Proof Portfolio

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Theorem 1. *A 3-regular graph G has a cut-vertex if and only if G has a bridge.*

Proof. Assume G is a 3-regular graph with a cut-vertex v. Then $G - v$ can be seperated into two components G_1 and G_2 or three components G_1, G_2, and G_3. Consider the case where there are two components. Since G was originally a 3-regular graph, without loss of generality v has a vertex u that is adjacent in G_1 and two vertices that are adjacent in G_2. As such, there is a bridge uv between the two components. Next, consider the case in which there are 3 components. Then, v has vertex u, x, and y that is adjacent in G_1, G_2, and G_3 respectively. As such, there are bridges uv, xv, and yv.

Assume G is a 3-regular graph with a bridge uv. Then the vertices of the bridge, u and v, are also cut-vertices since removing them would remove their edges too. The bridge is a part of the edges that will be removed. Next, assume G has bridges uv, xv, and yv. By the same logic used previously, the vertices of the bridges are also cut-vertices.

Thus we can conclude that a 3-regular graph G has a cut-vertex if and only if G has a bridge. ∎