Course Name: Engineering Calculus

Course Outcome

CO1- Compute n^{th} order derivative and study its application in Leibnitz theorem

CO2- Understand partial differentiation and its applications

CO3- Compute the Jacobian and its applications

CO4- Understand the expansion of functions

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Mid Term Examination, Odd Semester 2023-24

B. Tech., Year I, Semester I

Subject Code and Name-BMAS 0104 and Engineering calculus

Time: 2 Hours **Maximum Marks: 30**

Instruction for students:

All questions are compulsory. The symbols have their usual meanings.

Section – A

Attempt All Questions. 3 X 5 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
1	Find the n^{th} derivative of the following functions: (i) $y = \frac{1}{2x+3}$ (ii) $y = e^{3x-5}$ (iii) $y = \sin^2 x$ **OR,** If $u = f(y-z,\ z-x,\ x-y)$, prove that: $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$	3	1/2	R	C

2	Test if the function $$f(x,y) = \frac{x+y}{\sqrt{x}+\sqrt{y}}$$ is a homogeneous function in x and y or not? If yes, then find the following: (*i*) degree of f (x, y) (ii) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ (*iii*) $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial y\,\partial x} + y^2\frac{\partial^2 f}{\partial y^2}$	3	2	U	P
3	If $u = xyz,$ $v = xy + yz + zx,$ and, $w = x + y + z,$ then compute the Jacobian: $$\frac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)}$$	3	3	E	F
4	If **u** = e^{xyz} then prove that: $$\frac{\partial^3 u}{\partial x\,\partial y\,\partial z} = e^{xyz}(1 + 3xyz + x^2y^2z^2)$$ **OR,** Expand $e^x \sin y$ about the point (0, 0) as far as the terms of III degree.	3	2/4	An	C

5	A rocket moves in the space. Its position at any time t is given by $x = \log_e (t^2 + 3t + 2)$, Determine the velocity, acceleration and the rate of change of acceleration of the rocket at t = 5.	3	1	C	M

Section – B

Attempt All Questions. 5 X 3 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
6	(a) If u = $tan^{-1}\left(\frac{y^2}{x}\right)$, Use Euler's theorem to prove that: $(i)\ 2x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = \sin 2u$ $(ii)\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y\,\partial x} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin 2u\ \sin^2 u$ (b) Prove that: $e^x \log_e(1+y) = y + xy - \frac{1}{2}y^2 + \frac{1}{2}x^2y - \frac{1}{2}xy^2 + \frac{1}{3}y^3 + \cdots$	5	2, 4	E	P
7	If $x = r\cos\theta,\ y = r\sin\theta$ then, (i) Calculate $J_1 = \frac{\partial (x,\ y)}{\partial (r,\ \theta)}$ (ii) Calculate $J_2 = \frac{\partial (r,\ \theta)}{\partial (x,\ y)}$ (iii) Prove that: $J_1 J_2 = 1$ **OR,** If $y = (\sin^{-1}x)^2$, prove that $(i)\ (1-x^2)\,y_2 - x\,y_1 - 2 = 0$ $(ii)\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0.$ The suffixes denote differentiation with respect to x.	5	3/1	An	P

8	(a) In a project work, Rakesh was given a function $u(x,y) = \dfrac{x+y}{1-xy}$ while Mukesh was given another function $v(x,y) = \tan^{-1} x + \tan^{-1} y.$ They were asked by their teacher to establish a relation between these functions, if possible. Are these functions, functionally related? If yes, what should be the relation between u and v? (b) The curves $f(x,y) = 0$ and $\varphi(x,y) = 0$ touch each other. Show that at the point of contact, the following condition holds: $\dfrac{\partial f}{\partial x} \cdot \dfrac{\partial \varphi}{\partial y} = \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial \varphi}{\partial x}$	5	3	C	M