1. a. function $f : A \to B$ is one-to-one if and only if

$$\forall x_1, x_2 \in A \ (f(x_1) = f(x_2) \implies x_1 = x_2)$$

b. function $f : A \to B$ is not one-to-one if and only if

$$\exists x_1, x_2 \in A \ (f(x_1) = f(x_2) \wedge x_1 \neq x_2)$$

c. function $f : A \to B$ is onto if and only if

$$\forall y \in B \ \exists x \in A \ (f(x) = y)$$

d. function $f : A \to B$ is not onto if and only if

$$\exists y \in B \ \forall x \in A \ (f(x) \neq y)$$

2. Are the two functions f and g defined by:

a. $f : \{0,1\} \to \{0,1\}; f(x) = x^2$ and $g : \{0,1\} \to \{0,1\}; g(x) = x$ are equal, since $f(0) = g(0) = 0$ and $f(1) = g(1) = 1$.
b. $f : [0,1] \to [0,1]; f(x) = x^2$ and $g : [0,1] \to [0,1]; g(x) = x$ are not equal, since for $x = 0.5$, $f(0.5) = 0.25$ and $g(0.5) = 0.5$.

3.
- The codomain is $[0, 10]$.
- The range is $[0, 4]$, because x^2 for $x \in [0, 2]$ only reaches values between 0 and 4.
- The function is not surjective, because not every value in [0, 10] is mapped to. For example, there is no x $\in [0, 2] such that f(x) = 5$.

4. No, $f(x) = x^2$ is not injective on $\mathbb{R}$ because $f(x) = f(-x)$ for all $x \in \mathbb{R}$. For eg, $f(1) = f(-1) = 1$.

5. f is bijective because it has an inverse $f^{-1}(a, c) = (a, a \oplus c)$.
g and h are not bijective because they are not surjective; not all elements in $\{0,1\}^2$ are mapped to.
This relates to array storage, where injectivity and surjectivity impact whether memory addresses can be uniquely mapped.

6.
$$1 \leq 3x + 5 < 3$$

Subtract 5 from sides:

$$-4 \leq 3x < -2$$

Divide by 3:

$$-\frac{4}{3} \leq x < -\frac{2}{3}$$

Therefore, the solution is x $\in \left[-\frac{4}{3}, -\frac{2}{3}\right)$.

7. a. If a sequence is arithmetic and starts with $1, 2$, the next term is **3**.
   b. If a sequence is geometric and starts with $1, 2$, the next term is **4**.

8.

$$S = \sum_{k=0}^{7} 6^k = \frac{6^8 - 1}{5}$$

9. Simplifying the expression:

$$\sum_{k=2}^{1000} \frac{3^{2k+4}}{2^{3k+5}} = \sum_{k=2}^{1000} \left(\frac{9}{8}\right)^k \times \frac{81}{128}$$

This forms a geometric series, and we leave the large powers un-evaluated