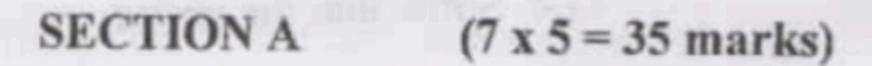
END Term, Even Semester Examination, 2018-2019

Sub.:- Engineering Mathematics II (BMAS-0102)

Time: - 3 Hrs. Course: - B. Tech. II Sem. Max. Marks: - 50

Note - Attempt BOTH Sections. The terms have their us meanings.



Note: ALL questions of this section are COMPULSORY. Each question of this section is of Five marks.

Q.1. Change the variables x, y, z to r, θ , φ by the equations,

$$x = r \sin \theta \cos \varphi,$$

$$y = r \sin \theta \sin \varphi,$$

$$z = r \cos \theta,$$

and evaluate the following integral

$$\iiint \frac{dx \, dy \, dz}{(x^2 + y^2 + z^2)}$$

taken throughout the volume of the sphere

$$x^2 + y^2 + z^2 = 4$$

Q.7. Solve the partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x \partial y} = y \cos x$$

OR,

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$$

SECTION B

(15 marks)

Note: Attempt ALL questions. Marks are shown against them.

Q.1. Solve the partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 2\cos x \cos y$$

Q.2. If $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ are the vectors joining the fixed points

(x1, y1, z1) and (x2, y2, z2) respectively to a variable point

(x, y, z); prove that,

$$div\left(\overrightarrow{V_1}\times\overrightarrow{V_2}\right)=0. \tag{2}$$

Q.3. Solve the partial differential equation:

$$(D-D'-2)(D-D'-1)z=e^{x+2y}$$
 (2)

where
$$D \equiv \frac{\partial}{\partial x}$$
 and $D' \equiv \frac{\partial}{\partial y}$.

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Q.2. Evaluate using Beta and Gamma functions:

(a)
$$\int_0^2 x (8 - x^3)^{\frac{1}{3}} dx$$
 (b) $\int_0^\infty \frac{dx}{1 + x^4}$

Q.3. Examine the convergence of the following infinite series:

$$\sum_{n=1}^{\infty} \frac{1^2 \cdot 3^2 \cdot 5^2 \cdot \dots \cdot (2n-1)^2}{2^2 \cdot 4^2 \cdot 6^2 \cdot \dots \cdot (2n)^2} x^{n-1}$$

Q.4. Show that the vector

$$\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational. Also, find the scalar potential φ such that

$$\vec{A} = \nabla \varphi$$

Q.5. Prove that: div (grad r^n) = $\nabla^2 r^n = n(n+1)r^{n-2}$

where, $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$ and $r = |\vec{r}|$. Also show that,

$$\nabla^2\left(\frac{1}{r}\right)=0$$

Q.6. Verify Gauss' Divergence theorem for

$$\vec{F} = 4xz\,\hat{\iota} - y^2\hat{\jmath} + yz\,\hat{k}$$

taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1.