1. Knowledge and the Universe

This chapter explores how the process of building knowledge in astronomy mirrors how we draw conclusions in everyday life. It introduces probabilistic inference, a framework for systematic decision making in many sciences. You will be invited to practice delineating prior assumptions from new evidence, understand how different individuals might draw different conclusions in light of the same evidence, and explore what it takes to change our minds in life and shift paradigms in science. The goal of this practice is to empower you to confidently expand your own knowledge and understanding of the universe we live in.

1.1 Drawing conclusions

Every day, throughout our lives, we are required to believe certain things and not to believe others. This applies not only to the “big questions” of life, but also to trivial matters, and everything in between. For example, on any given morning you may choose to board a bus in order to get to the university campus. When you do so, you board the bus with confidence that the bus will actually take you to campus, and not to the airport.

*How do you **know** the bus would not take you to the airport?*

Well, for starters, you may have taken the same bus many times before and it has always taken you to campus. If you haven’t taken the bus before, perhaps you received guidance from a friend or looked up information about the local bus schedule online. Another clue (or piece of evidence) may be that as the bus drove down the street, you may have been able to observe that the bus has a giant label reading “USC” on it—whereas a bus to the airport probably would have said “LAX.” None of this evidence proves that the bus would take you to USC, but it does make it very plausible. Given all these pieces of information, you can board the bus feeling quite certain that the bus will take you to USC. In fact, you feel so certain about this that the possibility of an unplanned trip to the airport never even entered your mind, perhaps even until you read this paragraph.

Our brains are very often able to accurately predict the correct answer to many questions of this nature (e.g. the destination of a bus), even though we don’t have all the available information that we would need to be 100% certain. We do this using a combination of (i) our past experience of the world/intuition (e.g., in the past, the bus always went where it said it would) and (ii) presently available clues/evidence (e.g., a bus sign). Usually, we come to conclusions without much conscious attention or dedicated problem solving. Most people will never think twice about how or why they are so confident; they simply take their mind's conclusion and move on.

While the bus example is fairly sterile, as you can fairly easily and confidently figure out the destination of a bus, this same strategy of drawing conclusions applies to much more complicated topics. For example, imagine that instead of the destination of a bus, you wish to know the answer to something more controversial:

> **Example 1.1 (Classes and pandemic.)** Is it safe to return to in-person classes while COVID-19 is still around? Think about how you would answer this question.

> **Example 1.2 (Curvature.)** Is it true that our Universe could be curved, like a surface of a ball? Think about what makes you believe/doubt this statement.

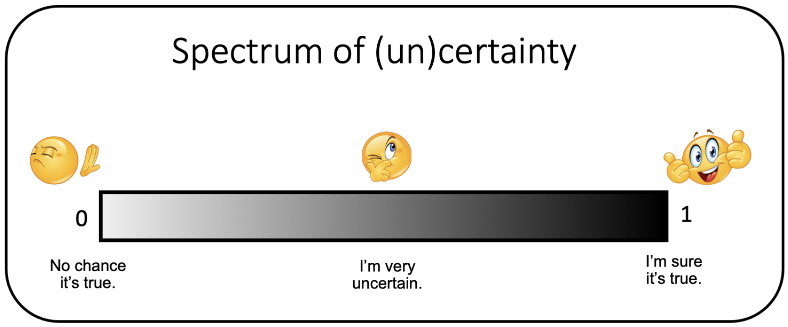
These are questions whose answers are not straightforward to simply look up online. Even worse, for some questions, you might find opposing answers on various websites and end up needing to somehow make sense of all the overwhelming and contradictory information that the internet provides. Such situations are common: juggling many pieces of evidence and taking into consideration opposing opinions and beliefs is often necessary to arrive at conclusions in life and science. Wouldn't it be convenient to have a tool that can churn all that with some good common sense and just pop out an answer that is most likely to be correct? And it would be extra nice if that answer came with a label like "take with 80% confidence." That way, we would have not only our "best bet," but also extra information to help us decide how sure we should be in the correctness of the answer (if the confidence is high, we may want to bet more money on it!).

In the following, we will get familiar with a tool called **probabilistic inference** that serves precisely this purpose in many sciences that rely on complex sets of data: it is a way to systematically assess all the factors that contribute to the process of drawing conclusions. But the most important message will be this: scientific inference is fundamentally a human endeavor; as such, it has to deal with the same issues we encounter in everyday life: uncertainty, assumptions, and change. Let's talk about uncertainty first.

1.2 (Un)certainty as probability

All of us have at some point said things like "I'm not 100% certain." The idea of using a number to describe how certain you are is quite natural. For example, contestants on the TV show "Who Wants to be a Millionaire" often say things like "I'm 75% sure the answer is A." There are some interesting things to notice about this statement. Firstly, it is a subjective statement. If someone else were in the seat trying to answer the question, she might say the probability that A is correct is 100%, because she knows the answer! A third person faced with the same question might say the probability is 25%, because he has no idea and only knows that one of the four answers must be correct.

In the process of scientific inference, **probability** is used to quantify our certainty that a statement is true. If the probability is 1, you are sure that the statement is true. So sure, in fact, that nothing could ever change your mind. If the probability is 0, you are sure that the statement is false. If the probability is 0.5, then you are as uncertain as you would be about the result of a fair coin flip. If the probability is 0.95, then you're quite sure the statement is true, but it wouldn't be hugely surprising if you found out the statement was false. See the illustration below for probabilities as degrees of (un)certainty.



It might sound like there is nothing more to this than just thinking about a question and then blurting out a probability that feels appropriate. Fortunately for us, there is more to it than that. To see why, think about how you change your mind when new evidence becomes available. For example, you may be on "Who Wants to be a Millionaire?" and not know the answer to a question, so you might think the probability that it is A is 25%. But if you call your friend using "phone a friend," and your friend says, "It's definitely A," then you would be much more confident that it is A. Your probability probably wouldn't go all the way to 100% though, because there is still a small possibility that your friend is mistaken. If you don't believe me, imagine the pit that would be in your stomach as you wait through the overly-dramatic extended pause before the host finally announces the correct answer. That feeling comes from your intuitive understanding that some uncertainty still remains.

Scientific knowledge is not immune to uncertainty either. Think how many times we've said in class things like "the mass of the Milky Way galaxy is *about* 900 billion solar masses." Or you may have read on Wikipedia that "the age of the Universe is 13.799 billion years, plus/minus 0.021 billion years." Or you may have noticed that "we are *confident* that the Universe is expanding today, but we *think* that the Moon formed by a collision between the Earth and another object." All these expressions of not being fully sure in a value of a measurement, or in a theory, or "truth," are present in science, just like in everyday life.

*Does this mean there is no such thing as **Truth**?*

No. Science assumes that truth is there, waiting to be discovered, but our knowledge of it is imperfect—and we are continually improving it. Remember, science is a human endeavor and thus admits uncertainty. And the more data we get about the solar system, the more confident we may become that the theory we construct about the Moon's origin is correct; the better our measurements of the cosmic microwave background radiation become, the more precisely we can infer the age of the Universe. But here is something really important: the better we understand this intrinsic uncertainty in the process of building knowledge, the more clear we can be about what we know and what we don't know, and the better we can decide how to move forward!

> **Example 1.3 (Comfort with uncertainty.)** Think of a situation where you had to make a choice, but the evidence you gathered made it only 80% likely that you're right. Is this enough? Can you think of situations where it is, and situations where it isn't? What do you think is the probability/level of certainty that is enough for scientists to announce a discovery?

> **Example 1.4 (Compare theories.)** Think of two theories mentioned in this course, and discuss how their uncertainty/probability compares. Which one would you be less surprised if it gets refuted? Why?

> **Example 1.5 (A safe bet.)** Imagine you're making a bet on the heads/tails result of a coin flip. How much money would you put on that bet? Imagine instead that you're making a bet that it will not rain tomorrow. How much money would you put this time? Finally, imagine that you are a National Science Foundation officer, deciding whether to fund a new CMB experiment, whose team claims that they would discover gravitational waves from the Big Bang, based on a theoretical prediction; they are asking for 50 million dollars. Would you be willing to make that bet?

1.3 Probabilistic inference

We will now demonstrate how probabilistic inference works. Our first example is very simple: Suppose there are two balls in a box. We know in advance that at least one of the balls is grey, but we're not sure about the color of the second ball. The second ball may be either grey or white. Our goal in this problem is to infer the contents of the box—after performing an experiment.

As the very first step, we will identify our hypotheses (or theories). Our hypotheses should cover all possible outcomes. In this case, our hypotheses are as simple as the problem setup itself: either both balls in the box are grey, or one ball is grey while the other is white. These are the only two possibilities we will consider. To keep things concise, we can label our two competing hypotheses. We could call them whatever we want, but let's call them GG and GW. So, at the beginning of the problem, we know that one and only one of the following hypotheses/theories/statement is true:

Hypothesis 1 – GG – Both balls are grey.

Hypothesis 2 – GW – One ball is grey, one ball is white.

These hypotheses are mutually exclusive, and exhaustive. In other words, only one of these possible hypotheses can be correct, and one of these possibilities must be correct. This means that when we add up their probabilities, they must add up to 1.

Now that we have identified our competing hypotheses, but before we even consider any data from an experiment that could help us determine the contents of the box, we start by applying any knowledge we might already have about the problem. With this, we can decide what probability is there for either hypothesis to be true. These probabilities that account for any information we have at the beginning of the problem are called **prior probabilities**, or simply **priors**. Since we are considering two hypotheses, there will therefore be two prior probabilities: one for GG and one for GW. For simplicity, we will assume that we don't have much of an idea which is true, and so we will use the following prior probabilities:

$P(GG) = 0.5$

$P(GW) = 0.5$

Here, we have introduced new notation, where P indicates probability. In each case, the argument inside the parentheses is the hypothesis we are testing. So, P(GG) is the probability of GG (the probability that both balls are grey) and P(GW) is the probability of GW (the probability that one ball is grey and one ball is white). In general, P(whatever) is the probability of "whatever." We assign both probabilities to 0.5 because, lacking additional information, we assume both outcomes are equally likely.

> Definition 1.1 **Prior** is the probability that a certain hypothesis is correct *before* (or prior to) considering any evidence, or data.

We're going to conduct an experiment that will help us get a better idea of which of these two hypotheses is true. The experiment is this: we will reach into the box without looking, pull out one of the balls, and observe its colour. But before we do that, let's make predictions about possible outcomes. If Hypothesis GG is correct, or "under Hypothesis GG," the probability of pulling out a white ball is zero, $P(W|GG) = 0$, and we are sure to pull out a grey ball, $P(G|GG) = 1$. We again have a new notation here: the sign "$|GG$" just means "under Hypothesis GG." If Hypothesis GW is true, then the probability of pulling out a grey ball is the same as the probability of pulling out a white ball, $P(G|GW) = P(W|GW) = 0.5$. Great! The predictions we have just made are actually probabilities of collecting any possible data, under each of the two hypotheses. And we're going to call that probability the **likelihood** of the data.

> Definition 1.2 **Likelihood** is the probability that a certain outcome (or data) would result *if* a certain hypothesis is true.

Why make these predictions? As you can imagine, a theory that better succeeds in predicting what we actually observe is more likely to be true. So, that's going to be useful information, once we collect our data. So let's do the experiment! The result of this experiment is the following data:

The result/data: The ball that was removed from the bag is grey.

Given this new data, or this new information, we have the opportunity to make a better inference about what's in the box. But let's set our expectations: by pulling out a grey ball, we can't know for certain which color the other one is. Still, by pulling out the grey one, one of the outcomes just became less likely. Can you guess which?

Answering that question quantitatively is our goal: we want to know the probability of Hypothesis GG and of Hypothesis GW, given all the priors *and* all the data, or information, we have collected. This will be called the **posterior probability** ("post" means "after," and refers to "after the experiment").

> Definition 1.3 **Posterior** is the probability that a hypothesis is true *after* considering all the information/evidence/data/experiments, and all the priors, and taking into account all the competing hypotheses.

To figure this out, and to show you a systematic way of figuring this out for any problem you may encounter in the future, let's construct something we will call the **Inference Box**. This will be a table with 5 columns:

- Hypothesis — In this column, we list all possible hypotheses. In our example, that is GG and GW.
- Prior — In this column, we put all corresponding priors. In our example, that would be P(GG) = 0.5 and P(GW) = 0.5
- Likelihood — In this column, we put all of the calculated likelihood values. In our example, that would be $P(G|GG) = 1$ and $P(G|GW) = 0.5$.
- h = Prior × Likelihood — In this column, we simply write down the product of the prior and the likelihood.
- Posterior — In this column, for each hypothesis, we write down h value divided by the total of all h values.

Let's begin:

Hypothesis	**Prior**	**Likelihood**	**h=Likelihood×Prior**	**Posterior**
GG	0.5	1	$1 \times 0.5 = 0.5$	?
WG	0.5	0.5	$0.5 \times 0.5 = 0.25$	?

No worries—we needed to do some simple number multiplication to get the value h, but that's as complicated as math will get in this chapter. Now, what we really want to know is that last column—the posterior. This will be what we get when we divide h for that hypothesis with the sum of all h values; that sum is $0.5 + 0.25 = 0.75$. So, we get this:

Hypothesis	**Prior**	**Likelihood**	**h=Likelihood×Prior**	**Posterior**
GG	0.5	1	0.5	0.666
WG	0.5	0.5	0.25	0.333

Ta-dah! The result of our probabilistic inference approach tells us that given equal priors and with one grey ball drawn out, there is a two-thirds chance that both balls in the bag are grey, and one-third chance that one ball is white.

The beauty of the Inference Box is that you can repeat it however many times you want, to incorporate any new information you get. For example, imagine putting the grey ball back into the box, shaking the box, and drawing a grey ball again. Now it's even more likely that the true hypothesis is GG. But how much more? What is the posterior this time? You can answer this by constructing a second Inference Box and using your previous posteriors as your new priors; that is, you can now start with P(GG) = 0.666 and P(WG) = 0.333 and take it away!

Example 1.6 (Fair coin?) Imagine that someone give you a coin, and you flip it, without looking. It's heads. Now, the person tells you that this is either a regular fair coin, or one that has 2 heads. After that one flip, what is the probability that this is a fair coin? And how does it change if you flip to heads twice in a row?

Example 1.7 (Unfair coin?) Imagine that someone gives you to pick one of 10 coins, without looking. And after you take one, she tells you that out of the 10, one was with double heads. You flip your coin, still without looking. It comes up heads. After that one flip, what is the probability that this is that unfair? How many times in a row do you need to get heads before you are fairly sure that you've drawn an unfair coin?

Example 1.8 (Three balls in a box.) Repeat the experiments with balls in a box, but instead of 2, use 3 balls; you will begin with one of the following combinations of balls in the box: GWW, WWG, WWW, or GGG. Imagine you draw G. Construct an Inference Box for this example. After you have drawn one G, what is the probability of each hypothesis?

1.4 Changing minds and shifting paradigms

Now that we're armed with understanding what priors are, and how to use them for probabilistic inference, let's talk about using priors in life. In the case of a well-labeled bus, most well-informed and reasonable riders will arrive at the same conclusions about the bus's destination. However, there are other examples where two observers can observe the same situation and reasonably arrive at different conclusions. For example, imagine that someone tells you that there's a traffic jam on the highway leading to your work.

*Is this person telling the **truth**?*

You may know this person, and you may know that they are both sane and trustworthy. You thus subconsciously assign a "high prior" to the hypothesis that they are telling the truth, even before they open their mouth. In that case, you will probably consider taking an alternate route to work that day. At the same time, your sister may not know this person and might decide that her prior is low, so she will check about the traffic on her phone before deciding which path to take.

Ultimately, determining an answer to the question "is this person telling the truth?" requires an individual to combine both the observations/evidence they have access to (likelihood) with their own personal assumptions (priors), which are often based on past experiences ("is this person generally trustworthy?"). Both fundamentally factor in to any conclusions we draw, or opinions we form. While we would hope that most rational individuals would be able to accurately and confidently select the right bus to get them to their desired destination, we would not necessarily expect that a varied group of people would arrive at the same conclusion about the truthfulness of another person's statement. And this often boils down to their distinct priors (or assumptions).

> **Example 1.9 (Bad priors.)** Can you think of an example situation where a prior assumption causes a problem?

> **Example 1.10 (Good priors.)** Can you think of an example situation where a prior assumption is helpful?

Take another example: if you read in a tabloid magazine that an alien spaceship was seen above the city on a Friday night, you may immediately discard this. Your prior on the trustworthiness of a tabloid was low to begin with, but the claim itself was also quite unbelievable. Some stronger evidence was needed to take that story seriously. This is a good thing—our intuition or priors help us efficiently navigate our internet-based information-drenched world more easily, by readily rejecting "nonsense" and only considering things we know could be true.

However, living and learning as human beings, we encounter times when we need to undergo a radical shift of perspective and dramatically change our minds, altogether throwing away our assumptions and forming new and fresh beliefs. Even in science, things like this can happen, and they are called **paradigm shifts**. Think about a shift from a Geocentric to a Heliocentric model that changed the way we see ourselves and our place in the Universe. Or think about what would happen if you discover that the "trustworthy" person from the above example actually lied to you; you may not hold them as trustworthy any more.

> **Example 1.11 (Changing minds.)** Consider something you have always believed to be true. Think about why, and what it would take to change your mind. This example can be something from life or from science.

> **Example 1.12 (Shifting paradigms.)** Think of a discovery you learned about in this class that shifted your understanding of the Universe. How uncomfortable are you with this new knowledge? Why do you believe it, or why do you still doubt? What would make you convinced?

1.5 What do we do with this?

Congratulations—if you read this chapter and honestly thought about its content, you've uncovered the process by which many sciences, including astronomy, sort out and improve theories that underpin our knowledge of the universe. You've seen the core of the "scientific process" as a probabilistic game of "betting" on explanations that are the most likely to be true, given all the evidence we have. And you've seen how this process is subject to uncertainty and change, just like the process of drawing conclusions in everyday life. With this, you may have some questions bubbling in your mind, so let's see if I can anticipate some of them:

*If this is how it works, what is then **scientific** about scientific inference?*

The answer has two parts. First, scientists do inference systematically. While you might rarely think of drawing an Inference Box to make a decision on whether to trust a piece of news or not, scientists routinely follow probabilistic inference, or a similar quantitative method, when they are trying to answer a scientific question. Second, scientists do not pick and chose which data to consider. They (informally) pledge not to reject any piece of evidence when testing a theory, as a part of our "code of ethics." The way we make sure this code of ethics is upheld is this: we expect, as a community, that any result or a discovery must be possible for other scientists to reproduce and test. And so we do. All big scientific discoveries and measurements have been tested over and over again, and often took a long time to actually sink into the community and become what we may call "paradigms."

Okay. Now, you may also wonder:

How can probabilistic inference apply in real life, without drawing the Box?

When you "walk out" of the zoom classroom, you may never want to draw another Inference Box again, and that is just fine. But here's a message you may want to keep with you: if honestly applied in broad strokes, the practice of noticing the difference between priors and evidence, like you've done in probabilistic inference, can not only lay bare our own biases, but also allow us to see and appreciate the biases that others around us weave into the conclusions they draw about the world. Moreover, we can see that biases/assumptions/priors are not intrinsically good or bad, but rather fundamental ingredients of conclusions, even those in science. The more clearly we understand them, the more we empower ourselves to construct an informed worldview and understand that of others.

And finally, I hope that you can now appreciate that some things (in life and in science) are known with a high degree of certainty, while others are not. As a result, scientific theories and human beliefs change and evolve, as our minds constantly churn the wheels of inference, whether we call it "probabilistic" or not. All of that is a healthy part of the very human process of building knowledge—and I hope you continue to partake in it with confidence.

> **Example 1.13 (Science and Truth)** Recently, a famous astrophysicist tweeted: "The good thing about Science is that it's true, whether or not you believe in it." Discuss why you do or do not agree with this statement.

> **Example 1.14 (Climate change.)** The following statement was made by a US senator a few years ago: "I'm a big believer that we should follow the science, and follow the evidence. If you look at global warming alarmists, they don't like to look at the actual facts and the data. The satellite data demonstrate that there has been no significant warming whatsoever for 17 years." The senator used this argument as proof that there is no global warming. Discuss why you do or do not agree with it.

1.6 References

For additional information and a great and accessible introduction to Bayesian Statistics, see the very nice course notes written by Brendon J. Brewer here: `https://www.stat.auckland.ac.nz/~brewer/stats331.pdf`. In accordance with the Creative Commons license, this text has been adopted from that original source, with significant changes as appropriate for this course. This chapter has also borrowed from the "Knowledge and the Universe" course taught by P. Torrey at the University of Florida. The changes made are abundant in nature, and are in no way endorsed by the original authors.