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Mid-Term Examination, Odd Semester 2021-22

B. Tech. I Year, I Semester

Subject Code: BMAS 1101 **Subject Name: Engineering Mathematics I**

Time: 02 Hours **Maximum Marks: 30**

Section- A

Note: Attempt ALL THREE Questions. 3 x 2 = 6 Marks

1. Test the convergence of the following infinite series:

$$\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{7}}+\sqrt{\frac{3}{10}}+\sqrt{\frac{4}{13}}+\cdots$$

2. Write n^{th} term of the following infinite series:

$$\frac{1}{2.3}+\frac{5}{3.4}+\frac{9}{4.5}+\frac{13}{5.6}+\cdots$$

3. If $u=x^2y+y^2z+z^2x$ then show that:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=(x+y+z)^2$$

Section- B

Note: Attempt ALL THREE Questions. 3 x 3 = 9 Marks

1. State Leibnitz's test. Test the following infinite series for its convergence:

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \cdots$$

2. If $u = \frac{x+y}{x-y}$ and $v = \frac{xy}{(x-y)^2}$,
then find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally dependent?
If so, find the relationship.

OR,

What is p – test? Test the convergence of the series: $\sum_1^{\infty} \frac{n^p}{(n+1)^q}$

3. If $u = f(P, Q, R)$ where,

$$P\,(x, y, z) = 2x - 3y,$$
$$Q\,(x, y, z) = 3y - 4z,$$
$$R\,(x, y, z) = 4z - 2x$$

Then, prove that:

$$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0.$$

Section- C

Note: Attempt ANY THREE Questions. 3 x 5 = 15 Marks

1. Test the convergence or divergence of the following infinite series:

(a) $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\frac{x^4}{17}+\cdots$

(b) $\sum_{n=1}^{\infty}\ (1+\frac{1}{n})^{n^2}$

2. Given that:

$$z = x^n f_1\left(\frac{y}{x}\right) + y^{-n} f_1\left(\frac{x}{y}\right),$$

Then prove that:

$$(x^2\frac{\partial^2 z}{\partial x^2}+y^2\frac{\partial^2 z}{\partial y^2}+2xy\frac{\partial^2 z}{\partial x\partial y})+(x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}) = n^2 z$$

3. (a) If $u\,(x,y,z) = x^3+y^3+z^3+3\,x\,y\,z$ show that,

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z} = 3\,u$$

(b) If $y_1 = \frac{x_2\,x_3}{x_1}$, $y_2 = \frac{x_1\,x_3}{x_2}$, $y_3 = \frac{x_2\,x_1}{x_3}$, then find

$$J\left(\frac{y_1,\ y_2,y_3}{x_1,\ x_2,x_3}\right).$$

4. If $u = \sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, then by using Euler's theorem, prove that

(a) $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u.$$

(b) $$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = -2\cot u\,(1 + 2\,cosec^2 u)\,.$$