Printed Pages: 04 University Roll No. :- ____________

Mid Term Examination, 2019 – 20

B. Tech. I Year I Semester

Sub.: Engineering Mathematics I (BMAS 0101)

Time: 2 Hrs. Note: Attempt ALL sections. **Total Marks: 30**

SECTION A (2 x 3 = 6 marks)

Instructions: This section has **3** questions. Attempt **ALL** questions. The symbols have their usual meanings.

Q.1. If $f(x,y,z) = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$,

find $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$.

Q.2. Determine the functional relationship between two functions:

$$u = \frac{x}{y}, \quad v = \frac{x+y}{x-y},$$

given that the Jacobian $\frac{\partial (u,v)}{\partial (x,y)} = 0$ and also $x \neq y$.

Q.3. If $V(x,y) = \frac{x^3y^3}{x^3+y^3}$, using Euler's theorem, evaluate:

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}.$$

SECTION B (3 x 3 = 9 marks)

Instructions: This section has **3** questions. Attempt **ALL** questions. The symbols have their usual meanings.

Q.1. If
$$x = u + v + w,$$
$$y = vw + wu + uv, \text{ and}$$
$$z = uvw,$$

and F is a function of x, y and z, show that,

$$u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z}.$$

Q.2. Expand the function

$$f(x,y) = x^2y + \sin y + e^x$$

in powers of $(x-1)$ and $(y-\pi)$ up to second degree terms.

Q.3. Find the rank of the matrix A by reducing to Echelon form if:

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

SECTION C (3 x 5 = 15 marks)

Instructions: This section has 4 questions. Attempt **ANY THREE** questions. The symbols have their usual meanings.

Q.1. Test the consistency for the following system of linear equations and if system is consistent, solve for x, y and z:

$$x + y + z = 6,$$

$$x + 2y + 3z = 14,$$

$$x + 4y + 7z = 30.$$

Q.2. If $u^3 + v^3 + w^3 = x + y + z,$

$$u^2 + v^2 + w^2 = x^3 + y^3 + z^3, \text{ and}$$

$$u + v + w = x^2 + y^2 + z^2,$$

then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

Q.3. Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$, using Lagrange's method of undetermined multipliers.

Q.4. If $log_e u = \frac{x^3+y^3}{3x+4y}$, Use Euler's theorem to prove that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u\, log_e u,$$

Hence also evaluate

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y}.$$