

30 March

University Roll No. _____

II Mid Term Examination, 2014-15

Mathematics - II
Time: - 90 Min.Paper Code - AHM 102
Max. Marks:-20

Section - A

Note: Attempt All Questions.

(1×5 = 05 Marks).

- 1) A particle moves on the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find its speed at $t = 1$.
- 2) If \vec{F} and \vec{G} are irrotational, show that $\vec{F} \times \vec{G}$ is solenoidal.
- 3) Find a vector in the direction in which $F(x, y, z) = xyz^2$ decreases most rapidly at the point $P(1, 0, 3)$.
- 4) A particle moves so that its position vector is given by $\vec{r} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$. Find the angle between velocity \vec{v} of the particle and position vector \vec{r} .
- 5) Find 'a_n' in the Fourier series of the function $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$

Section - B

Note: Attempt Any Three questions.

(3×2 = 6 Marks)

- 1) Show that the gradient field describing a motion is irrotational.

- 2) Find directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $P(1, -2, -1)$ in the direction $\vec{v} = 2\hat{i} - \hat{j} - 2\hat{k}$.
- 3) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$.
- 4) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at $(0, 0, 0), (2, 0, 0), (2, 3, 0)$.

Section - C

Note: Attempt Any Three questions.

(3×3 = 9 Marks)

- 1) Prove that: $\text{div}(\text{grad } r^n) = n(n+1)r^{n-2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
- 2) Vector field is given $\vec{A} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$. Show that the field is irrotational and find the scalar potential.
- 3) Verify Green's theorem in the plane for $\oint_C (x^2 + xy)dx + (x^2 + y^2)dy$ where C is the square formed by the line $x = \pm 1$ and $y = \pm 1$.
- 4) Obtain the Fourier series for $f(x) = x^2$, for $-\pi \leq x \leq \pi$.