

GLA University, Mathura

First- Mid Term Examination, 2013-2014

Course: - B.Tech

Subject: -Mathematics-I (AHM -101)

Time: - 1 Hr. 30 Minutes

Notes:-

- 1) Attempt ALL groups.
- 2) Attempt ANY FOUR questions from Group A, ANY THREE questions from Group B and ANY TWO questions from Group C.
- 3) All questions of a group should be answered at one place.
- 4) Answer should be brief and to-the-point.
- 5) Any missing or wrong data may be assumed suitably giving proper justification.
- 6) Figures on the right-hand side margin indicate full marks.

I-Year, I-Sem.

Uni. Roll No:-

Total Marks: - 20

Section -A (1x4 = 4 marks)

Attempt any four questions:

Q.1 If $y = \frac{1}{x}$ then find $\frac{d^{10}y}{dx^{10}}$.

Q.2 If $u = \log\left(\frac{x^2}{y}\right)$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.

Q.3 If $u = x^4 \cos\left(\frac{y}{x}\right)$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.

Q.4 Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for the functions $u = e^x \sin y$ and $v = e^x \cos y$.

Q.5 Find $\frac{dy}{dx}$, when $x^y + y^x = c$.

Section -B (2x3 = 6 marks)

Attempt any three questions

Q.1 Find the n^{th} derivative of $e^{3x} \sin^2 2x$

Q.2 If $u = e^{xyz}$ find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$

Q.3 If $u = \frac{y^2}{2x}$ and $v = \frac{x^2 + y^2}{2x}$ find $\frac{\partial(u,v)}{\partial(x,y)}$

Q.4 Test the maxima or minima given that $F(xy) = x^2 + 2xy + 2y^2 + 2x + y$

P.T.O

Section -C (5x2 = 10 marks)

Attempt any two questions

Q.1 If $x = \sin\left(\frac{\log y}{a}\right)$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ and also find $y_n(0)$.

Q.2 If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$

Q.3 If $y = \frac{x}{x^2+a^2}$, Prove that $y_n = \frac{(-1)^n n!}{a^{n+1}} \sin^{n+1} \phi \cos(n+1)\phi$, where $\phi = \tan^{-1} \frac{x}{a}$

Q.4 If $x+y+z=u$, $y+z=uv$ and $z=uvw$ then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$