CO4- Learn the use of change of variables in solving multiple integrals
CO5- Find the gradient of a scalar field and divergence, curl of a vector field
CO6- Know various integral theorems related to line, surface and volume integrals

**Printed Pages: 3** **University Roll No. ........................**

**Mid Term Examination, Odd Semester 2022-23**
**B. Tech. (All sections), First Year, First Semester**
**Subject Code & Subject Name- BMAS 0104 & ENGINEERING CALCULUS**

**Time: 2 Hours** **Maximum Marks: 30**

Instructions for students:

1. *Attempt all questions.*
2. *Answers should be brief and lucid.*

**Section – A**
*Attempt All Questions.* 3 X 5 = 15 Marks

| No. | Detail of Question | Marks | CO | BL | KL |
|---|---|---|---|---|---|
| 1 | Write down the $n^{th}$ derivative of the following functions: (i) $y = \sin 2x$ (ii) $y = \log_e x$ (iii) $y = e^{3x}$ | 1 + 1 + 1 | 1 | U | P |
| 2 | If $z = \log_e (x^2 + xy + y^2)$ then prove that: $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$ | 3 | 2 | A | F |
| 3 | If $V = \frac{x^4 y^4}{x^4 + y^4}$ then using Euler's theorem, prove that: (i) $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 4V$ (ii) $x^2\frac{\partial^2 V}{\partial x^2} + 2xy\frac{\partial^2 V}{\partial x\,\partial y} + y^2\frac{\partial^2 V}{\partial y^2} = 12V$ | 3 | 2 | A | C |
| 4 | Expand $f(x,y) = e^x \sin y$ in powers of $x$ and $y$ as far as the terms of the third degree. | 3 | 2 | A | P |
| 5 | If $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_1x_3}{x_2}, y_3 = \frac{x_1x_2}{x_3}$ then show that: $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4.$ | 3 | 2 | U | F |

**Section – B**
*Attempt All Questions* 5 X 3 = 15 Marks

| No. | Detail of Question | Marks | CO | BL | KL |
|---|---|---|---|---|---|
| 6 | Prove that: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$ if, $u = \sin^{-1}\left(\frac{x^3 + y^3 + z^3}{ax + by + cz}\right)$ | 5 | 2 | An | M |

| | | | | | |
|---|---|---|---|---|---|
| 7 | If $w = f(x,y)$<br>where, $x = e^u \cos v$ and $y = e^u \sin v$<br>then, show that,<br>$$y\frac{\partial w}{\partial u} + x\frac{\partial w}{\partial v} = e^{2u}\frac{\partial w}{\partial y}$$<br>**OR,**<br>If<br>$$u^3 + v^3 = x + y, \qquad u^2 + v^2 = x^3 + y^3$$<br>then evaluate:<br>$$\frac{\partial\,(u,v)}{\partial\,(x,y)}$$ | 5 | 2 | A | P |
| 8 | Using Lagrange's method of undetermined multipliers, show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.<br>**OR,**<br>If $y = \sin\,(m\,sin^{-1}x)$, then prove that<br>(i) $(1-x^2)y_2 - xy_1 + m^2y = 0$<br>(ii) $(1-x^2)y_{n+2} - (2n+1)\,x\,y_{n+1} = (n^2 - m^2)y_n$<br>Here, m is a constant. | 5 | 2 | C | M |

CO – Course Outcome, BL – Abbreviation for Bloom's Taxonomy Level (R-Remember, U-Understand, A-Apply, An-Analyze, E-Evaluate, C-Create), KL – Abbreviation for Knowledge Level (F-Factual, C-Conceptual, P-Procedural, M-Metacognitive).