First Mid Term Examination 2013-14

Title of Question Paper: Mathematics-II | **Paper Code: AHM-102**

Time: 90 Minutes | **Max Marks: 20**

Notes:-

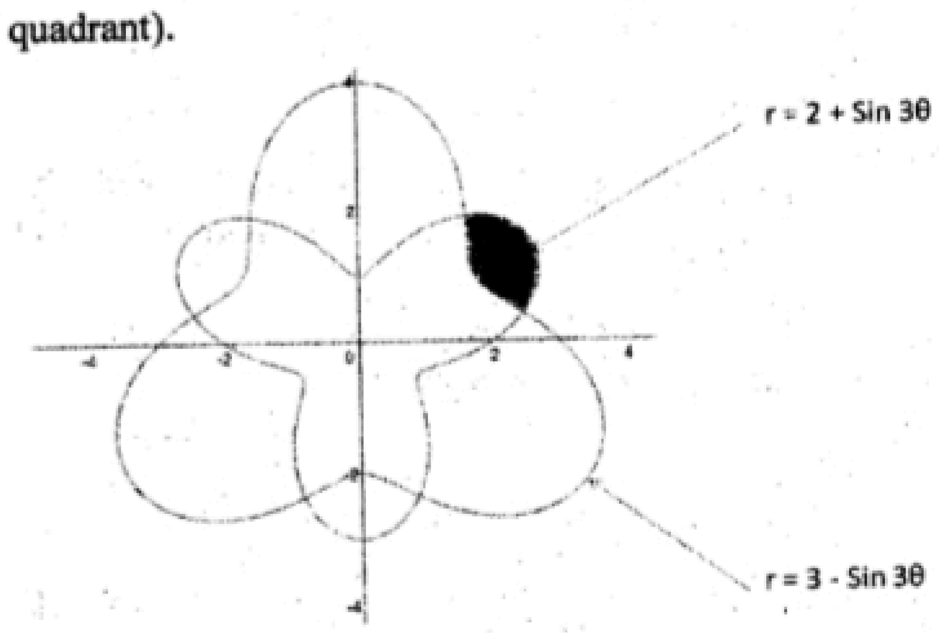
1. Attempt all sections.
2. All questions of the particular section should be answered collectively at one place.
3. Answers should be to-the-point and wherever required, be supplemented with neat sketches.
4. Any missing data may be assumed suitably giving proper justification.
5. Figures on the right hand side margin indicate marks.

Section A (Attempt all questions) **01*05=05**

1. If R is any region in the plane (R^2), what does the double integral $\iint_R 1\,dA$ represent?
2. Find the value of $\beta(2,1)+\beta(1,2)$.
3. Find the value of $\Gamma(-1/2)$.
4. Evaluate $\int_0^1\int_0^1\int_0^1 xyz\,dx\,dy\,dz$.
5. While expanding a function in fourier series in $[-\pi,\pi]$ what will be four Dirichlet's conditions?

Section B (Attempt any three questions) **02*03=06**

1. Consider the region bounded by the curves determined by $-2x+y^2=6$ and $-x+y=-1$
   a. Sketch the region R in the plane.
   b. Set up and evaluate an integral of the form $\iint_R dA$ that calculates the area of R.
2. Let f be continuous on [0,1] and let R be the triangular region with vertices (0,0), (1,0) and (0,1). Show that $\iint_R f(x+y)dA=\int_0^1 uf(u)du$
3. Evaluate $\int_0^\infty \frac{x^a}{a^x}dx$. where a > 1.
4. The region inside the curve $r=2+Sin3\theta$ and outside the curve $r=3-Sin3\theta$ consists of three pieces. Evaluate the double integral $\iint r\,dr\,d\theta$ in the shaded region (one of these pieces in first quadrant).



Section C (Attempt any three questions) **03*03=09**

1. Sketch the graph of f(x)= |x| and compute the fourier series for f(x) on $-\pi\le x\le\pi$.
2. Apply Dirichlet's integral to find the volume of the solid bounded by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
3. Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
4. The function $P(x)=e^{-x^2}$ is fundamental in probability.
   a. Sketch the graph of P(x). Explain why it is called a "bell curve".
   b. Evaluate $I=\int_{-\infty}^{\infty}e^{-x^2}dx$ using the following brilliant strategy of Gauss.
      i. Instead of computing I, compute $I^2=\left(\int_{-\infty}^{\infty}e^{-x^2}dx\right)\left(\int_{-\infty}^{\infty}e^{-y^2}dy\right)$.
      ii. Rewrite I^2 as in integral of the form $\iint_R f(x,y)dxdy$ where R is the entire Cartesian plane.
      iii. Convert that integral to polar coordinates.
      iv. Evaluate to find I^2.