Proof Portfolio

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Theorem 1. *Let a be a fixed real number. Then*

$$\sum_{i=0}^{n}(a+i) = \frac{1}{2}(n+1)(2a+n)$$

for every nonnegative integer n.

Proof. We proceed by induction.
Let $n = 0$ be the smallest nonnegative integer. Then,

$$\sum_{i=0}^{0}(a+i) = a = \frac{1}{2}(0+1)(2a+0).$$

Thus, the result holds when $n = 0$.
Assume that

$$\sum_{i=0}^{k}(a+i) = \frac{1}{2}(k+1)(2a+k)$$

for a nonnegative integer k. We show that $\sum_{i=0}^{k+1}(a+i) = \frac{1}{2}(k+2)(2a+k+1)$. Observe that

$$\begin{aligned}
\sum_{i=0}^{k+1}(a+i) &= (a+k+1) + \sum_{i=0}^{k}(a+i) && (1)\\
&= (a+k+1) + \frac{1}{2}(k+1)(2a+k) && (2)\\
&= \frac{1}{2}(2ak+4a+k^2+3k+2) && (3)\\
&= \frac{1}{2}(k+2)(2a+k+1), && (4)
\end{aligned}$$

where in (2), we use the Inductive Hypothesis.

Thus, by the Principle of Mathematical Induction, we conclude that $\sum_{i=0}^{n}(a+i) = \frac{1}{2}(n+1)(2a+n)$ for all nonnegative integers n. ∎