5LMB0 - Model Predictive Control

Report - Homework: Assignment 2

Quartile 3 - 2023-2024

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1 Quasi-LPV Model Predictive Control design

This assignment aims to design and implement a constrained quasi-Linear Parameter-Varying (qLPV) Model Predictive Control (MPC) controller for a pair of interconnected synchronous generators exhibiting nonlinear coupling. The designed controller must effectively stabilize the system at a zero equilibrium point from the given initial conditions.

1.1 LPV formulation and scheduling variable definition

LPV models are used to represent linear plant models that depend on time-varying parameters called the scheduling variables (ρ). The scheduling variables can also be used to model nonlinear dynamics, and in this case, they will be functions of state or input variables, and the model is referred to as a quasi-LPV (qLPV) model [1]. In this assignment, two interconnected synchronous generators with nonlinear coupling is represented with the discrete-time state space model, i.e.,

$$x(k+1) = Ax(k) + B_1u(k) + B_2 sin(x_1(k) - x_3(k)), \quad k \in \mathcal{N} \tag{1}$$

Here, the state vector $\mathbf{x}$, defined as $\mathbf{x} := [x_1(k), x_2(k), x_3(k), x_4(k)]^T \in \mathbb{R}^4$, represents the angular and frequency deviations of the first and second generators, respectively. The input vector $\mathbf{u}$, denoted as $\mathbf{u} := [u_1(k), u_2(k)]^T \in \mathbb{R}^2$, represents the mechanical power inputs to the generators. The nonlinear system model described in Equation (1) can be reformulated as a quasi-LPV model, i.e.,

$$x(k+1) = Ax(k) + B_1u(k) + B_2\rho(k) \tag{2}$$

The scheduling variable (ρ) at each time step depends on the state vectors at that time and is given by,

$$\rho(k) = sin(x_1(k) - x_3(k)) \tag{3}$$

1.2 Closed-loop simulation of the designed MPC controller

For the closed-loop simulation of the designed MPC controller, the scheduling variable $\rho(k)$ is predicted over N steps into the future using the LPV model to formulate an iterative linear MPC Quadratic Programming (QP) problem [2]. With fixed scheduling variables, the predicted states X_k are linearly dependent on the control inputs U_k [1, 2]. The predicted state at time i, using the measured state $x(k)$, i.e., $x_{i|k}$ satisfies:

$$x_{i+1|k} = Ax_{i|k} + B_1u_{i|k} + B_2\rho_{i|k} \ , \quad \forall i = 0, \dots, N-1, \tag{4}$$

$$\vdots$$

$$x_{N|k} = A^N x_{0|k} + A^{N-1}B_1u_{0|k} + A^{N-1}B_2\rho_{0|k} + \dots + B_1u_{N-1|k} + B_2\rho_{N-1|k} \tag{5}$$

The individual predictions over multiple time steps can be systematically arranged into a matrix framework to obtain the compact form representation of the prediction model:

$$X_k = \Phi x(k) + \Gamma_1 U_k + \Gamma_2 P_K \tag{6}$$

where,

$$X_k = \begin{pmatrix} x_{1|k} \\ x_{2|k} \\ \vdots \\ x_{N|k} \end{pmatrix}, \quad U_k = \begin{pmatrix} u_{0|k} \\ u_{1|k} \\ \vdots \\ u_{N-1|k} \end{pmatrix}, \quad P_k = \begin{pmatrix} \rho_{0|k} \\ \rho_{1|k} \\ \vdots \\ \rho_{N-1|k} \end{pmatrix}, \quad x(k) = x_{0|k} \tag{7}$$

and the matrices Φ, Γ_1 and Γ_2 are defined as:

$$\Phi = \begin{pmatrix} A \\ A^2 \\ \vdots \\ A^N \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} B_1 & 0 & \cdots & 0 \\ AB_1 & B_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B_1 & A^{N-2}B_1 & \cdots & B_1 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} B_2 & 0 & \cdots & 0 \\ AB_2 & B_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B_2 & A^{N-2}B_2 & \cdots & B_2 \end{pmatrix} \tag{8}$$

Here, X_k, U_k and P_k represent the complete sequence of predicted states, scheduling variables and control inputs, respectively, while Φ, Γ_1 and Γ_2 are the prediction matrices.

The following quadratic cost function is used for the assignment:

$$J(x(k), U_k) = \sum_{i=0}^{N-1} \left(x_{i|k}^T Q x_{i|k} + u_{i|k}^T R u_{i|k} \right) \tag{9}$$

The cost function can be written in a compact form as,

$$J(x(k), U_k) = x(k)^T Q x(k) + X_k^T \Omega X_k + U_k^T \Psi U_k \tag{10}$$

Substituting Equation (6) in the Equation (10) and eliminating the terms not dependent on U_K gives:

$$J(x(k), U_k) = \frac{1}{2} U_k^T G U_k + U_k^T F \tag{11}$$

where,

$$G = 2(\Psi + \Gamma_1^T \Omega \Gamma_1), \quad F = 2\Gamma_1^T \Omega (\Phi x(k) + \Gamma_2 P_k) \tag{12}$$

$$\Omega = diag([Q, \ldots, 0]), \quad \Psi = diag([R, \ldots, R]) \tag{13}$$

As with the linear MPC framework outlined in **Lecture 2**, all the constraints can be stacked together to yield:

$$\mathcal{D}x(k) + \mathcal{M}X_k + \mathcal{E}U_k \leq c \tag{14}$$

Replacing Xk in Equation (14) with the prediction matrices from Equation (6) gives the compact formulation of the constraint matrices:

$$LU_k \leq c + W \tag{15}$$

where,

$$L = \mathcal{M}\Gamma_1 + \mathcal{E}\ , \quad W = -(\mathcal{D} + \mathcal{M}\Phi)x(k) + (-\mathcal{M}\Gamma_2)P_k \tag{16}$$

The approach for constructing the future parameter vector P_k draws from methodologies discussed in **Lecture 6** and the research paper [1]. The optimization is iterated until the Euclidean norm between successive predicted input sequences, U_k and U_{k-1} is less than a predefined tolerance (e) set by the user or until the number of iterations surpasses a predetermined limit (Paper [1] indicates that convergence is generally achieved within two to three iterations). The first element of the input sequence is then applied to the system. The procedure is repeated at each sampling instant.

Figure 1 shows the simulated state and input trajectories of the closed-loop system. The trajectories are converging to the origin indicating that the origin is a stabilizing equilibrium point.

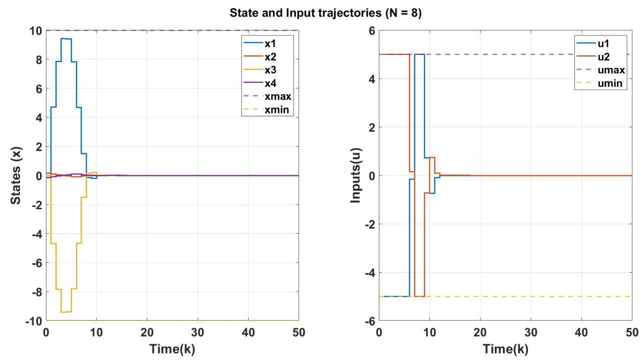
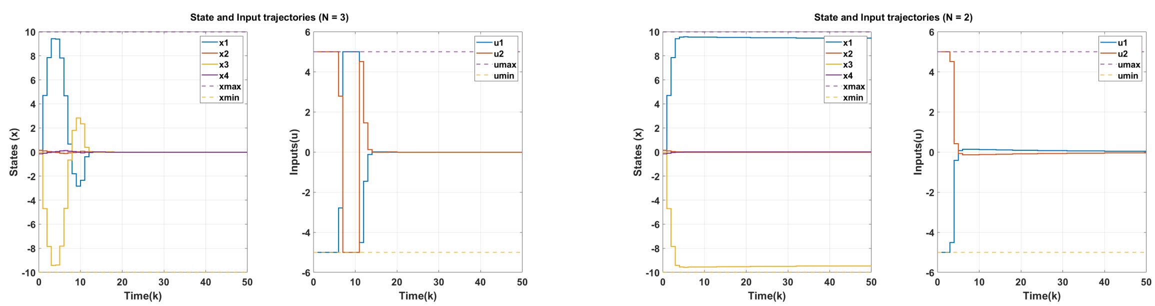


Figure 1: State and Input trajectories of Constrained MPC for Prediction Horizon (N) = 8

1.3 Shortest prediction horizon for the stabilizing quasi-LPV MPC

The shortest prediction horizon N for which the quasi-LPV MPC can stabilize the system to the origin is **3**. The state and input trajectories are plotted in Figure 2. Additionally, Figure 2b illustrates that for $N = 2$, the MPC control law fails to achieve asymptotic stability of the system. When trajectories in Figure 2 are compared with the trajectories for a longer prediction horizon of $N = 8$ (see Figure 1), it becomes evident that a shorter prediction horizon leads to increased angular deviation trajectory overshoot.



(a) Trajectories for N = 3

(b) Trajectories for N = 2

Figure 2: State and Input trajectories of Constrained MPC

2 Quasi-LPV Model Predictive Control design with stability and recursive feasibility guarantees

In order to compute the terminal ingredient, the value of P must be computed from the given closed-loop inequality (17),

$$(A + B_1K)^\top P(A + B_1K) - P \preceq -(Q + K^\top RK) \tag{17}$$

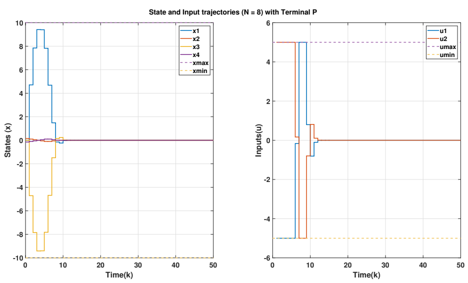
Since inequality is non-linear, by taking the schur complement of (17), turns the problem into a LMI which is further computed using $YALMIP$ in MATLAB. Let $P = O^{-1}$ and $K = YO^{-1}$ such that

$$\begin{bmatrix} O & (AO + B_1K)^\top & O & Y^\top \\ (AO + B_1K) & O & 0 & 0 \\ Y & 0 & Q^{-1} & 0 \\ Y & 0 & O & R^{-1} \end{bmatrix} \tag{18}$$

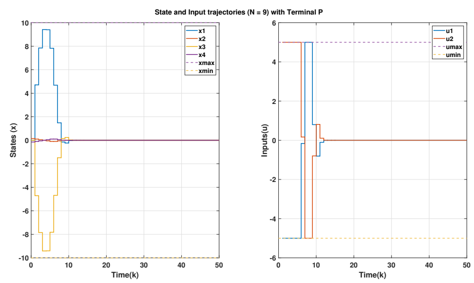
Via computing (18), P is obtained, for which the cost function can be formulated as

$$J(x(k), U_k) = x^T_{N|k} P x_{N|k} + \sum_{i=0}^{N-1} \left(x^T_{i|k} Q x_{i|k} + u^T_{i|k} R u_{i|k} \right) \tag{19}$$

Using the value of K, a terminal set $\mathbb{X}_T = \{\xi \in \mathbb{R}^n : M_N \xi \leq b_N\}$ is computed which holds to be invariant, $M_N(A + BK)\xi \leq b_N$ and constraint admissible $(M + EK)\xi \leq b$ for the closed-loop system.
The designed closed-loop system does not lead to recursive feasibility when using the modified cost function with terminal ingredients for the prediction horizon $N = 2$. Recursive feasibility is guaranteed for N=6 and after.



(a)



(b)

Figure 3: (a) State and input trajectories for prediction horizon N=8 (b) State and input trajectories for prediction horizon N=9

References

[1] P. S. G. Cisneros, S. Voss, and H. Werner, "Efficient nonlinear model predictive control via quasi-lpv representation," in *2016 IEEE 55th Conference on Decision and Control (CDC)*, 2016, pp. 3216–3221.

[2] "Lectures slides and supporting material from the Model Predictive Control course (5LMB0)," 2024, Eindhoven University of Technology.