University Roll No. ____________________

Mid Term Examination, 2016 – 2017
B. Tech. I Year II Semester

Sub. Name: Engg. Mathematics - II **Sub. Code: AHM – 2101**
Time: 90 Min. **M.M.: 20**

Note - Attempt **ALL** Sections. All symbols have usual meanings. Make neat sketches, wherever required.

SECTION A (1 x 5 = 5 marks)

Note: ALL questions of this section are **COMPULSORY.**

Q.1. Use Dirichlet's integral to evaluate $\iiint xyz\,dx\,dy\,dz$ throughout the volume bounded by x = 0, y = 0, z = 0 and x + y + z = 1.

Q.2. Changing the order of integration in the double integral

$$I = \int_0^a \int_{\frac{y^2}{a}}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}\,dx\,dy$$

leads to

$$I = \int_0^a \int_p^q \frac{y}{(a-x)\sqrt{ax-y^2}}\,dy\,dx$$

What are p and q?

Q.3. Evaluate: $\int_0^1 \sqrt{\frac{1-x}{x}}\,dx$ using Beta and Gamma functions.

Q.4. Change $\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$ into polar coordinates.

Q.5. Evaluate: $\int_0^3 \int_0^1 (x^2+y^2)\,dx\,dy$

SECTION B (2 x 3 = 6 marks)

Note : Attempt any **THREE** questions.

Q.1. Prove that: $\int_0^\infty e^{-x^6} dx = \frac{\left(\left|\frac{1}{3}\right.\right)^2}{(2)^{\frac{4}{3}}(3)^{\frac{1}{2}}\left|\frac{1}{2}\right.}$

Q.2. Show that $\iint_R r^2 \sin\theta dr d\theta = \frac{2}{3}a^3$ where R is the region bounded by the semi – circle $r = 2a\cos\theta$ above the initial line $\theta = 0$.

Q.3. Evaluate the following by changing into polar coordinates:

$$\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$$

Q.4. Evaluate: $\int_0^\infty x^2 e^{-x^2} dx$ using Beta and Gamma functions.

SECTION C (3 x 3 = 9 marks)

Note : Attempt any **THREE** questions.

Q.1. Using Dirichlet's integral, find the volume of the solid bounded by the coordinate planes and the surface

$$\left(\frac{x}{a}\right)^{1/2} = 1 - \left(\frac{y}{b}\right)^{1/2} - \left(\frac{z}{c}\right)^{1/2}$$

Also find its mass if the density at any point is k xyz. Here k, a, b and c are all non - zero constants.

Q.2. (a) Evaluate $\iint xydxdy$ over the positive quadrant of circle $x^2 + y^2 = a^2$, where a is any constant.

(b) Evaluate: $\int_0^1\int_0^1\int_0^1 e^{x+y+z} dxdydz$

Q.3. Evaluate $\iint_R (x^2+y^2)dxdy$ by change of variables where R is the region in the first quadrant bounded by $x^2 - y^2 = a$, $x^2 - y^2 = b$, $2xy = c$ & $2xy = d$, $0 < a < b$, $0 < c < d$. Here a, b, c and d are constants.

Q.4. Evaluate by changing the order of integration:

$$\int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y}{\sqrt{(2-x^2)(1-x^2y^2)}} dxdy$$