No. of printed pages: 04 **University Roll No. :** ______________

END Term, Even Semester Examination, 2018–2019

Sub.:- Engineering Mathematics II (BMAS–0102)

Time: - 3 Hrs. **Course:- B. Tech. II Sem.** **Max. Marks:- 50**

Note - Attempt **BOTH** Sections. The terms have their us[illegible] meanings.

SECTION A (7 x 5 = 35 marks)

Note: ALL questions of this section are **COMPULSORY.** Each question of this section is of **Five** marks.

Q.1. Change the variables x, y, z to r, θ, φ by the equations,

$$x = r \sin\theta \cos\varphi,$$
$$y = r \sin\theta \sin\varphi,$$
$$z = r \cos\theta,$$

and evaluate the following integral

$$\iiint \frac{dx\,dy\,dz}{(x^2+y^2+z^2)}$$

taken throughout the volume of the sphere

$$x^2 + y^2 + z^2 = 4$$

Q.7. Solve the partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x\, \partial y} = y\cos x$$

OR,

$$\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right) z = \sin(x + 2y)$$

SECTION B (15 marks)

Note: Attempt **ALL** questions. Marks are shown against them.

Q.1. Solve the partial differential equation: (2)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 2\cos x \cos y$$

Q.2. If $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ are the vectors joining the fixed points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively to a variable point (x, y, z); prove that,

$$div\left(\overrightarrow{V_1} \times \overrightarrow{V_2}\right) = 0.$$ (2)

Q.3. Solve the partial differential equation:

$$(D - D' - 2)(D - D' - 1)\, z = e^{x+2y}$$ (2)

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

Q.2. Evaluate using Beta and Gamma functions:

(a) $\int_0^2 x\,(8 - x^3)^{\frac{1}{3}}\,dx$ (b) $\int_0^\infty \frac{dx}{1+x^4}$

Q.3. Examine the convergence of the following infinite series:

$$\sum_{n=1}^{\infty} \frac{1^2.3^2.5^2 \ldots\ldots (2n-1)^2}{2^2.4^2.6^2 \ldots\ldots (2n)^2}\, x^{n-1}$$

Q.4. Show that the vector

$$\vec{A} = (6\,xy + z^3)\,\hat{\imath} + (3\,x^2 - z)\,\hat{\jmath} + (3xz^2 - y)\hat{k}$$

is irrotational. Also, find the scalar potential φ such that

$$\vec{A} = \nabla\varphi$$

Q.5. Prove that: div (grad r^n) = $\nabla^2 r^n$ = n (n + 1) r^{n-2}

where, $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ and $r = |\vec{r}|$. Also show that,

$$\nabla^2\left(\frac{1}{r}\right) = 0$$

Q.6. Verify Gauss' Divergence theorem for

$$\vec{F} = 4xz\,\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$$

taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1.

P. T. O.