

Mid Term Examination, 2015-16

Mathematics - II

Paper Code -AHM 2001

Time: - 90 Min.

Max. Marks:-20

Section - A

Note: Attempt All Questions.

(1 × 5 = 05 Marks)

- 1) A fluid motion is given by $\vec{v} = (x^2 - 3x)\hat{i} + (y + 2)\hat{j}$. Find the points in xy plane where the velocity is zero.
- 2) If $\phi = xyz$ is increases and decreases most rapidly in the direction of \vec{v} and \vec{u} respectively. Find the angle between \vec{v} and \vec{u} .
- 3) If $\vec{A} = (xz + 3)\hat{i} + (xy + 2)\hat{j} + (yz + 5)\hat{k}$, then find where the given vector is irrotational.
- 4) Evaluate $\iint_S \vec{a} \cdot \hat{n} ds$ where \vec{a} is constant vector and S is any closed surface.
- 5) Calculate Fourier coefficient 'a₀' for $f(x) = |x|$; $-\pi < x < \pi$.

Section -B

Note: Attempt Any Three questions. (3 × 2 = 6 Marks)

- 1) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the points (2, -1, 2)

- 2) Find directional derivative of $\phi(x, y, z) = x^2 - y^2 + 2z^2$ at $(1, 2, 3)$ in the direction $\vec{v} = 4\hat{i} - 2\hat{j} + \hat{k}$.
- 3) If $\vec{\omega}$ is a constant vector and \vec{r} is position vector. Then show that \vec{v} is solenoidal, where \vec{v} is normal to the both $\vec{\omega}$ and \vec{r} .
- 4) Evaluate $\oint_C (e^x dx + 2y dy - dz)$ where C is closed curve bounded by $x^2 + y^2 = 4$ and $z = 2$.

Section - C

Note: Attempt Any Three questions. $(3 \times 3 = 9 \text{ Marks})$

- 1) Prove that: $\text{div}(r^n \vec{r}) = (n+3)r^n$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
- 2) Show that the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational and find the scalar potential.
- 3) Verify Green's theorem in the plane for $\int_C (x^2 y dx + x^2 dy)$ where C is boundary of the triangle with vertices $(0,0)$; $(1,0)$; $(1,1)$.
- 4) Obtain the Fourier series for $f(x) = x^2$, for $0 \leq x \leq 2\pi$.