I Term Examination, Even - Semester, 2017-18

Program:- B. Tech   Year:- I   Sem:- II
Subject :-Engineering Mathematics-II   Code:- AHM-2201

Time: - 60 Min.   Max. Marks: 15

Section- A

<u>**Note : Attempt ALL Questions.**</u>   (2x3=6 Marks)

Q.1.(a) Find: $\Gamma\left(\dfrac{1}{4}\right)\Gamma\left(\dfrac{3}{4}\right)$.

   (b) Using Beta and /or Gamma function, evaluate

$$\int_0^1 \sqrt{\frac{1-x}{x}}\,dx.$$

Q.2.   What is p- series test? Test the convergence of the series:

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+...$$

Q.3.   Test the series whether it is convergent or divergent.

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^1+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^2+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^3+...$$

Note: Attempt ALL Questions.   (3x3=9 Marks)

Q.1. Test the series for convergence and divergence:

$$1+\frac{x}{2}+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+... \quad (x>0).$$

Q.2. Show that the series:

$$x+\frac{2^2 x^2}{2!}+\frac{3^3 x^3}{3!}+\frac{4^4 x^4}{4!}+\frac{5^5 x^5}{5!}+... \quad (x>0).$$

is convergent for $x<\frac{1}{e}$ and divergent if $x\geq\frac{1}{e}$.

Q.3. (a) Prove that :

$$\int_0^\infty x^n e^{-a^2 x^2}dx=\frac{1}{2a^{n+1}}\Gamma\left(\frac{n+1}{2}\right), \ n>-1 \text{ and}$$

a is non-zero constant.

Hence deduce that:

$$\int_{-\infty}^\infty e^{-x^2}dx=\sqrt{\pi}.$$

(b) Show that the series

$$\log\frac{2}{1}-\log\frac{3}{2}+\log\frac{4}{3}-\log\frac{5}{4}+.......$$

is convergent.