Proof Portfolio

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Theorem 1. *Let $f : A \to B$ be a function and let A_1 and A_2 be subsets of A. Prove that if f is one-to-one then*

$$f(A_1 \cap A_2) = f(A_1) \cap f(A_2).$$

Proof. Suppose $f : A \to B$ is an injective function.

If $x \in f(A_1 \cap A_2)$, then there exists some $a \in A_1 \cap A_2$ such that $f(a) = x$. Since $a \in A_1 \cap A_2$, then a is also an element of A_1, which means $f(a)$ is an element of $f(A_1)$. In the same way, $f(a)$ is also an element of $f(A_2)$. As a result, $f(a)$ is an element of $f(A_1) \cap f(A_2)$ which implies $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$.

Next, let y be an element of $f(A_1) \cap f(A_2)$. Then, there is an $a_1 \in A_1$ and $a_2 \in A_2$ such that $f(a_1) = y$ and $f(a_2) = y$. From this we can conclude that $a_1 = a_2$ since $f(a_1) = f(a_2)$ and by the function being injective. This implies $f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2)$.

The only way $f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2)$ and $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$ is if $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$. ∎