1],\ X_3 = [4,\ 2,\ 1]$$

Q.2. Let A be a square non – singular matrix of order 3. It is given that $|A| = 6$ and the sum of its principal diagonal elements is 0. Also if $|A+2I| = 0$ where I is the identity matrix of order 3 then find the eigen values of A.

Q.3. Define: (a) Diagonal Matrix (b) Unitary Matrix

Q.4. Show that the matrix $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew – Hermitian.

Q.5. The quadratic form of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 4 \\ 4 & 4 & 3 \end{bmatrix}$ is $3x^2 + 3z^2 + 4xy + 8xz + 8yz$. The canonical form of this quadratic form is obtained as $3y_1^2 - \frac{4}{3}y_2^2 - y_3^2$ by using suitable non – singular transformations.

Find the index and the signature of this quadratic form.

SECTION B **(2 x 3 = 6 marks)**

Note : Attempt any **THREE** questions.

Q.1. Find the eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

Q.2. The non – singular square matrix $B = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ satisfies the relation $BA = I_3$ where I_3 is unit matrix of order 3. Find the matrix A.

Q.3. Prove that the characteristic roots of an idempotent matrix are either zero or unity.

Q.4. Find the values of a non – zero real number k for which the following system of equations has a non – trivial solution.

$$(3k-8)x + 3y + 3z = 0$$
$$3x + (3k-8)y + 3z = 0$$
$$3x + 3y + (3k-8)z = 0$$

SECTION C **(3 x 3 = 9 marks)**

Note : Attempt any **THREE** questions.

Q.1. Reduce the matrix A to its normal form and hence find its rank, where $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

Q.2. Investigate for consistency of the following system of equations

and if consistent, find the solution (s):

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2,$$

and $$x - y + z = -1$$

Q.3. Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

Also find the matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 7A^2 - 2A + 10I$$

Q.4. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}$; a, b and c being all real.

Discuss the following cases in finding the rank of A:

$(i) a \neq b \neq c, a+b+c \neq 0$ $(ii) a \neq b \neq c, a+b+c = 0$ $(iii) a = b \neq c$