

Mid Term Examination, Even Semester 2022-23

B.Tech. Year - I, Semester - II

BMAS 1105: Matrices, Differential Equations and Laplace transform

Time: 2 Hours

Maximum Marks: 30

Section - A

Attempt All Questions.

3 X 5 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
1	<p>Obtain the complete solution of the following linear differential equation :</p> $(D^2 - 2D + 1)^2 y = 0,$ <p>where, <math>D \equiv \frac{d}{dx}</math></p> <p style="text-align: center;">OR,</p> <p>Explore the complete solution of the second order linear differential equation,</p> $\frac{d^2y}{dx^2} - y = 1$ <p>which vanishes when <math>x = 0</math> and tends to a finite limit as <math>x \rightarrow -\infty</math>.</p>	3	3	E	C

2	<p>Reduce the following matrix A into either normal form or echelon form.</p> $A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \\ -2 & 4 & -6 & 2 \end{bmatrix}$ <p>hence find the rank of matrix A. What will happen to the rank of matrix A if all the elements of matrix A are taken as 1?</p>	3	1	An	P
3	<p>Show that the following system of equations:</p> $x + 2y = 2u,$ $2x - y - u = 0,$ $x + 2z - u = 0,$ $4x - y + 3z - u = 0$ <p>do not have a non-trivial solution.</p>	3	1	A	C
4	<p>Solve the following differential equations of first order and first degree:</p> <p>(i) <math>\frac{dy}{dx} = \frac{x^2 - 4y}{4x - y^2}</math></p> <p>(ii) <math>(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0</math></p>	3	3	U	F

5	<p>Define complex matrix and give an example of complex matrix of order <math>3 \times 3</math>. Show that the matrix <math>A</math> where,</p> $A = \begin{bmatrix} 2 & 3 - 4i \\ 3 + 4i & 2 \end{bmatrix}$ <p>is Hermitian. Also verify if <math>iA</math> is Skew-Hermitian matrix?</p>	3	1	R	C
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### Section – B

*Attempt All Questions.*

**$5 \times 3 = 15$  Marks**

No.	Detail of Question	Marks	CO	BL	KL
6	<p>Find the eigen values and eigen vectors of the matrix</p> $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ <p>Also, without finding <math>A^{-1}</math>, find the eigen values of <math>A^{-1}</math>.</p>	5	2	E	C
7	<p>Solve the following differential equations:</p> <p>(i) <math>(y - x y^2) dx - (x + x y^2) dy = 0</math></p> <p>(ii) <math>\frac{d^4y}{dx^4} - y = e^x</math></p> <p><b>OR,</b> Check the consistency of the following system of simultaneous linear equations and hence solve, if consistent.</p> $\begin{aligned} -x + y + 2z &= 2, \\ 3x - y + z &= 6, \\ -x + 3y + 4z &= 4 \end{aligned}$ <p>Also, if the constants of the right-hand side (R.H.S.) of each of the above equations are made zero, will the new system have a unique trivial solution or an infinite number of non-trivial solutions? Give reasons.</p>	5	3	An	P

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8	<p>Obtain the complete solution of the following differential equations:</p> <p>(i) <math>(D^2 + 16 - 8D) y = e^{4x} - e^{-4x} + 3</math></p> <p>(ii) <math>y dx - x dy + \log x \ dx = 0</math></p> <p><b>OR,</b></p> <p>(i) Find the characteristic equation of the matrix</p> $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ <p>Hence find the real latent roots of <math>A</math>.</p> <p>(ii) Show that <math>A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 &amp; 1 &amp; 1 \\ 1 &amp; \omega &amp; \omega^2 \\ 1 &amp; \omega^2 &amp; \omega \end{pmatrix}</math> is a unitary matrix where <math>\omega</math> is a complex cube root of unity. It is known that <math>\omega^3 = 1</math> and <math>1 + \omega + \omega^2 = 0</math>.</p>	5	3, 2	C	M
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