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G.L.A. UNIVERSITY, MATHURA

B.TECH. I YEAR II SEM. SECOND MID – TERM EXAMINATION, APRIL 2011

M.M.: 40 **MATHEMATICS - II (AHM - 102)** **Time : 90 Min.**

General Instructions :

(1) This question paper contains three parts – A , B and C. Instructions for each part are given against them separately.

(2) Solve the questions of a part at one place only.

(3) Assume $p = \partial z/\partial x$ and $q = \partial z/\partial y$ wherever required.

(4) Assume $D \equiv \partial / \partial x$ and $D' \equiv \partial / \partial y$ wherever required.

PART A

Attempt ALL questions of this Part. **(8 x 2 = 16 Marks)**

Q1. What is the two dimensional heat flow equation in steady state ?

Q2. Classify the one dimensional heat flow equation.

Q3. Show that the one dimensional wave equation $y_{tt} = c^2 y_{xx}$ is hyperbolic.

Q4. Solve : $y z p - x z q = xy$

Q5. Solve : $p \tan x + q \tan y = \tan z$

Q6. Solve : $\partial^2 z / \partial x^2 + \partial^2 z / \partial x \partial y = 0$

Q7. Solve : $D D' (D + 2D' + 1) z = 0$

Q8. Solve : $\partial^3 z / \partial x^3 = 0$

P.T.O.

PART B

Attempt any THREE of the following: **(3 x 5 = 15 Marks)**

Q9. Solve the partial differential equation : $(y^2 + z^2)\,p - xy\,q = -zx$

Q10. Solve the partial differential equation : $(D - D')^2 z = \tan(y + x)$

Q11. Show that the partial differential equation

$$z_{xx} + 2x\,z_{xy} + (1 - y^2)\,z_{yy} = 0$$

is elliptic for the values of x and y in the region $x^2 + y^2 < 1$, parabolic on the boundary and hyperbolic outside this region.

Q12. Solve by the method of separation of variables:

$$\partial u / \partial x = 4\ \partial u / \partial y \ ;\ u(0, y) = 8e^{-3y}$$

PART C

Attempt any ONE of the following : **(1 x 9 = 9 Marks)**

Q13. The vibrations of an elastic string is governed by the partial differential equation :

$$\partial^2 y / \partial t^2 = \partial^2 y / \partial x^2$$

The length of the string is π and ends are fixed. The initial velocity is zero and initial deflection is $y(x, 0) = 2\ [\sin x + \sin 3x]$. Find the deflection $y(x, t)$ of the vibrating string at time t.

Q14. Solve the partial differential equation :

$$(D^2 + D\,D' - 6\,D'^2)\,z = y \cos x$$

Q15. A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function u (x, t).