Printed Pages: 03 **University Roll No. :-** ____________

Mid Term Examination, 2019 – 20
B. Tech. I Year II Semester

Subject Name and Code: Engineering Mathematics II (BMAS 0102)

Time: 2 Hrs. Note: Attempt **ALL** sections. **Total Marks: 30**

SECTION A **(2 x 3 = 6 marks)**

Instructions: This section has 3 questions. Attempt **ALL** questions. The symbols have their usual meanings.

Q.1. What is Leibnitz test? Test the following infinite series for convergence: (2)

$$\frac{2}{1^3}-\frac{3}{2^3}+\frac{4}{3^3}-\frac{5}{4^3}+\cdots$$

Q.2. Use Beta and Gamma functions to evaluate the integral:

$$\int_0^1 x^5 (1-x^3)^{10}\, dx$$ (2)

Q.3. Show that: (2)

$$\iint_R r^2 \sin\theta \, dr\, d\theta = \frac{2a^3}{3}$$

where R is the region bounded by the semi-circle $r = 2a\cos\theta$ above the initial line.

SECTION B **(3 x 3 = 9 marks)**

Instructions: This section has 3 questions. Attempt **ALL** questions. Marks are indicated against each question.

Q.1. Test the following infinite series for convergence and divergence:

$$\frac{(1+a).(1+b)}{1.2.3}+\frac{(2+a).(2+b)}{2.3.4}+\frac{(3+a).(3+b)}{3.4.5}+\cdots$$

where a and b are non-zero, fixed and finite numbers. (3)

Q.2. Prove that: (1.5 + 1.5)

(a) $\beta(m+1,n) + \beta(m,n+1) = \beta(m,n)$

(b) $\int_0^\infty \frac{e^{-st}}{\sqrt{t}} dt = \sqrt{\frac{\pi}{s}}$, $s > 0$

Q.3. Evaluate the triple integral: (3)

$$\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$$

SECTION C **(3 x 5 = 15 marks)**

Instructions: This section has 4 questions. Attempt **ANY THREE** questions. Marks are indicated against each question.

Q.1. (i) Test the convergence of the infinite series: (4)

$$\frac{1^2}{4^2} + \frac{1^2.5^2}{4^2.8^2} + \frac{1^2.5^2.9^2}{4^2.8^2.12^2} + \frac{1^2.5^2.9^2.13^2}{4^2.8^2.12^2.16^2} + \cdots$$

(ii) State Cauchy's root test for determining the convergence and divergence of the infinite positive term series. (1)

Q.2. (i) Use Beta and Gamma functions to evaluate: (3)

$$\int_0^\infty \frac{x^2}{(1+x^4)^3} dx$$

(ii) Prove that: $\Gamma\frac{1}{2} = \sqrt{\pi}$ (2)

Q.3. Change the order of integration of the following double integral and hence evaluate the same:

$$\int_0^a \int_y^a \frac{x}{x^2+y^2} dx \, dy$$ (5)

Q.4. (i) Evaluate $\iint xy \, dx \, dy$ over the positive quadrant of the circle

$x^2 + y^2 = a^2$. (3)

(ii) Calculate the volume of the solid bounded by (2)

$x = 0, \ y = 0, z = 0$ and $x + y + z = 1$.