ANSWERS CHAPTER ONE

EXERCISE 1A

1. Explanatory variable: Gender
Response variable: Church attendance
2. Explanatory variable: Age of the car
Response variable: Asking price.
3. Explanatory variable: Smoking classification
Response variable: Cancer
4. Explanatory variable: Gender
Response variable: Favourite season.
5. Explanatory variable: Gender
Response variable: Frequency of binge drinking.
6. Explanatory variable: Gender
Response variable: With whom it is easiest to make friends.
7. Explanatory variable: Lack of sleep
Response variable: Performance in a team game.
8. Explanatory variable: Type of flight.
Response variable: Arrival time
9. Explanatory variable: Age of people
Response variable: Station listened to.
10. Explanatory variable: Played a sport
Response variable: Name on honour board
11. Explanatory variable: Age of person
Response variable: Preferred choice of beverage.
12. (a)

	Job	No Job	Total
Had license	70	110	180
No license	20	100	120
Total	90	210	300

(b) 120 (c) 210 (d) 78% (answer to nearest 1%) (e) 17%
(f) 39% (g) 40%

13. (a)

	Female	Male	Totals
Positive response	80	50	130
Negative response	70	40	110
Totals	150	90	240

(b) 90 men (c) 70 women (d) 54% (nearest %) (e) 53%
(f) 36% (g) 44%.

14. (a)

	Female	Male	Totals
Worked part time	67	88	155
Did not work part time	31	37	68
Totals	98	125	223

(b) 223 students (c) 68% (d) 70% (e) 43% (f) 30% (g) 54%

15. (a)

	Scooter	No scooter	Totals
Bicycle	23	25	48
No bicycle	3	5	8
Totals	26	30	56

(b) 23 students (c) 3 students. (d) 45% (e) 48% (f) 9%
(g) 17%

EXERCISE 1B

1. (a)

	Maths	Science	English	Totals
Females	13.1%	14.5%	20.9%	48.5%
Males	20.7%	18.5%	12.4%	51.6%
Totals	33.8%	33.0%	33.3%	100.1%

(b)

	Maths	Science	English	
Females	39%	44%	63%	
Males	61%	56%	37%	
Totals	100%	100%	100%	

(c)

	Maths	Science	English	Totals
Females	27%	30%	43%	100%
Males	40%	36%	24%	100%

2. (a)

	Choc	Vanilla	Straw	Mango	Totals
Children	11%	3%	12%	2%	28%
Teenagers	10%	6%	9%	6%	31%
Adults	9%	11%	6%	14%	40%
Totals	30%	20%	27%	22%	99%*

* Total 99% instead of 100% due to rounding.

(b)

	Choc	Vanilla	Straw	Mango	
Children	37%	16%	46%	11%	
Teenagers	34%	28%	33%	29%	
Adults	29%	56%	21%	61%	
Totals	100%	100%	100%	101%	

(c)

	Choc	Vanilla	Straw	Mango	Totals
Children	38%	11%	43%	8%	100%
Teenagers	32%	19%	29%	21%	101%
Adults	22%	29%	14%	35%	100%

3. (a)

	Age of respondent in years					
Opinion	18 to 29	30 to 41	42 to 53	54 to 65	Over 65	Totals
In favour	2.6%	2.6%	7.1%	5.7%	2.0%	20.0%
Not in favour	4.3%	6.9%	15.7%	17.4%	12.9%	57.2%
Undecided	5.7%	5.7%	5.4%	4.6%	1.4%	22.8%
Totals	12.6%	15.2%	28.2%	27.7%	16.3%	100%

(b)

	Age of respondent in years					
Opinion	18 to 29	30 to 41	42 to 53	54 to 65	Over 65	
In favour	20%	17%	25%	21%	12%	
Not in favour	34%	45%	56%	63%	79%	
Undecided	45%	38%	19%	16%	9%	
Totals	99%	100%	100%	100%	100%	

(c)

	Age of respondent in years					
Opinion	18 to 29	30 to 41	42 to 53	54 to 65	Over 65	Totals
In favour	13%	13%	36%	29%	10%	101%
Not in favour	8%	12%	28%	31%	23%	102%
Undecided	25%	25%	24%	20%	6%	100%

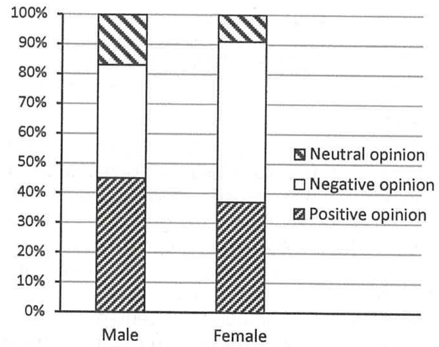
EXERCISE 1C

1. As we move across the explanatory variable categories "Male" and "Female" the big differences between columns in the percentages in the response variable categories indicate that there is an association between gender and opinion.
The significant differences in the percentages suggest a strong association. Males tend to be against the proposal and females tend to be in favour of the proposal.
2. The male – female proportions (percentages) are identical for the response variable. That is, with regard to opinion in this survey it does not matter what gender the respondent is. As opinion is not influenced by what gender we are considering this indicates that there is no association between gender and opinion in this survey. Males and females are both very strongly against the proposal.
3. As we move across the explanatory variable categories "Male" and "Female" the big differences between columns in the percentages in the response variable categories indicate that there is an association between gender and opinion.
The significant differences in the percentages suggest a strong association. Males tend to be in favour of the proposal and females tend to be against the proposal.
4. The male – female proportions (percentages) are nearly identical for the response variable. That is, with regard to opinion in this survey it does not matter what gender the respondent is. As opinion is not influenced by what gender we are considering this indicates that there is no association between gender and opinion in this survey. Males and females are approximately evenly split on this proposal.
5. As we move across the age bracket categories the percentage of people in favour gradually decrease from 25% to 9% (25%, 20%, 17% 13% 9%). The percentage of people against increase significantly from 30% to 82% (30%, 44%, 58%, 67%, 82%) and the percentage that are undecided decrease from 45% to 9%.
These changing percentages across the age brackets indicate a strong association between opinion and age brackets.
6. The opinion percentages show very little change as we move across the age brackets of the respondents.
The figures from this survey suggest that there is no association between the age bracket and the opinion for this survey.

7. As we move across the categories of age brackets the percentage of people in favour decrease progressively from 70% to just 25%. At the same time as we move across the age bracket categories the percentage of people against the proposal increase from 22% to 67% (22%, 40%, 47%, 58% 67%)
Even though there is very little variation in the percentages that are undecided across the age brackets the changing percentages for the in favour and against indicate a strong association between opinion and age brackets.
8. (a) Explanatory variable: Age in years; Response variable: Film type.
(b) As we move across the age categories changes in the percentages of most preferred film are very evident in the table. For example the comedy film category increases from being the film most preferred by 21% of people in the 18 to 30 age group to 46% of people in the 31 to 43 age group and to 63% of people in the 44 to 56 age group. The western film category decreases from being the film most preferred by 58% of people in the 18 to 30 age group to just 5% of people in the 44 to 56 age group.
These changes in the percentages as we move across the age groups indicates that there is an association between age and preferred choice of film.
9. (a) Explanatory variable: Gender. Response variable: Opinion on train travel.
(b) Columns: Response variable; Rows: Explanatory variable
(c)

Gender	Positive opinion	Negative opinion	Neutral opinion	Totals
Male	45%	38%	17%	100%
Female	37%	54%	9%	100%

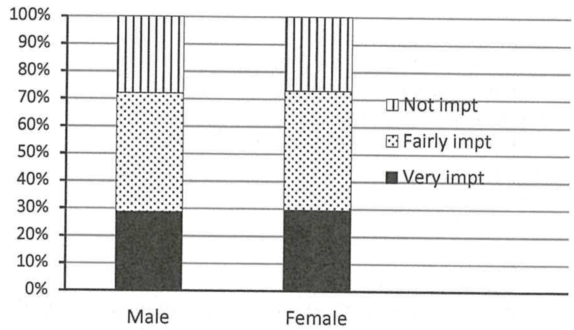
(d)



(e) As we move across the gender groups male through to female the percentage of train passengers with a neutral opinion decreases from17% to 9%; the percentage of train travellers having a negative opinion increases considerably from 38% to 54% and those having a positive opinion decreases from 45% to 37%. The changing percentages suggest a strong association between gender and train travel opinion. Females are more likely to have a negative opinion.
10. (a) Explanatory variable is the gender of the respondents and the response variable is the importance of religion in a persons life.
(b) 28.7% (c) 12.3% (d) 27.4% (e) 57.2%
(f) Row percentages as we wish to examine the patterns of the importance of the religion categories across gender. Or Row percentages as the explanatory variable categories have been assigned the rows in the two-way table.
(g)

	Very important	Fairly important	Not important	Total
Male	28.7%	43.3%	28.0%	100%
Female	29.5%	43.5%	27.0%	100%

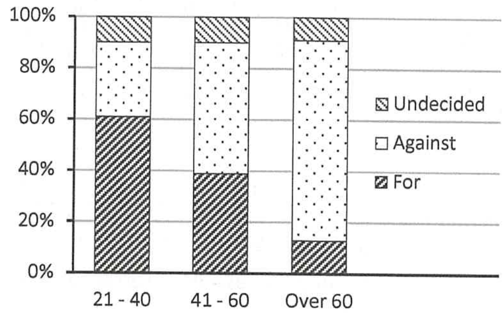
(h)



(i) No. As we move across the columns of the proportional column graph there is very little variation in the proportions of the response for males and females. The absence of change in the proportions across the gender categories suggests that there is no association or relationship between the importance of religion in the lives of these people and gender.
11. (a) 109 respondents voted for daylight saving, 108 were against daylight saving and 23 were undecided.
(b) 51.4% (c) 15% (d) Over 60 age group (e) 52.2%
(f) Explanatory variable: Age. Response variable: Support for introducing daylight saving.
(g)

	Age of respondents in years			
	21 - 40	41 - 60	Over 60	
For	61%	39%	13%	
Against	29%	51%	78%	
Undecided	10%	10%	9%	
Totals	100%	100%	100%	

(h)



(i) Yes. As we move across the age categories the big differences between columns in the percentages in the response variable categories indicate that there is a strong association between the two variables.
The percentage of people for daylight saving decreases progressively from 61% for the 21 to 40 age category, to 39% for the 41 to 60 age category to just 13% for the over 60 age category. As the vote for daylight saving decreases over the age categories the vote against daylight saving increase from 29% to 51% to 78%.
The percentage undecided about daylight saving is approx. 10% across all of the age categories.
12. To answer this question we need to construct a column percentaged two-way frequency table.

	Male	Female
Worked part time	68.6%	69.1%
Did not work part time	31.4%	30.9%
Totals	100%	100%

The given data shows that there does not appear to be any association between gender and whether students are engaged in part-time work for the surveyed respondents.
Consider the column percentage two-way frequency table:
As we move across the gender categories there is very little variation in the percentages for students that work part-time. 68.6% of the males in this survey work part-time and 69.1% of females in this survey work par-time. The percentage of students engaged in part-time work for this group across gender is approximately the same indicating that there is no association between gender and working part-time.
13. To answer this question we need to construct a row percentaged two-way frequency table.

	In favour of gun law	Not in favour of gun law	Totals
Owns a gun(s)	62.2%	37.8%	100%
Does not own a gun	84.9%	15.1%	100%

As we move across the gun ownership categories "owns a gun" and "does not own a gun" the respondents in favour increases from 62.2% of respondents that own guns to 84.4% of respondents that do not own a gun. Also those that are not in favour of a gun law decrease from 37.8% of respondents that own guns to 15.1% of respondents that do not own a gun. These big differences between the percentages indicates that there is an association between gun ownership and whether the respondent is in favour or not in favour of the registration law. Hence the data supports the conjecture.

CHAPTER ONE REVIEW EXERCISE

1. (a) (a) Explanatory variable is the gender of the children and the response variable is status regarding being immunised against contracting whooping cough.

(b)

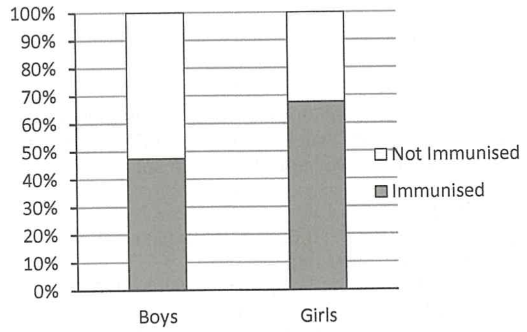
	Boys	Girls	Totals
Immunised	63	147	210
Not immunised	70	70	140
Totals	133	217	350

(c) 217 (d) 147 (e) 47.4% (f) 40%

(g)

	Boys	Girls	Totals
Immunised	47.4%	67.7%	
Not immunised	52.6%	32.3%	
Totals	100%	100%	

(h)



(i) The percentages across gender are quite different indicating an association between immunisation and gender. Girls have a higher percentage of being immunised (67.7%) than boys (47.4%)

2. (a)

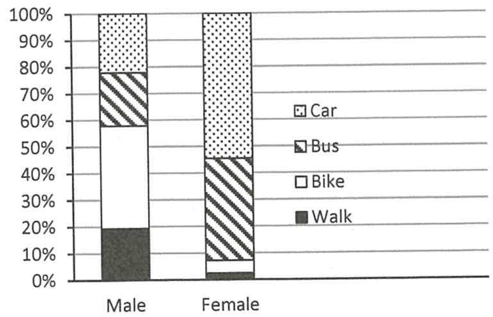
	Walk	Bike	Bus	Car	Totals
Male	26	52	27	30	135
Female	4	8	63	90	165
Totals	30	60	90	120	300

(b) 75% (c) 21% (d) 38.2%

(e)

	Walk	Bike	Bus	Car	Totals
Male	19.3%	38.5%	20.0%	22.2%	100%
Female	2.4%	4.8%	38.2%	54.5%	99.9%
Totals					

(f)



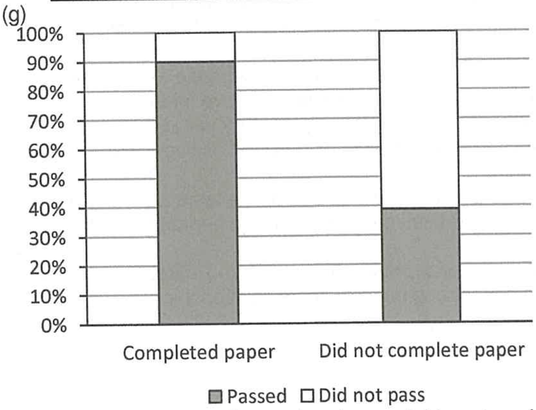
(g) As we are investigating whether mode of transport can be explained by gender, gender is the explanatory variable and mode of transport the response variable. Moving across the gender categories from male to female we find that percentage of students that walk decreases very significantly from 19.3% to 2.4% also the percentage that rode a bike decreases from 38.5% to just 4.8%. Furthermore we find that the percentages using the bus and car to get to school increase as we move across the gender categories from male to female. These changing percentages indicate a strong association between mode of transport and gender.

3. (a) Explanatory variable: Completed last year's examination paper. Response variable: Examination result. (b) 94 students (c) 68% (d) 12% (e) 10%

(f)

Examination results	Last year's examination paper: Completed paper	Did not complete paper
Passed	90%	39%
Did not pass	10%	61%
	100%	100%

(g)



(h) As we move across the explanatory variable categories, that is "Completed last year's examination paper" and "Did not complete last year's examination paper " changes in the proportions of examinations results are very evident. The percentage of students that passed the examination in the group that completed last year's examination paper declined from 90% to just 39% for those that did not complete last year's examination paper. This significant change in the percentages as we move across the segmented column graph suggests that there is an association between variables for these students.

(i) Not necessarily, we cannot make a general statement based on a small statistical study. Common sense indicates that if students complete the previous year's examination paper on similar content they will be more familiar with the material to be examined and hence will most likely do better on the actual examination.

4. (a) Explanatory variable: Duration of oestrogen use. Response variable: State of the womb

(b)

State of womb	Duration of oestrogen use: Never used	Under 1 year	1 to 5 years	More than 5 years	
Healthy womb	59%	39%	34%	8%	
Womb cancer	41%	61%	66%	92%	
Totals	100%	100%	100%	100%	

(c)

(d) As we examine and move across the categories of the explanatory variable, that is "Never used', "Under 1 year", "1 to 5 years" and "More than 5 years" the big differences between the columns in the percentages in the response variable categories suggest that there is an association between the variables.
The proportion of women with womb cancer increase quite dramatically from those that never used oestrogen. The proportion goes from 41% for those women who never used oestrogen to 92% for those women that had used it for more than five years.

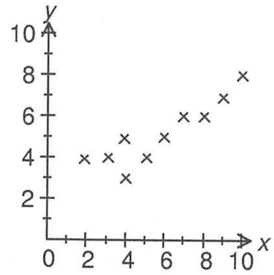
ANSWERS CHAPTER TWO

EXERCISE 2A

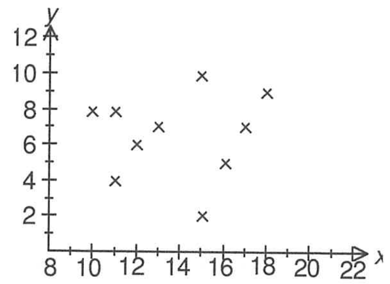
1. Graph A (b) Graph B (h) Graph C (a) Graph D (g) Graph E (c) Graph F (f) Graph G (e) Graph H (d)

2. (a) (i) Height is the explanatory variable and the weight is the response variable. (ii) moderate positive linear relationship
(b) (i) Kilometres travelled is the explanatory variable and the depth of tread the response variable. (ii) strong negative linear relationship
(c) (i) Number of litres is the explanatory variable and the price paid the response variable (ii) perfect positive linear relationship
(d) (i) Year of birth is the explanatory variable and the age is the response variable. (ii) perfect positive linear relationship
(e) (i) Salaries earned is the explanatory variable and income tax is the response variable. (ii) strong positive linear relationship
(f) (i) (i) Age of car is the explanatory variable and the price paid is the response variable. (ii) strong negative linear relationship

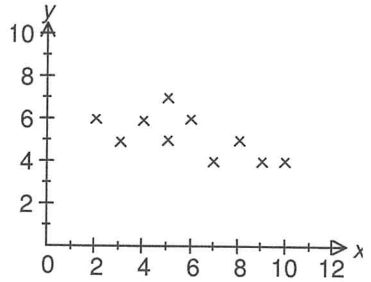
3. (a) Shape: linear, Strength: very strong, Direction: positive
Hence a very strong positive linear relationship



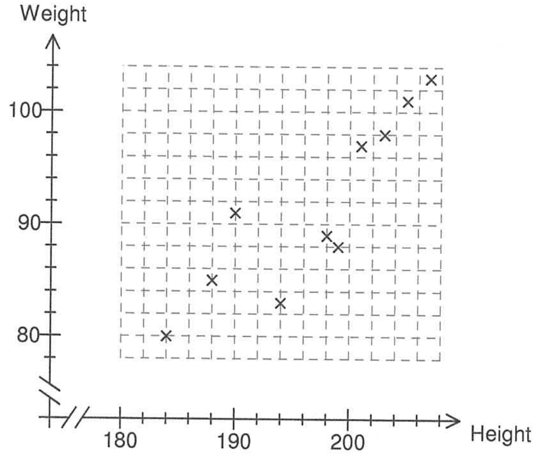
(b) Shape: not linear, Strength: none, Direction: none
Hence no significant linear relationship



(c) Shape: Linear, Strength: moderate, Direction: negative
Hence a moderate negative linear relationship

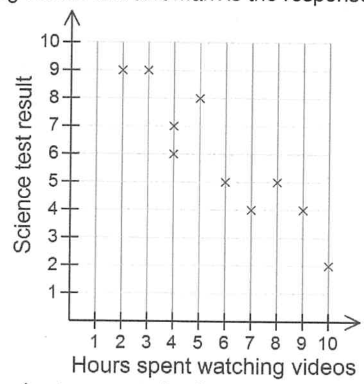


4. (a)



(b) Strong positive linear relationship.
(c) Draw a "line of best fit" and use it to read off the desired information. 94 kg (d) Not valid unless the fan is as fit and athletic as the football players.

5. (a) Explanatory variable is the number of hours spent watching videos and test mark is the response variable.
(b)



(c) There is strong negative linear relationship between the number of hours spent watching videos and Science test results. There are no outliers in this data set.

EXERCISE 2B

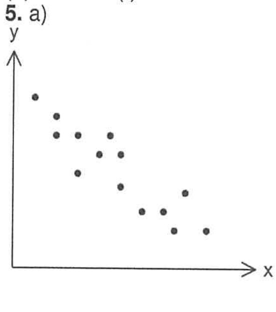
1. (a) (i) 7.25, 1.3919 (ii) 9.625, 4.7153 (iii) r = 0.7761
(iv) strong positive linear relationship
(b) (i) 7.25, 3.9607 (ii) 11.625, 3.1598 (iii) r = 0.8065
(iv) strong positive linear relationship
(c) 43.5, 6.7082 (ii) 43.375, 5.6111 (iii) r = 0.8120
(iv) strong positive linear relationship
(d) (i) 2.11, 0.57 (ii) 1.4, 0.3521 (iii) r = -0.1794
(iv) no significant linear relationship.
(e) (i) -0.556, 0.1976 (ii) 16.7, 3.6620 (iii) r = -0.9533
(iv) strong negative linear relationship.

2. (a) $r_{xy} = 0.9533$ sign has changed. (b) 0.9533 sign has changed. (c) -0.9533 no change

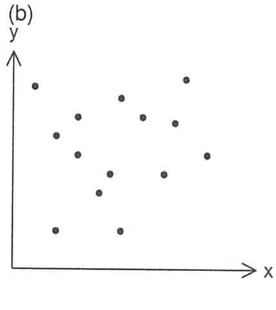
3. Graph A (i) $\overline{x} = 5$, $s_x = 2.7080$ (ii) $\overline{y} = 5.5$, $s_y = 2.5658$ (iii) $r_{xy} = 0.9235$ Graph B (i) $\overline{a} = 5.3$, $s_a = 2.7586$ (ii) $\overline{b} = 8.6$, $s_b = 4.9639$ (iii) $r_{ab} = -0.9187$
Graph C (i) $\overline{m} = 4.6$, $s_m = 2.5768$ (ii) $\overline{n} = 5.2$, $s_n = 1.6$ (iii) $r_{mn} = 0.3347$

4. (a) r = -0.95 (b) r = 1 (c) r = 0.14 (d) r = -0.32
(e) r = -1 (f) r = 0.84

5. a)



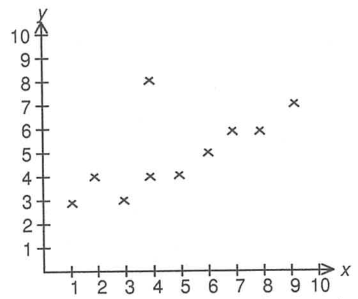
(b)



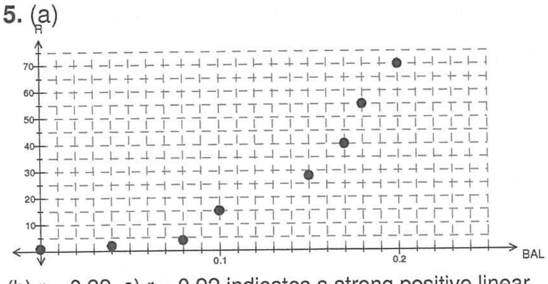
6. (a) Response variable is test 2,the explanatory variable is test 1 as test 1 will be completed before test 2. (b) 0.77
(c) The value indicates a strong positive linear relationship between the results of test 1 and the results of test 2. (d) The score attained by a student in test 2 should increase as the score in test 1 increases.
7. The statement is valid for graphs 1 and 2 only. For graphs 3 and 4 the concept of a correlation coefficient has no meaning because we do not have a situation in which two variables vary. In graph 3 the value of y is always constant that is it does not vary and in graph 4 the value of x does not vary. That is the data in graphs 3 and 4 is not *bivariate.*
8. (a) Explanatory variable is the distance and the response variable is the airfare.
(b) The scatterplot show that there is a strong positive linear relationship between distance and return airfare.
(c) Correlation coefficient = 0.937 which indicates that there is a strong positive linear relationship between distance and airfares.
(d) r = 1. There is a perfect linear relationship between "peak" fares and "off-peak" fares as each "off peak" has been multiplied by 1.4 to obtain the "peak" fare.

EXERCISE 2C

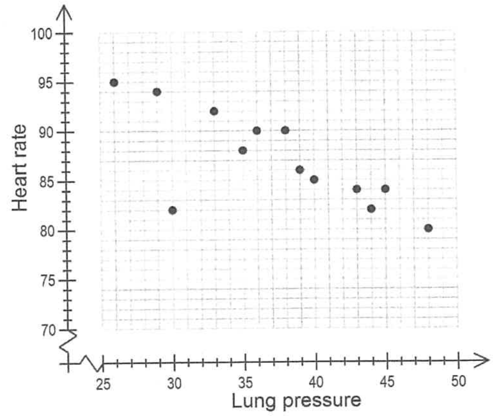
1. (a) (i) 5.5, (ii)5.9, (iii) 2.5, (iv) 2.4678, (v) 0.6078
(b) (i) $5.\dot{8}$, (ii) $5.\dot{6}$, (iii) 2.3307, (iv)2.4944,(v) 0.8728
(c) The correlation coefficient changed from indicating a moderate linear relationship between x and y to indicating a strong linear relationship on the removal of the outlier.
2. (a) Response variable is Science test mark, explanatory variable is hours spent watching videos. Science mark depends on the number of hours watching videos as the more hours spent watching videos reduced the number of hours available for studying for the test. Furthermore saying that the hours spent watching videos depends on the Science test mark make no sense.
(b) 6.6 marks, 5.8 hours (c) 6.5 marks (d) Weak negative linear relationship as r ≈ -0.44 (e) 5.9 marks, 5.5 marks (f) Linear relationship altered dramatically as now r ≈ -0.92 which indicates a strong negative linear relationship.
3. (a) 0 (b) 1 (c) −1 (d) r_{xy} is undefined as x is constant.
(e) r_{xy} is undefined as y is constant (f) −0.5 (g) 0.5 (h) 0
4. (a)



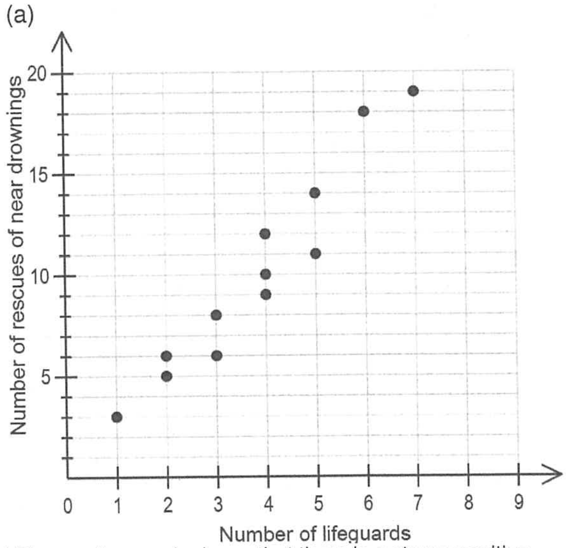
(b) As r ≈ 0.65 moderate positive linear relationship.
(c) As r ≈ 0.81 strong positive linear relationship.
(d) By making the correction the resulting linear relationship has increased significantly.
5. (a)



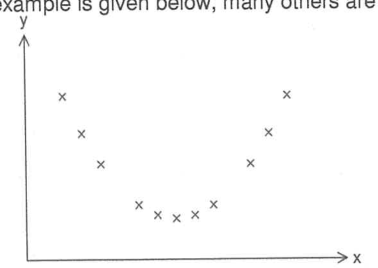
(b) r = 0.92 c) r = 0.92 indicates a strong positive linear relationship between blood alcohol level and risk factor.
(d) The correlation coefficient is not useful in this case to describe the relationship between BAL and R because the given data is clearly not linear. The correlation coefficient describes the strength and direction of a linear relationship.
6. (a) r = -0.758 (3 d.p.) indicating a moderately strong negative linear relationship. There is evidence to support that an increase in lung pressure generally means a decrease in heart rate.
(b)



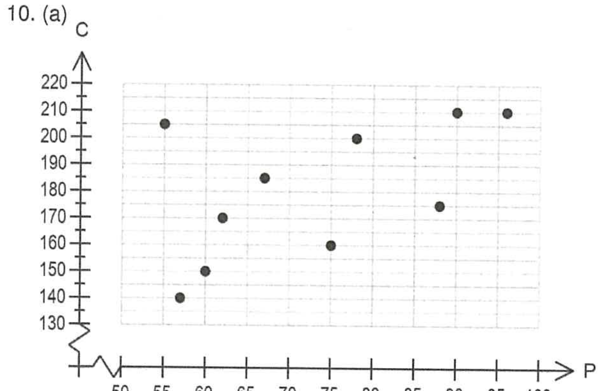
(c) Child numbered 7 with data (30, 82).
(d) With the removal of data for child 7 the value of r would get closer to -1 (r = -0.964 3 d.p.) indicating a very strong linear association between lung pressure and heart rate.
7. (a)



(b) The scatter graph shows that there is a strong positive linear relationship between the number of lifeguards on duty and the number of rescues of near drownings.
(c) The number of lifeguards on duty does not cause the number of rescues of near-drownings. The high correlation of 0.96 shows the relationship or association of the number of lifeguards and the number of rescues of near drownings and NOT that one is the cause of the other. It is obvious that at least one other factor must be involved in this situation. This third factor may be the number of people on the beach. The greater the number of people on the beach means that more lifeguards will be needed to be on duty and also the greater the number of people on the beach the greater will be the number of people finding themselves in need of being rescued from drowning.
8. No, having more mobile phones per household does not increase life expectancy. There is some lurking variable that is having an effect here. This lurking variable is more than likely to be the economic standing of the countries that were surveyed. More wealth means more mobile phones per household and more money means better health care, good hospitals, access to clean water and food which all contribute to a longer life.
9. An example is given below, many others are possible.



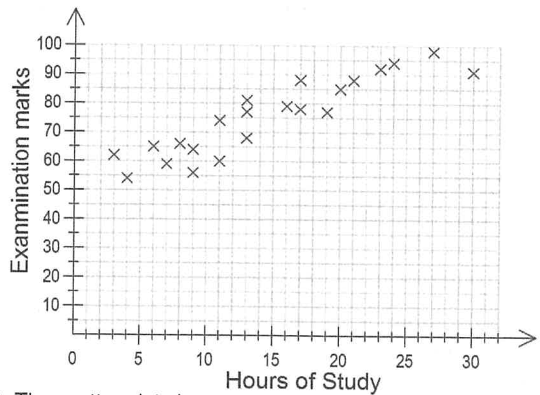
10. (a)



(b) The scatter graph shows a weak positive linear relationship between performance and cost. The graph also shows an unusual data point (55,205) a possible outlier.
(c) r = 0.54 (2 d.p.) and this represents a weak to moderate positive linear relationship between performance and cost.
(d) Type H (55,205). Performance of type H is the lowest and its cost nearly the highest which is not the trend of all of the other calculators.
(e) r = 0.80. (f) Removal of the outlier type H results in a much stronger correlation between performance and cost.
(g) A strong correlation between performance and cost does not necessarily mean that there is a causal relationship between them. As there does not appear to be an obvious lurking variable we call the situation confounding because it is confusing to determine how the variables performance and cost are related. There may be many other factors involved.
11. It does not make any sense to conclude that the number of police officers would be causing crimes to be committed. Cities with high crime rates would actually need higher numbers of police officers. It is more than likely that both of these variables increase in cities with higher populations, lower economic status, and other possible lurking variables.
12. It makes no sense to conclude that television sets cause people to live longer. There is some lurking variable that is having an effect here on both variables. It is likely that the economic status of the countries surveyed is causing the number of television sets and life expectancy to change. If a country is wealthy than it is much more likely that its citizens would own television sets and also if a country is wealthy it is much more likely to have better health care system, good and sufficient number of hospitals, a higher standard of education, access to clean water and food etc.
13. (a) r= -0.0791, the value of the correlation indicates a negative linear relationship of no significance.
(b) (1, 6), (2, 7) and (6, 1)
(c) r= 0.8783, the value of the correlation coefficient for the cropped data indicates a strong positive linear relationship between x and y.
(d) The outliers may be removed from the data set if it can be shown that these points have special characteristics which are not present in the rest of the distribution.

CHAPTER TWO REVIEW EXERCISE

1. (a) Explanatory variable is weight of the cars in kg and the response variable is the fuel consumption of the cars in litres/km
(b) The relationship between weights and fuel consumption is strong, linear and negative. As the weight of the cars increases the fuel consumption decreases There are no unusual points i.e. no outliers.
2. (a) There is no explanatory-response relationship between height and IQ scores. It is not reasonable to state that there is an association between height and IQ scores as height does not depend on IQ score nor does IQ score depend on height.
(b) The scatter plot shows that there appears to be no relationship between height and IQ scores. The graph shows no form and no direction and is of zero strength. There is no apparent trend or pattern between the height of these students and their IQ scores. Also there are no apparent outliers.
3. (a) r = 0.99, (b) r = -0.31, (c) r = -0.02, (d) r = 1.0,
(e) r = -0.48, (f) r = 0.7, (g) 1.05, -1.99, (h) -1.0
4. (a) Graph A r = 0.9 Graph B r = 0.4 Graph C r= -0.4
Graph D r = 0.7
(b) Graph A (i) r = 0.9 which indicates a strong positive linear relationship (ii) The response variable should increase as the explanatory variable increases.
Graph B (i) r = 0.4 which indicates a weak positive linear relationship (ii) There is limited evidence to suggest that the response variable should increase as the explanatory variable increases.
Graph C (i) r = -0.4 which indicates a weak negative linear relationship (ii) There is limited evidence to suggest that the response variable should decrease as the explanatory variable increases.
Graph D (i) r = 0.7 which indicates a moderate positive linear relationship (ii) There is some evidence to suggest that the response variable should increase as the explanatory variable increases.
5. (a) Explanatory variable is the time, in hour, spent studying and the response variable is the examination mark attained.
(b)
Hours of study vs Examination Marks



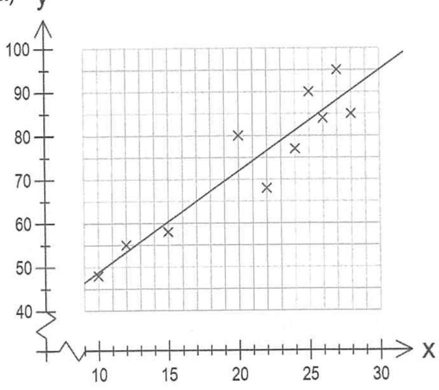
(c) The scatter plot shows a strong positive linear relationship between hours of study and examination marks. The given data does not have any outliers.
6. (a) Explanatory variable is the temperature and the attendance is the response variable.
(b) The value of r = 0.82 indicates a strong positive relationship between temperature and attendance.
(c) The strong positive relationship between temperature and attendance indicates that the attendance should increase as the temperature increases.
(d) Explanatory variable is the price of admittance and the response variable is the attendance.
(e) The value of r = -0.91 9 indicates a strong negative relationship between price and attendance.
(f) The strong negative relationship between attendance and price indicates that the attendance should decrease as the price of admittance increases.
(g) Price is the better predictor because there is a stronger relationship between price and attendance than between temperature and attendance as indicated by the magnitude of the correlation coefficients.
7. (a) Examining a scatterplot of the given data set reveals that person 9 is most likely to be Jackson. The ATAR for person 9 is about average but the income is high indicating a very successful business. (b) 0.8114 (4 d.p.)
(c) There is a strong positive linear relationship between ATAR and salary. (d) Remove person 9 as person 9 may be considered to be an outlier in this group of people.
8. It clearly makes no sense to conclude that increased divorce rates cause the price of new cars to increase. It also make no sense to say that increase in new car prices causes higher divorce rates. It seems that it is a coincidence that both divorce rates and the price of new cars are increasing.
9. We cannot say that the ailment is caused as a result of age there are other factors involved. Smoking may be a contributing factor in this case. The high value of the correlation coefficient tells us that there is a strong positive linear relationship between age and the respiratory ailment and not that one causes the other.
10. (a) The amount of sunshine depends on the percentage of the sky covered in cloud and hence the explanatory variable is percentage cloud cover and the response variable is mean daily hours of sunshine.
(b) The evaporation rate depends on the percentage of the sky covered in cloud and hence the explanatory variable is the percentage of sky covered in cloud and the evaporation rate is the response variable.
(c) The evaporation rate depends on the amount of sunshine and hence the explanatory variable is the mean daily hours of sunshine and the evaporation rate is the response variable.

(d) (i) -0.96, (ii) 0.98, (iii) -0.97
(e)(i)True. The magnitude of r for all three pairings is close to 1.
(ii) False. r_{SE} = 0.98 and the magnitude of the strength of the correlation is 0.98. r_{CE} = -0.97 and the magnitude of the strength of the correlation is 0.97.
Hence the correlations are nearly the same. (iii) False. The value of r_{CE} is negative and this informs us that the evaporation rate should decrease as the percentage of sky covered with cloud increases.

ANSWERS CHAPTER THREE

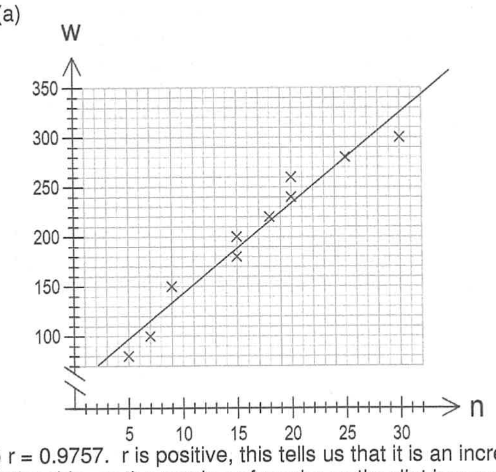
EXERCISE 3A

1. (a) Slope is 0.54. According to the regression line, for an increase of 1 cm in the father's height there is an expected increase of 0.54 cm in the son's height.
(b) The y intercept is 89.58 cm. When the father's height is 0 cm the expected height of the son is 89.5 cm. Although the y intercept exists it is meaningless in this context.
2. (a) Height is explanatory variable and shoe size the response variable. (b) Slope is 0.25. According to the regression line for an increase of 1 cm in height there is a corresponding expected increase of 0.25 in shoe size.
(c) The y intercept is -33.4. When height is 0 cm the expected shoe size of the person is -33, which does not make any sense. Although the y intercept exists it is meaningless in this context.
3. (a) Explanatory variable is number of rainy days and the response variable is the number of hours of sunshine.
(b) Slope is -7.4. The slope of the regression line predicts that the number of hours of sunshine per year will decrease by approx. (or on average) 7.4 hours for each additional day of rain.
(c) The y intercept is 2920. The y intercept of the regression line predicts that if there were no days of rain we could expect 2920 hours of sunshine for the year. This of course is an unreliable estimate as there may be no rainy days during the year but sufficient cloud cover to reduce the number of hours of sunshine below or even well below 2920 hours.
4. (a) Slope is 2.74. The slope of the regression line predicts that the heart weight of this species will increase by an average of 2.74 mg for each additional gram increase in body weight.
(b) The y intercept is 45.28. The y intercept of the regression line predicts that the heart weight of this species is 45.28 mg when the body weight is 0 grams. This of course is not possible. In this context the y intercept makes no sense and should not be considered.
5. (a) The slope is -8.54. The slope of the regression line predicts that, on average, response time decreases by 8.54 minutes for a 1 mg increase in drug dose.
(b) The y intercept is 45.55. The y intercept of the regression line predicts that on average the response time when no drug is administered is about 45.55 minutes.
6. (a)



(b) r = 0.9353. r is positive, this tells us that it is an increasing relationship, as x increases so does y. r is close to 1 hence this relationship between x and y is strong.
(c) $\hat{y}$ = 2.31x + 25.67 see (a).
(d) The slope is 2.31. It means that for every increase of one unit in x there is and average increase of 2.31 units in y.
(e) The y intercept is 25.67.It means that when the value of x is zero, we would expect that the predicted value of y would be 25.67.

7. (a)



(b) r = 0.9757. r is positive, this tells us that it is an increasing relationship, as the number of weeks on the diet increases so does the weight gain of these chickens. r is very close to 1 hence this relationship is very strong.
(c) $\hat{w}$ = 9.12n + 51.47
(d) The gradient is 9.12. It means that for every week these chickens spend on this special diet there is on average a weight increase of 9.12 grams.
(e) The vertical axis intercept is 51.47. It means that when these chickens start this special diet their weight gain is 51.47 grams which of course makes no sense in context.
8. (a) Explanatory variable is monthly maximum average temperature and the response variable is the monthly profit.
(b) A scattergraph shows that there is a strong positive linear relationship between monthly maximum average temperature and monthly profit.
(c) r = 0.967. r is positive and this tells us the relationship is increasing, that is, as the monthly average temperature increases so does the monthly profit. r is very close to 1, which tells us that the relationship is very strong.
(d) Predicted profit = 1.265 x monthly average maximum temp – 4.481 or $\hat{p}$ = 1.265t -4.481.
(e) Gradient is 1.265. The gradient tells us that for every increase of 1 degree in the monthly average maximum temperature there is an average increase of $126.50 in the monthly profit.
(f) The vertical axis intercept is -4.481 which tells us that if monthly average maximum temperature was 0°C then the predicted monthly profit would be -$448.10, that is a loss of $448.10. in this context the vertical axis intercept makes no sense.
9. (a) Explanatory variable is the price per kg and the response variable is the quantity sold in kg
(b) A scattergraph shows that there is a strong negative linear relationship between quantity sold and price paid per kg.
(c) r = -0.953. r is negative and this tells us that the relationship is decreasing, that is, as the price per kilogram of potatoes increase then the quantity purchased decreases. r is close to -1 which tells us that the relationship between price per kg and quantities purchase is strong.
(d) Predicted quantity sold = -147.15 x price per kg + 470.77 or $\hat{p}$ = -147.15d + 470.77
(e) Gradient is -147.15. The gradient tell us that for every increase of 1 dollar in the price per kg we could expect that there will be on average a decrease of 147 kg of potatoes sold
(f) The vertical axis intercept is 470.77. This means that if the price per kg was $0 then we would expect 470.77 kg would be sold given away. This does not make sense in this context. We would expect that if there was no charge for potatoes more than 470 kg would be taken by shopping public.
(g) The horizontal axis intercept is 3.20. This means that if the price per kilogram was $3.20 we would expect that no potatoes would be sold. This is highly unlikely as we would expect that some people would still purchase some potatoes at this price.
10. (a) Explanatory variable is the age in years of this model car and the response variable is the selling price.
(b) A scattergraph shows that there is a strong negative linear relationship between selling price and age of these cars.
(c) r = -0.9696. r is negative, this tells us that it is a decreasing relationship, as age increases the price decreases. r is very close to -1 hence this relationship very strong.
(d) Predicted price = -662.5 x age + 13 112.5

(e) Slope is -662.5. The slope tells us that for every increase of 1 year in the age of these cars we would expect that there would a decrease of approx. $660 in the selling price.
(f) Vertical axis intercept is 13 112.5. This means that if the age of the car was 0 years we would expect the selling price to be approx. $13000. In this context $13 000 would be the price of a new car of this model.
(g) Horizontal axis intercept is 19.8. This tells us that a car of this model that is 20 years old has an expected selling price of $0 i.e. has no value. The selling price of cars of this model and 20 years old may have some value as scrap metal or may be worth more than expected depending on their condition.

EXERCISE 3B

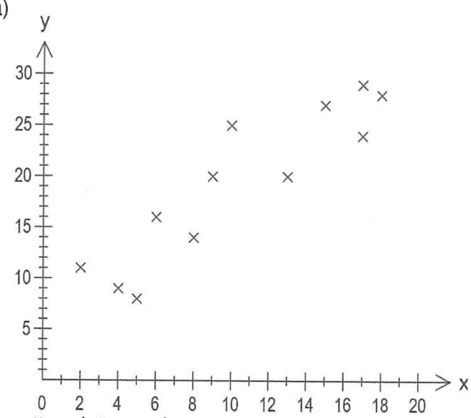
1. (a) (i) $r^2 = 0.336$ (ii) 33.6% of the variation in the response variable can be explained by the variation in the explanatory variable.

(b) (i) $r^2 = 0.503$ (ii) 50.3% of the variation in the response variable can be explained by the variation in the explanatory variable.

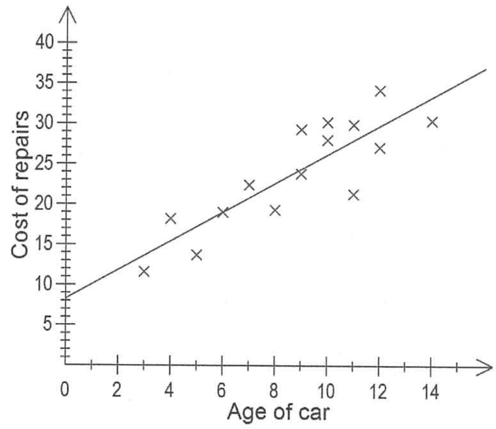
(c) (i) $r^2 = 0.177$ (ii) 17.7% of the variation in the response variable can be explained by the variation in the explanatory variable.

(d) (i) $r^2 = 0.810$ (ii) 81.0% of the variation in the response variable can be explained by the variation in the explanatory variable.

2. (a)

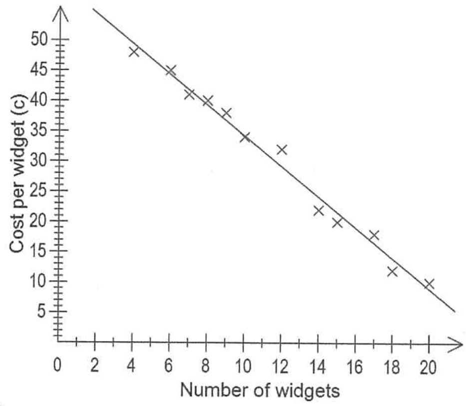


The scatter plot reveals a strong positive linear relationship between x and y.
(b) r = 0.9071 (4 d.p.) (c) When r = 0.9071 it indicates a strong positive relationship between x and y.
(d) $r^2 = 0.8229$ (4 d.p.) (e) 82.29% of the variation in y can be explained by the variation in x. (f) $\hat{y} = 1.2299x + 6.5406$
(g) The slope is 1.2299. The slope of the regression line predicts that on average we can expect an increase of 1.2299 units in y for a unit increase in x. (h) Yes it is an appropriate model due to the high value of r^2. Only 17.71% of the variation between the variables is unexplained.
3. (a) The age of the cars in years is the explanatory variable and the cost of repairs in $'00s is the response variable.
(b)

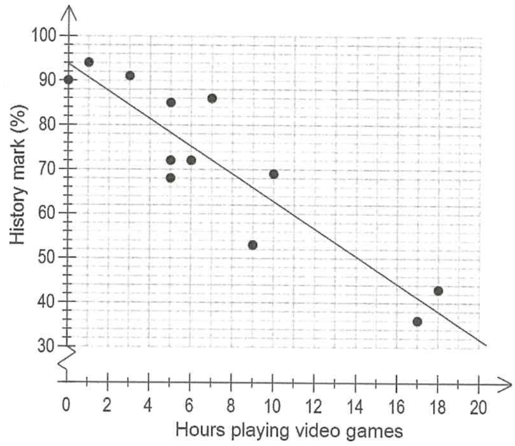


(c) $r^2 = 0.7344$ (d) 73.44% of the variation in the cost of repairs can be explained by the variation in the age of these cars. (e) Repair costs = 1.7881 x age of car + 8.2776; see graph

(f) Yes. The relationship is linear as indicated by the scatter plot, is strong and positive. The value of r^2 indicates that only 26.56% of the variation between the variables is unexplained.
4. (a) Explanatory variable is number of widgets purchased, response variable is cost per widget in cents.
(b)



(c) $r^2 = 0.9855$ (d) 98.55% of the variation in the cost per widget can be explained by the variation in the number of widgets purchased. (e) Cost per widget = -2.5287 x Number of widgets purchased + 59.5011; see graph
(f) Slope is -2.5287, the slope of the least squares regression line predicts that on average we can expect a decrease of 2.5287 cents in the price of each widget for every extra widget purchased.
(g) Y intercept is 59.50. This indicates that the expected cost per widget is 59.50 cents when zero widgets are purchased. In this context the y intercept has no meaning.
(h) Yes. The relationship is linear as indicated by the scatter plot, and is very strong and negative. The value of r^2 indicates that only 1.45% of the variation between the variables is unexplained. However it must be noted that the y-intercept is meaningless in this context.
5. (a) Hours spent playing video games is the explanatory variable and the response variable is the History mark.
(b)



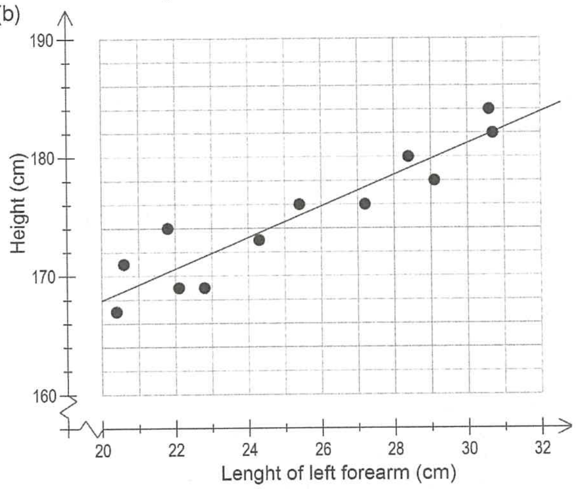
(c) The relationship is linear, negative and strong.
(d) -0.9072, strong negative relationship. We can conclude that History marks will decrease with increased hours playing video games.
(e) 0.8230, 82.3% of the variation in the History marks can be explained by the variation in the number of hours of playing video games.
(f) History mark = 93.7876 – 3.0983 x video game hours
(g) The gradient is -3.0983 which tells us that History marks decrease by 3.0983 marks for every 1 hour increase in the number of hours spent playing video games.
(h) The y-intercept is 93.7876 which tells us that the expected History mark is 94 when the number of hours spent playing video games is 0 hours.
(i) Model is appropriate, the y-intercept is logical in this context and the r^2 value is high indicating that only 17.7% of the variation in the variables is unexplained.
6. (a) Explanatory variable is the number of people in the family and the response variable is the monthly medical car expense-
(b) Monthly medical care expense = 16.83 x number in the family + 110.47 (c) 48.36%

(d) Not appropriate. The correlation coefficient indicates a positive moderate relationship which may or may not be linear as we cannot view a scatter plot of the data set. Also the value of r^2 is less than 0.5. The value of r^2 indicates that 51.64% of the variation between the variables is **unexplained.**

7. $r = -0.8289$ **8.** $r^2 = 0.25$. The value of the coefficient of determination tells us the 25% of the variation in variable b is explained by the variation in variable a.

9. (a) Response variable is the height (cm) and the explanatory variable is the length of the left forearm (cm).

(b)



(c) $r = 0.9308$ (4 d.p.) the correlation coefficient indicates a positive and strong to very strong relationship between the forearm length and height. (d) $r^2 = 0.8665$ (4 d.p.) The coefficient of determination tells us that 86.65% of the variation in the height can be explained by the variation in the length of the left forearm. (e) Height = 1.3193 x forearm length + 141.56 (f) Gradient = 1.3193 and tells us that height increases by 1.3193 cm for every 1cm increase in the length of the left forearm. (g) Y-intercept is 141.56 which tells us that the expected height of a student would be 141 6 cm if the length of the student's forearm is 0 cm. In this context the y-intercept has no logical meaning. (h) see graph (i) The value of the coefficient of determination is high and tells us that only 13.35% of the variation in the variables is unexplained. This informs us that the model appears to be reliable. However the y-intercept is meaningless in context. Provided forearm lengths range from approx. 20 cm to 31 cm we can conclude that the mode is appropriate. **10.** 81% **11.** $r = 0.7$

EXERCISE 3C

The answers to questions 1 to 4 parts (c) and (d) have been given to the degree of accuracy as indicated by the given data. The answer in brackets is given so that students can readily check their calculated answer.

1.(a) $\hat{y} = 2.6290x - 9.4355$ (b) $\hat{x} = 0.2291y + 5.0450$

(c) 14 (14.2255) (d) 7 (7.336)

2.(a) $\hat{y} = 0.6434x + 6.9602$ (b) $\hat{x} = 1.0110y - 4.5023$

(c) 11 (10.8206) (d) 8 (7.6297)

3. (a) $\hat{y} = 0.6792x + 15.8313$ (b) $\hat{x} = 0.9707y - 0.5464$

(c) 50 (49.7913) (d) 48 (48.48036)

4. (a) $\hat{y} = 0.1108x + 1.6338$ (b) $\hat{x} = -0.2903y + 2.5165$

(c) 1.4 (1.40112) (d) 2.0 (2.05202)

5. (a) $\hat{y} = 0.68x + 0.64$ (b) 4 (4.05) (c) 10 (9.51)

(d) The estimate for x = 5, because it is an interpolation and r_{xy} close to 1 and hence the correlation is strong.

The estimate for x = 13 is an extrapolation and is rather removed from the given data hence a poor estimate.

6. (a) $\hat{x} = 1.02y + 1.09$ (b) 3 (3.14) (c) 14 (14.40)

(d) The estimate for y = 2 because it is an interpolation and r_{xy} is close to 1 and hence the correlation is strong. The estimate for y = 13 is an extrapolation and well outside the given data, hence a poor estimate.

7. (a) $r_{pq} = -0.86$ (b) $\hat{p} = -0.89q + 79.30$

(c) $\hat{q} = -0.83p + 75.42$ (d) 53 (52.71) (e) 51 (50.63)

8. (a) $\hat{n} = 2m + 20$ (b) $\hat{m} = 0.5n - 10$

(c) $r_{mn} = 1$. The equations are identical. (e) If r = -1

(f) (i) 50 (ii) 15 (g) q. Regression lines are the same.

9. (a) Explanatory variable is the height in cm and the response variable is the weight in kg.

(b) $r_{hw} = 0.88$. There is a strong positive linear relationship between weight and height

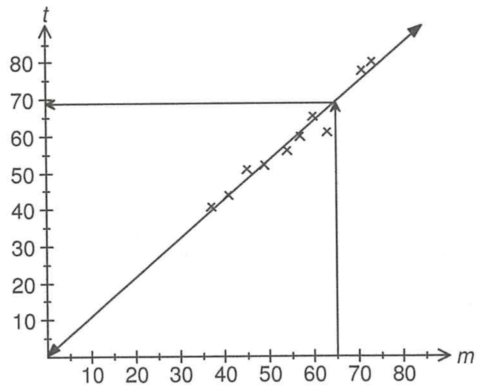
(c) The coefficient of correlation, that is $r_{hw} = 0.88$, indicates that there is a strong positive correlation and the scatter diagram indicates the a linear relationship exists between height and weight.

(d) $\hat{h} = 0.855w + 118.623$ (e) 204 cm

(f) Using the weight on height regression line, that is

$\hat{w} = 0.9066h - 87.0093$ predicted weight to the nearest kg is 112 kg Even though the correlation is high the prediction is questionable because the predictions is an extrapolation and the number of data points in the data set is very small.

10. (a)



The scatter diagram indicates a linear relationship exists between the two variables and the correlation coefficient of $r_{mt} = 0.979$ indicates a very strong positive relationship between the Mock ATAR and the ATAR.

(b) & (c) See graph ATAR mark near 69.

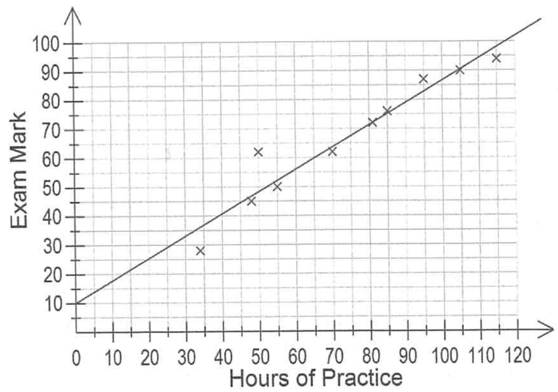
(d) The answer will not be the same. Fitting a line of best fit by eye will result in different lines being drawn and hence the predictions will not be the same.

(e) $\hat{t} = 1.0589m + 0.4533$ (f) 69 (g) Using the mock ATAR on ATAR regression line, that is

$\hat{m} = 0.9059t + 1.8319$ predicted Mock ATAR is 47. (h) As the predicted Mock ATAR mark is an interpolation, the correlation coefficient is very high between the Mock ATAR and ATAR marks and the relationship is linear indicating a high degree of reliability in the prediction. However the number of students marks used to determine the statistic is very small which give rise to questioning the reliability.

11. (a) Response variable is the exam mark and the explanatory variable is the number of hours of practice.

(b)



(c) The relationship between exam marks and hours of practice can be described as linear, positive and very strong.

(d) r 0.963. There is a very strong positive linear relationship between exam results and hours of practice.

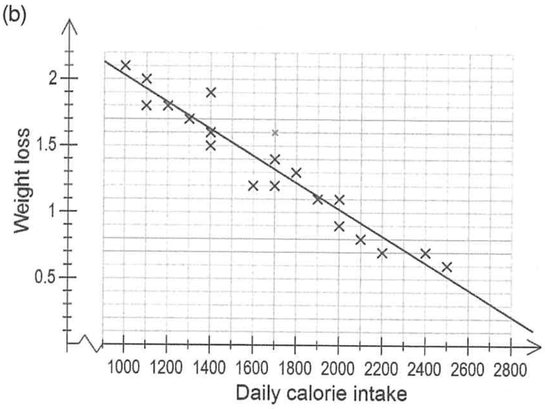
(e) $\hat{m} = 0.7665h + 10.0317$ see graph above

(f) Slope is 0.7665, so for every 1 hour increase in practice we would expect the exam mark to increase by 0.7665 marks.

(g) The intercept is 10.0317. So if a music student did no practice then we would expect the exam mark for that student to be about 10 marks.

(h) Predicted mark is 25. r is high, but 20 hours is outside the range of the known data (extrapolation) hence prediction is

unreliable. Also the number of students in this data set is small which also supports an unreliable prediction.
(i) Point of intersection is (73.8, 66.6). 73.8 is the mean of h and 66.6 is the mean of m.
12. (a) Explanatory variable is daily calorie intake and weight loss is the response variable. Weight loss depends on calorie intake hence weight loss is the dependent or response variable.
(b)

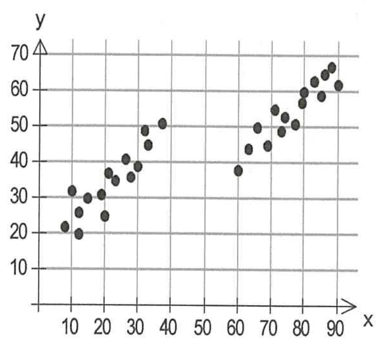


(c) The scatter plot shows a linear relationship between weight loss and daily calorie intake. Linear relationship is strong and negative.
(d) r = -0.9575, very strong negative linear relationship therefore we can conclude that the weight loss decreases as daily calorie intake increases.
(e) $\hat{w}$ = -0.001008c + 3.0376 see graph above
(f) Vertical axis intercept is 3.0376. If the daily calorie intake was 0 calories then the weight loss would be approx. 3 kg.
(g) Horizontal axis intercept is 3013.5 calories. If the daily calorie intake was approx. 3000 calories there would be no weight loss.
(h) The gradient is -0.001008, so for every 1 calorie increase in daily calorie intake the weight loss would be expected to decrease by 0.001008kg or by approx. 1g.
(i) Using the regression line predicted weight loss is -2.0024kg. This represents a weight GAIN of about 2kg. The prediction is an extrapolation well outside the range of the given data set and hence cannot be considered as being reliable. We cannot assume that weight loss will follow the same trend.
(j) Using calorie intake on weight loss regression line predicted calorie intake is 1993.5 calories. To lose a kilogram the person should restrict daily calorie intake to 1900 calories.
(k) Prediction is an interpolation, the value of r is high, the association between weight loss and calorie intake is linear hence predicted value may be considered to be reliable. However with a larger sample size than the one given we would be more confident of the reliability of the prediction.
13. (a) r_{pm} = 0.769, r_{ms} = 0.706 r_{ps} = 0.853
(b) $\hat{m}$ = 0.8002p+14.5606
(c) $\hat{p}$ = 0.7388m+19.6080
(d) $\hat{s}$ = 0.5862p + 17.6783
(e) $\hat{m}$ = 1.0699s + 8.4984
(f) The Probability test is the better indicator because the correlation between probability and matrices is 0.769 which is slightly higher than that between statistics and matrices, 0.706.
(g) 85%
14. (a) Speed = 0.2629 x temperature – 1.3631
(b) 4.9 m/sec
(c) When temperature is 24 degrees. The prediction for 24 degrees is from temperatures within the range given i.e. an interpolation, whereas a prediction for 8 degrees is outside the given range i.e. and extrapolation which is less reliable. (or There is no guarantee that the trend before the given trend was the same)
(d) The speed increases at a rate of 0.2629 m/sec for every degree increase in surrounding temperature.
(e) 98%

EXERCISE 3D

1. (a) (i) 5.4 (ii) 5 (iii) 2.444 (iv) 0.676
(v) $\hat{y}$ = 0.616x + 1.473 (vi) 0.457 (b) (8,1) (c) (i) $5\frac{3}{14}$
(ii) 5 (iii) 2.425 (iv) 0.944 (v) $\hat{y}$ = 0.799x + 0.906 (vi) 0.892

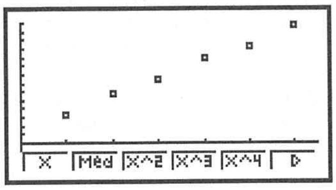
2. (a) (i) 5.6 (ii) 10 (iii) 2.776 (iv) -0.661
(v) $\hat{y}$ = -1.254x + 17.024 (vi) 0.437 (b) (9, 18)
(c) (i) $5\frac{5}{14}$ (ii) $9\frac{3}{7}$ (iii) 2.715 (iv) -0.920
(v) $\hat{y}$ = -1.687x + 18.467 (vi) 0.846
3. (a) r_{xy} = -0.7726, there is moderate negative linear relationship between x and y (b) $\hat{y}$ = -0.52x + 68.72
(c) (48, 68) (d) r_{xy} = -0.9356, there is strong negative linear relationship between x and y
(e) $\hat{y}$ = -0.57x + 69.04
4. (a) r_{xy} = 0.5412, a positive, moderately weak relationship between x and y. The scatter plot indicates that the relationship is linear
(b) $\hat{y}$ = 0.426x + 1.771
(c) 3.3 (d) (3.8, 1.2)
(e) r_{xy} = 0.9010, without the outlier there is strong positive linear relationship between x and y
5. (a) r_{xy} = -0.0791, the value of the correlation indicates a negative relationship of no significance.
(b) (1, 6), (2, 7) and (6, 1)
(c) r_{xy} = 0.8783, the value of the correlation coefficient for the cropped data indicates a strong positive linear relationship between x and y.
(d) The outliers may be removed from the data set if it can be shown that these points have special characteristics which are not present in the rest of the distribution.
6. (a) r_{xy} = 0.90395, the value of the correlation coefficient indicates that there is a strong positive linear relationship between x and y.
(b) $\hat{y}$ = 0.415x + 24.231
(c) 45 (44.981)
(d) The scatterplot reveals that the points of the scatterplot form two distinct groups or clusters. Each group shows a strong positive linear relationship between x and y.
(e) The correlation coefficient found in (a) did not identify the nature of the clustered distribution and should always be considered in conjunction with a scatter plot to justify any relationship between two variables. Correlation coefficient is not valid in this case as the form of the data set is not linear. The regression line found in (b) is not appropriate for making predictions as it has been determined by the "averaging out " of the squares of the residuals from the two distinct groups and its validity hence is questionable. The predicted value for x = 50 in (c) is questionable as it was calculated using a questionable regression line and examination of the scattergram it appears that a score of 50 for x is not in the domain of variable x.



EXERCISE 3E

1. Graph A: Linear regression model not suitable as residual plot shows a pattern or residuals are not randomly distributed.
Graph B: linear regression model suitable as no discernible pattern shown by the residuals or the residuals are randomly distributed.
Graph C: Linear regression model not suitable as residual plot shows a pattern or residuals are not randomly distributed.
Graph D: linear regression model suitable as no discernible pattern shown by the residuals or the residuals are randomly distributed
Graph E: Linear regression model not suitable as residual plot shows a pattern or the residuals are not randomly distributed.

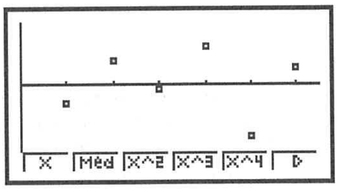
Graph F: Linear regression model not suitable as residual plot shows a pattern or the residuals are not randomly distributed.
2. Graph A: Linear regression model suitable as r indicates a strong linear relationship and the residuals exhibit no discernible pattern.
Graph B: Linear regression model suitable as r indicates a strong linear relationship and the residuals exhibit no discernible pattern.
Graph C: linear regression model not suitable as r indicates no significant correlation even though the residual plot exhibits no discernible pattern.
Graph D: linear regression model not suitable as r indicates no significant correlation even though the residual plot exhibits no discernible pattern.
Graph E: Linear regression model not suitable as the residual plot shows a pattern even though r indicates a strong negative linear relationship.
Graph F: : Linear regression model not suitable as the residual plot shows a pattern even though r indicates a strong positive linear relationship.
3. (a) (i)



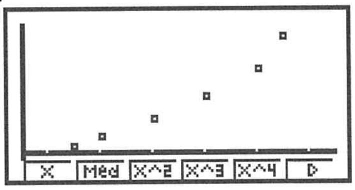
The scatter diagram indicates a strong positive linear relationship. (ii) $r_{xy} \approx 0.9975$ the correlation coefficient indicates a very strong positive linear relationship.
(iii) $\hat{y} = 2.04x + 1.42\dot{6}$
(iv)

x	1	2	3	4	5	6
y	3.3	5.7	7.5	9.9	11.2	13.8
$\hat{y}$	$3.4\dot{6}$	$5.05\dot{6}$	$7.54\dot{6}$	$9.58\dot{6}$	$11.62\dot{6}$	$13.\dot{6}$
$y-\hat{y}$	$-0.1\dot{6}$	$0.19\dot{3}$	$-0.04\dot{6}$	$0.31\dot{3}$	$-0.42\dot{6}$	$0.1\dot{3}$

(v)



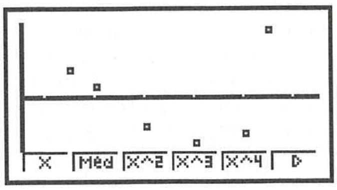
The residual plot indicates no discernible pattern hence the linear regression model is a suitable predictor of the response variable y.
(b) (i) The scatter graph indicates a strong positive linear relationship



(ii) $r_{mn} \approx 0.981$ the correlation coefficient indicates a very strong positive linear relationship.
(iii) $\hat{n} = 14.69x - 264.87$
(iv)

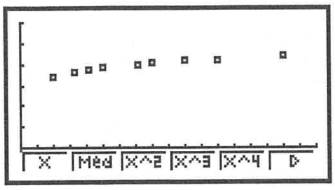
m	20	30	50	70	90	100
n	90	200	410	670	980	1350
$\hat{n}$	28.97	175.90	469.74	763.59	1057.44	1204.36
$n-\hat{n}$	61.03	24.1	-59.74	-93.59	-77.44	145.64

(v)



The residual plot shows a definite pattern and even though the value of r is high the linear regression model is not suitable for prediction purposes.

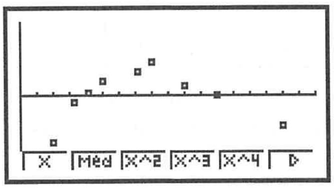
(c) (i) The scatter graph indicates a strong positive linear relationship.



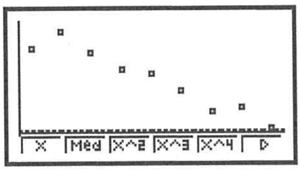
(ii) $r_{pq} \approx 0.951$, the correlation coefficient indicates a very strong positive linear relationship.
(iii) $\hat{q} = 0.071p + 3.397$
(iv)

p	2	3.25	4.21	5	7.1	8	9.95	12	16
q	3.35	3.6	3.7	3.81	4	4.1	4.14	4.24	4.4
$\hat{q}$	3.54	3.63	3.69	3.75	3.90	3.96	4.10	4.24	4.53
$q-\hat{q}$	-0.19	-0.03	0.01	0.06	0.10	0.14	0.04	0	-0.13

(v)



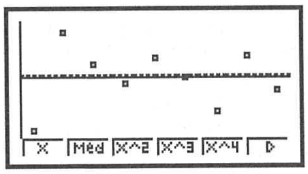
The residual plot shows a definite pattern and even though the value of r is high the linear regression model is not suitable for prediction purposes.
(d) (i) The scatter graph indicates a strong negative linear relationship.



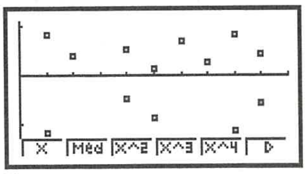
(ii) $r_{st} \approx -0.965$ the correlation coefficient indicates a very strong negative linear relationship.
(iii) $\hat{t} = -2.71\dot{6}s + 262.5$
(iv)

s	90	80	70	60	50	40	30	20	10
t	10	60	50	100	140	150	190	240	200
$\hat{t}$	18	45.2	72.3	99.5	126.7	153.8	181	208.2	235.3
$t-\hat{t}$	-8	14.8	-22.3	0.5	13.3	-3.8	9	31.8	-35.3

(v)



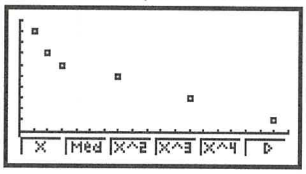
The residual plot indicates no discernible pattern hence the linear regression model is a suitable predictor of the response variable t.
4. (a) The scatterplot shows a linear association between x and y, r = 0.8508 There is a strong positive linear relationship between the variables x and y.
(b) $\hat{y} = 0.4237x + 1.7512$
(c)



The linear regression model is suitable because there is no discernible pattern exhibited by the residuals and there exists a strong positive correlation between x and y.
(d) 5.988 This extrapolation may be considered to be reliable as it is only just outside the range of the given data and the correlation between x and y is strong. The size of the data set however detracts from the reliability of the prediction as it is too small.

(e) 10.225 This extrapolation is not reliable as it is too far removed from the given data even though the correlation of the given data is strong,

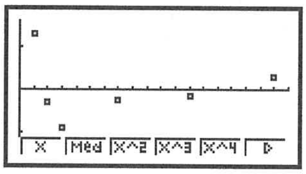
5. (a) The scatter plot shows that there appears to be a strong negative linear relationship between x and y.



(b) $r_{xy} \approx -0.966$, the correlation coefficient indicates that there is a very strong negative linear relationship between x and y.

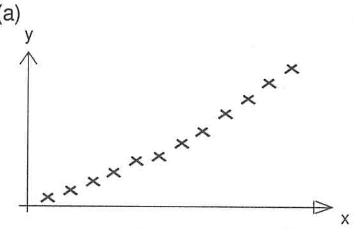
(c) $\hat{y} = -0.4136x + 8.1309$

(d)



Using the linear regression model for prediction would not be suitable because the residuals exhibit a pattern, residuals are positive for high and low values of x and negative for values in between.

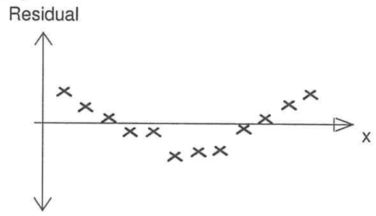
6. (a)



There appears to be a strong positive linear relationship between x and y.

(b) $r_{xy} \approx 0.9899$, the correlation coefficient indicates that there is a very strong positive linear relationship between x and y.

(c) $\hat{y} = 531.58x - 559.94$



(d) Using the linear regression model of y on x for prediction would not be suitable because the residuals are not randomly distributed, residuals are positive for low values of x negative for middle values of x and positive for higher values of x.

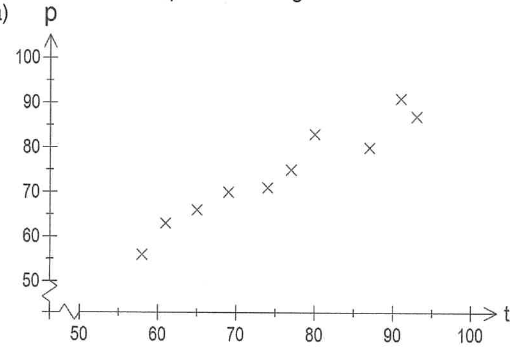
7. (a) There appears to be a moderate negative linear relationship between x and y.

(b) $r_{xy} \approx -0.7685$, the value of the correlation coefficient confirms that there is moderate negative linear relationship between x and y.

(c) $\hat{y} = -11.998x + 117.556$

(d) Using the linear regression model of y on x for prediction would not be suitable because the residuals are not randomly distributed, residuals are positive for low values of x negative for middle values of x and positive for higher values of x.

8. (a)



The scatter graph and associated $r_{pt} \approx 9.98$ indicate a very strong positive linear relationship between practical and music theory.

(b) $\hat{p} = 0.86776t + 8.684153$

(c) (i) =0.86776 x B2 + 8.684153 (ii) =C2 – D2
(iii) =1.07336 x C2 – 4.143295 (ii) =B2 – F2

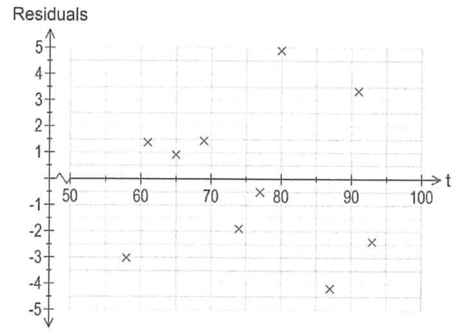
(d)

	A	B	C	D	E	F	G
1	Student	Theory (t)	Practical (p)	$\hat{p}$	$p-\hat{p}$	$\hat{t}$	$t-\hat{t}$
2	1	58	56	59.014	-3.01	55.96	2.04
3	2	61	63	61.617	1.38	63.48	-2.48
4	3	65	66	65.089	0.91	66.70	-1.70
5	4	69	70	68.560	1.44	70.99	-1.99
6	5	74	71	72.898	-1.90	72.07	1.93
7	6	77	75	75.502	-0.50	76.36	0.64
8	7	80	83	78.105	4.90	84.95	-4.95
9	8	87	80	84.179	-4.18	81.73	5.27
10	9	91	91	87.650	3.35	93.53	-2.53
11	10	93	87	89.386	-2.39	89.24	3.76

(e) Column E contains the vertical residuals, that is the difference between practical exam marks and the predicted exam marks using the regression line in (b) for the ten students.

(f) The residuals are used to determine whether the relationship between practical and theory marks is linear or non-linear. If the residual indicate that the relationship is linear then the linear regression line may be used to make predictions.

(g)

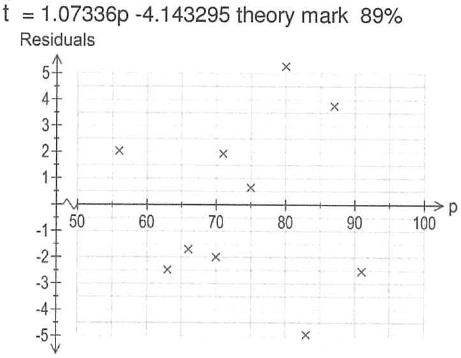


The residual plot indicates no discernible pattern hence the relationship between practical and theory marks is linear and the linear model is a suitable predictor of practical marks.

(h) Using $\hat{p} = 0.86776t + 8.684153$ practical mark 87%

(i) Using $\hat{t} = 1.07336p - 4.143295$ theory mark 89%

(j)

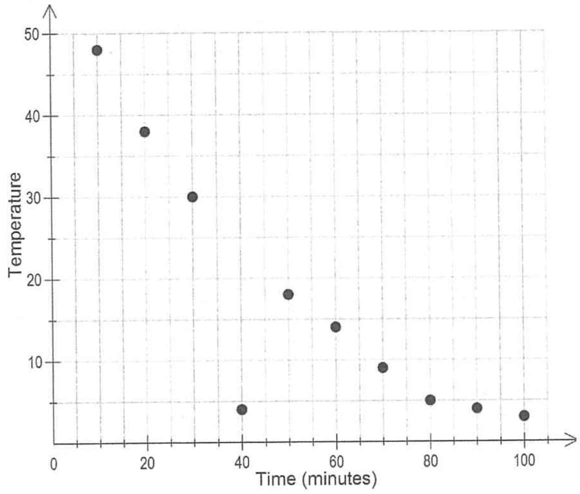


The residual plot indicates no discernible pattern hence the linear model is a suitable predictor of theory marks.

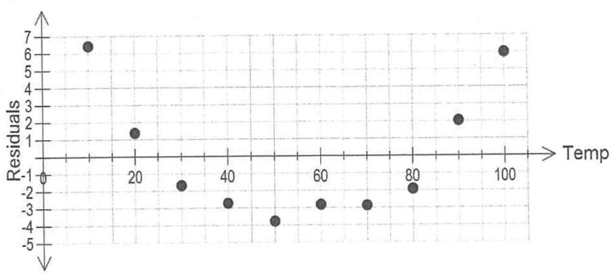
9. (a) $r^2 = 0.97^2 = 0.9409$, that is approx.. 94% of the variation in obesity can be explained by the variation in the number of hours spent using a computer.

(b) The result of this study does not mean that increasing the number of hours spent by teenagers on the computer causes their obesity level to increase. Correlation shows association but does not prove that the association is causal. Amongst teenagers we would expect obesity levels to more than likely increase with an increase in time spent on the computer because the increase in computer time means that the teenagers will have less time for physical activity and hence we would expect levels of obesity to increase. In this study the lurking variable or confounding factor would be the decrease in physical activity.

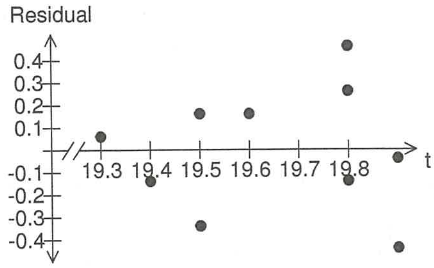
10.(a)



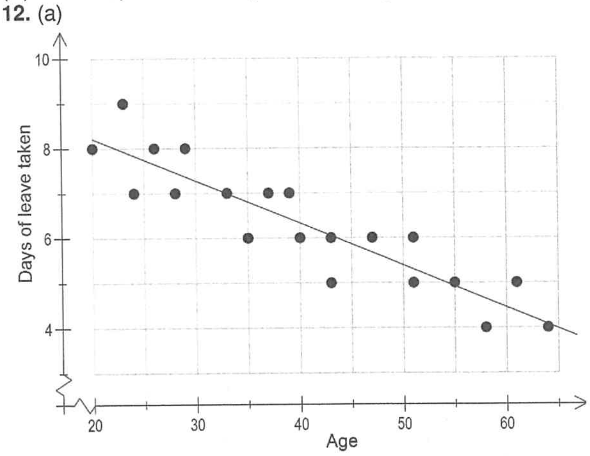
(b) Excluding the point with coordinates (40,4) there appears to be a strong negative linear relationship between the variables time and temperature.
(c) r = -0.8648 (4 d.p.) indicating a strong negative linear relationship between elapsed time and temperature. We can conclude that temperature should decrease as the lapsed time increases.
(d) (40, 4)
(e) The value of the correlation coefficient would be closer to -1 if the recording error is corrected.
(f) r = -0.9699 (4 d.p.)
(g) Temperature = -0.495 x elapsed time + 46.533
(h) The y-intercept is 46.533 °C indicating the temperature of the food before it was put in the refrigerator. This makes no sense because the temperature of the food after being in the refrigerator for 10 minutes was 48°C an increase of over one degree which makes no sense. The y-intercept should be ignored.
(i) The gradient is -0.495. This indicates that on average the temperature of the food decreases on average by 0.5 degrees for every minute it is in the refrigerator.
(j) $r^2 = (-0.9699)^2 = 0.9407$. The correlation coefficient of -0.9699 informs us that there is a nearly perfect negative linear relationship between elapsed time and temperature. The coefficient of determination supports this as approximately 94% of the variation in temperature is explained by the variation in elapsed time. Hence the linear regression model based on the above is appropriate for making predictions
(k) 16.8°C
(l) Residual = 14°C − 16.8°C = −2.8°C
(m)



(n) No.The linear regression model is not an appropriate model for this data and should not be used to make predictions because the data is not linear as the points in the residual plot make a clear curved pattern.
11. (a) Explanatory variable is the average weekly temperature and the response variable is the weekly average number of eggs laid by each hen.
(b) $r_{nt} \approx 0.9532$, very strong positive correlation between n and t. It can be concluded that the weekly average number of eggs laid by each chicken should increase as the average weekly temperature increases. (c) n = 4t – 71.56
(d) Gradient is 4. For every one degree increase in average weekly temperature the average rate of increase of the weekly average of eggs laid by a hen is 4 eggs over this period of ten weeks.
(e) Vertical intercept is -71.56. In this case the vertical intercept has no practical meaning as the weekly average number of eggs laid by a hen cannot be negative. Although there is no practical application the vertical axis intercept exists and it is where the regression line will intersect the vertical axis if extended.
(f)



The residual plot indicates that the residuals are randomly distributed indicating that the linear model is appropriate for making predictions as the data is linear. The residual plot shows that the magnitude of the residuals is not large giving further support to the reliability of predictions using the linear relationship n = 4t – 71.56 for prediction.
(g) 8.44 (h) Prediction reliable because:
(i) $r_{nt} \approx 0.9532$, indicating a very strong positive correlation between n and t. (ii) residual plot shows that residuals are randomly distributed.
(iii) magnitude of residuals is very small.
(iv) the extrapolation is very close to the given data.
12. (a)



Line graphed using the points (25, 7.740 and (60,4.45)
(b) Gradient is -0.094. The decrease in the average number of days sick leave taken by these employees is 0.094 days for each yearly increase in age. There is an inverse relationship between age of these employees and the number of days sick leave taken.
(c) 10.093 days. In this case the vertical axis intercept has no practical meaning.as the number of days sick leave taken by an employee who is 0 years old makes no sense. The intercept exists because this is where the regression line will cross the vertical axis but it has no meaning in context.
(d) There is an inverse relationship between the age of these employees and the number of days of sick leave taken.
As age increase the number of days sick leave taken decreases, hence the slope is negative. Now the correlation coefficient is a measure of this relationship and hence it also must be negative.
(e) 0.8208 (4 d. p.). 82 08% of the variation in the number of days sick leave taken can be explained by the variation in the age of these employees.
(f) 5 days (4.923)
(g) The residual plot reveals that the residuals are randomly distributed indicating that the data is linear and that using a linear model is appropriate for making predictions. Prediction may be considered to be reliable as it is an interpolation, that is, within the range of the data and the correlation coefficient indicates a strong negative linear relationship between age and number of days sick leave taken.
(h) 5 – 4.923 = 0.077 days
(i) The residual value, 0.077 days, is the difference between the actual number of days sick leave taken and the predicted number of sick leave as given by the equation of the least squares regression line.

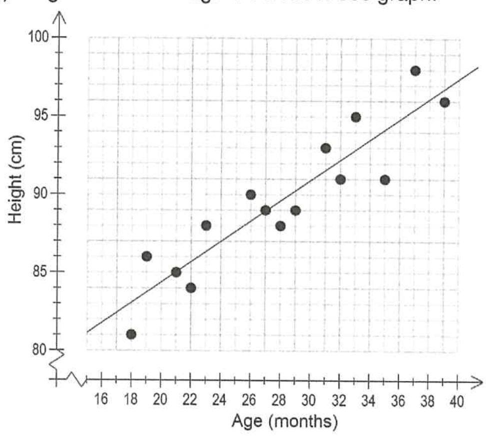
CHAPTER THREE REVIEW EXERCISE.

1.

Scatter plot	Matching number of r^2	Value of r_{xy}
A	(iii)	-0.4
B	(ii)	0.9
C	(i)	-1
D	(iv)	0.8

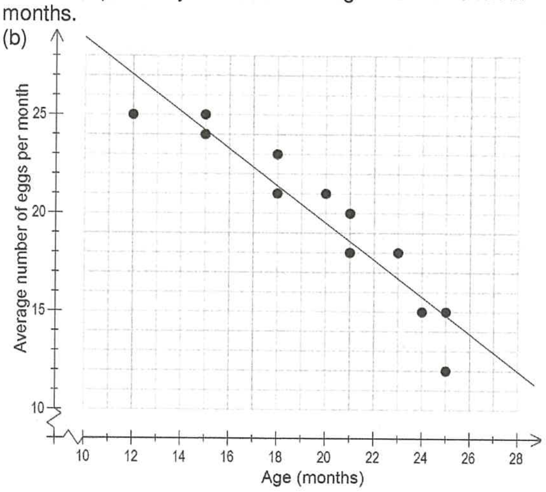
2. (a) Response variable is the "quantity of grapes sold in kilograms". The Explanatory variable is the "price in dollars".
(b) Quantity sold = -45.786 x price of grapes + 469.285 (3 d.p.)
(c) Approx. 23 kg decrease in sales for every 50 cent increase in the price per kilogram.
(d) Approx. 9 kg increase in sales for every 20 cent decrease in price.
(e) -0.9446 (4 d.p.) (f) 0.8924 (4 d.p.) (g) 10.76%
(h) Approx. 263 kilograms. Reasonably reliable as the prediction is an interpolation and there exists a strong linear relationship between price and quantity sold.

3. (a) Height in cm (b) Strong positive linear relationship between age and height. The height of these boys increases as their age increases. (c) Estimate: r = 0.8 data points tending to form a line and r is positive because as age increases so does height. (d) r = 0.9183; the value of r informs us that there is a strong positive linear relationship between age and height. As age increase so does height.
(e) Height = 0.6522 x age + 71.3391. see graph.



(f) Y-intercept is 71.34 cm, so that if a boy was 0 months old we would expect the boy to be 71.34 cm tall. This is outside the range of human birth lengths and may be considered to be meaningless. (g) The gradient of the line is 0.6522. This tells us that on average the height of a boy increases by about 0.65 cm for each one month increase in age.
(h) 94.82 cm (i) Predicting the height of a 3 year old boy involves interpolation. Predicting the height of a 4 year old boy involves extrapolation and hence less reliable. (j) 0.8433
(k) 84.33% of the variation in the boy's height can be explained by the variation in the ages of the boys.
(l) $1 - r^2 = 1 - 0.8433 = 0.1567 = 15.67\%$ i.e. 15.7% (1 d.p.) of the variation is unexplained and is due to chance and/or other factors (variables).

4. (a) Response variable is the number of eggs laid per month and the explanatory variable is the age of the chicken in months.
(b)

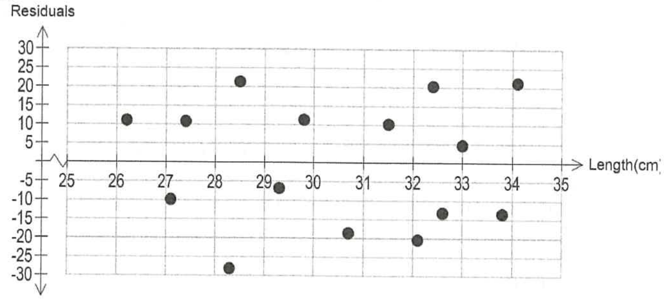


(c) There is a very strong negative linear relationship between the age of these chickens and the average number of eggs they lay each month. The average number of eggs laid each month deceases with the age of the chickens.
(d) r = -0.9 data points tend to form a line and r is negative because as age increases average number of eggs laid deceases.
(e) r = -0.9420
(f) The negative sign indicates that as the age of these chickens increases the average number of eggs laid per month decreases.
(g) Average no. of eggs per month = -0.9420 x age + 38.3544
(h) Approx. 23 eggs laid per month. Prediction is considered to be reliable as the prediction is an interpolation and the value of the correlation coefficient is very close to -1.
(i) Mean of explanatory variable = 19.75 months; Standard deviation of explanatory variable = 4.0646 months (4 d.p.). Mean of response variable = 19.75 eggs per month. Standard deviation of response variable = 4.0646 eggs per month (4 d.p.).
(j) They are the same. The scores for each data set are identical and the association is negative, i.e. low age scores are associated with high average number of eggs per month scores and high age scores are associated with low average number of eggs per month scores.
(k) The value of the coefficient of determination, gives the variation in the average number of eggs laid per month that is explained by the variation in the in the age of these chickens. In this example it is 88.73%. Therefore the unexplained variation is 100% - 88.73% which is 11% to the nearest percent.

5. r = -0.8

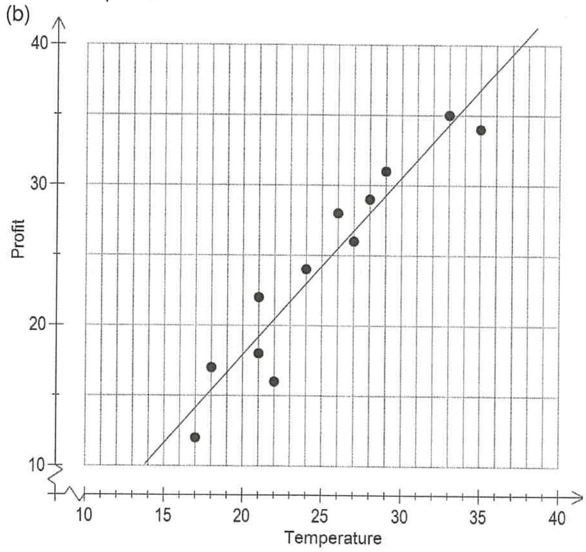
6. 0.9 or -0.9

7. (a) Weight = 17.704 x length – 203.893 (b) The weight increases at a rate of 17.704 grams per 1 cm increase in length
(c) r = 0.9391, $r^2 = 0.8819$ (d) The linear regression model appears to be appropriate as the data has a strong positive relationship and 88.19% of the variation in weight is explained by the variation in length. (e) 354 gms (f) 10.2 gms
(g)

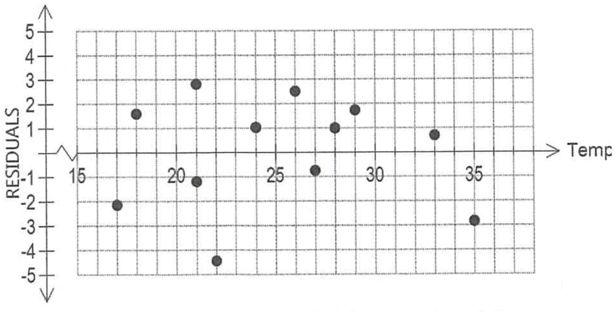


(h) Yes. The regression model is appropriate as the data is linear because the points in the residual plot are randomly distributed above and below zero. That is, there is no pattern that is evident in the residual plot.

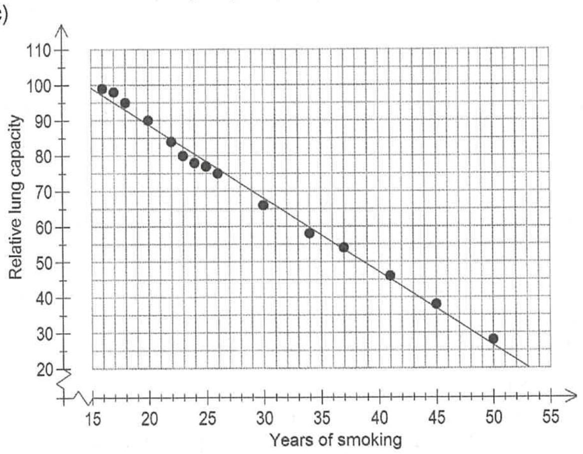
8. (a) Response variable is Profit and the explanatory variable is the Temperature.
(b)



(c) P = 1.26T – 7.27 (d) See graph (e) Estimate r = 0.9 All data points are close to the line of best fit so r must be close to 1 and as temperature increase so does the profit hence r is positive. (f) r = 0.9525 (4 d.p.)
(g) r = 0.9525 indicates a very strong positive linear relationship between the variables. Therefore we can conclude that the profit made should increase as the average monthly maximum temperature increases.
(h) Missing residuals are 1.59, -2.15, 2.81, 1.03, -0.75, 1.73
(i)

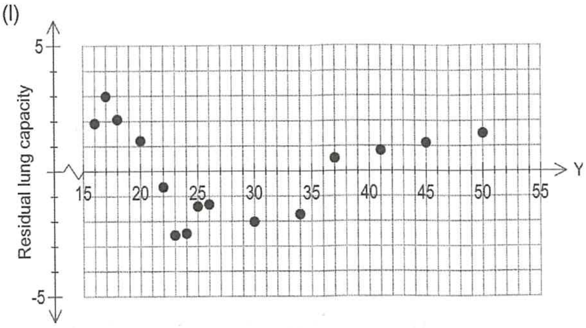


(j) Yes, the least squares regression line is a good model as the data has a very strong positive line relationship and the points in the residual plot are randomly scattered above and below the temperature axis indicating that the original data is linear.
9. (a) The assumption that must be made is that these males contracted emphysema from smoking and not from any other cause e.g. asbestosis.
(b) Explanatory variable is year so smoking and the response variable is relative lung capacity.
(c)

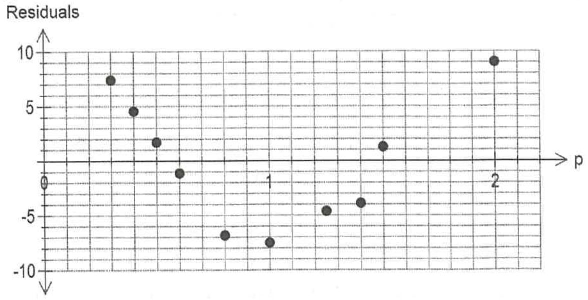


(d) The scatterplot shows a very strong, almost perfect negative linear relationship between years of smoking and relative lung capacity.
(e) C = -2.076Y + 130.305 see graph
(f) The y-intercept is 130.3 units (1 d.p.) and it has no meaning in this context. The y-intercept gives the relative lung capacity of a person with emphysema and not having smoked. Such a person does not comply with our assumption that a person has contracted emphysema as a result of smoking and hence is not valid.
(g) The slope is -2.076. The slope or gradient of the line of best fit informs us that on average the lung capacity of males with emphysema decreases by approx. 2.076 units for every year that they have been smoking. This rate of decrease in lung capacity is only valid for males smoking for at least 16 years.
(h) 87 units to the nearest integer.
(i) r = -0.9966 (4 d.p.) which indicates an almost perfect negative linear relationship between years of smoking and relative lung capacity. Therefore we can conclude that lung capacity should decrease with an increase in smoking years.
(j) The prediction is an interpolation and the value of the correlation coefficient is almost -1 indicating that the prediction is very reliable.
(k) $r^2 = 0.9932$ (4 d.p.) informing us that approx. 99% of the variation in relative lung capacity can be explained by the number of years of smoking. This strongly supports the answer to (j).

(l)

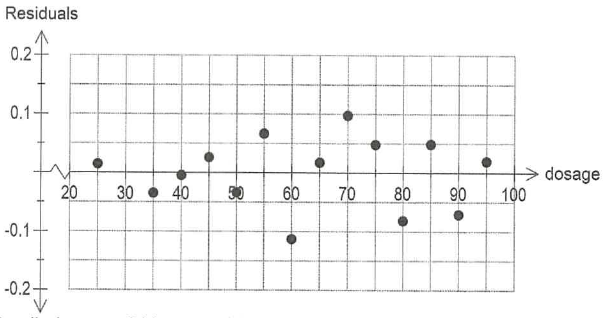


The least squares regression line is not a good model for this data and should not be used to make predictions because the data is not linear as the points in the residual plot make a clear curved pattern.
10. (a) Slope is -1.4, The regression line predicts that on average life expectancies in these countries will decrease by 1.4 years for an increase in birth rate of 1 birth per 1000 people.
(b) Y intercept is 98.6. The regression line predicts that on average the life expectancy for countries with a zero birth rate is 98.6 years. This prediction is highly unlikely to be reliable as only countries with birth rates of more than 30 births per thousand were investigated and we cannot assume that the given regression line will apply to countries with birth rates lower than 30 births per thousand people.
11. (a) Response variable is the "Number of glasses sold per day" and the explanatory variable is "Price charged per glass".
(b) (i) 90 (ii) 75 (iii) 1.26 (iv) 9.04 (c) -0.9895 (4 d.p.) The value of the correlation coefficient indicates a very strong negative linear relationship between price and the number of glasses sold. Therefore we can conclude that the number of glasses sold will decrease as the price of each glass increases.
(d) 25 glasses
(e) The correlation coefficient indicates a very strong negative linear relationship, and $1.80 is within the range of the data (interpolation) so it would be reasonable to conclude that the prediction is reliable.
(f)



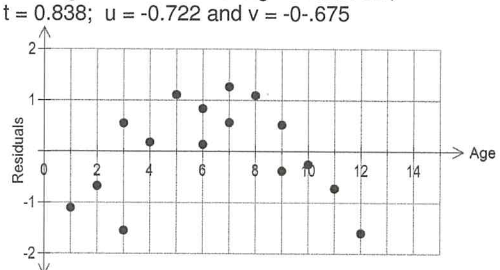
(h) The residual plot shows that the residuals are not randomly distributed about the horizontal axis, they make a clear curved pattern. This suggests that the data is not linear and it would not be appropriate to use the given linear model for prediction purposes. A quadratic or some other model may be appropriate in this case. As the linear model is not appropriate any further comment on the estimate made in(d) using a linear model would not be valid.
12. (a) Response variable is "Average reaction time in seconds".
(b) There appears to be a strong positive linear relationship between dosage of the drug and average reaction time.
(c) r = 0.5952 The value of the correlation coefficient indicates a moderate positive linear relationship between dosage and average reaction time.
(d) Average reaction time = 0.0040 x dosage + 0.9714
(e) Predicted reaction time is 1.14 seconds. Although the prediction is an interpolation its reliability is questionable as the correlation coefficient indicates a low moderate positive linear relationship and the sample of data is small.
(f) (30, 1.45)
(g) r = 0.9047 the strength of the linear relationship increases and without the outlier is strong and positive.
(h) 1.08 seconds.

(i)



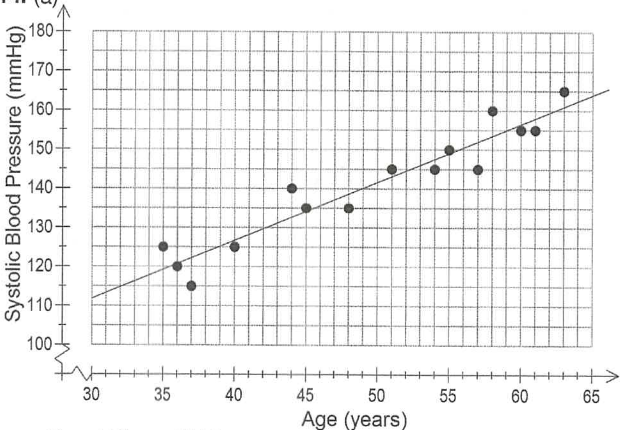
(j) Prediction would be considered to be reliable as it is an interpolation and the residual plot shows that the residuals are randomly scattered above and below the dosage axis indicating that the original data is linear and a linear least squares regression model is appropriate.

13. (a) p = 10.5; q = 7; r = 4
(b) Response variable is "Weight in kilograms".
(c) r = 0.9387 (4 d.p.) The value of the correlation coefficient indicates a strong positive linear relationship between age and weight.
(d) 0.8812 (4 d.p.) (e) 88.12% of the variation in the weight of these boys can be explained by the variation in the age of the boys.
(f) Weight = 0.7719 x age + 3.9309
(g) Slope is 0.7719. For every I month increase in the age of these boys their weight on average increase by 0.7719 kgs.
(h) Vertical axis intercept is 3.9309. The vertical axis intercept tells us that he weight of a boy is on average 3.9309 kgs when the age of the boy is 0 months, that is at birth.
(i) Weight is 6.2 kg (1 d.p.)
(j) The prediction in part (i) may be considered to be reliable as it is an interpolation and the correlation coefficient is high indicating . The linear model used for prediction is appropriate as the value of the coefficient of determination is high in value as only 11.88% (100% – 88.12%) of the variation between age and weight is unexplained.
(k) s = -1.547
(l) The actual weight of this 3 month old boy is 1.547 kg below the weight of a 3 month old boy as predicted by the least squares regression line for the given data set,
(m) t = 0.838; u = -0.722 and v = -0-.675
(n)

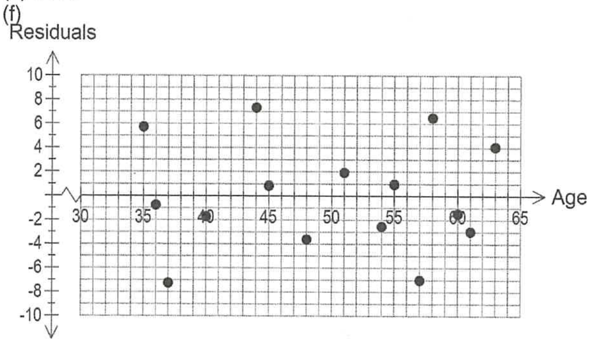


(o) The residual plot shows that the residuals are not randomly distributed about the horizontal axis, they make a clear curved pattern. This suggests that the data is not linear and it would not be appropriate to use the given linear model in part (f) for prediction purposes. A quadratic or some other model may be appropriate in this case.
(p) A baby's age and weight have a high correlation which suggests that as age increases weight also increases. However, a change in age does not necessarily cause a change in the weight of the baby. There may be other factors such as health, diet etc which will affect the change in weight.

14. (a)



(b) Blood pressure = 1.4881 x age + 67.1887
(c) (i) 142 mmHg (ii) 171 mmHg
(d) The prediction for the 50 year old is an interpolation and may be considered to be reasonably reliable whereas the prediction for the 70 year old is an extrapolation and may be considered to be unreliable.
(e) 91%
(f)



(g) The residual plot tests the suitability of a linear regression model for prediction purposes.
(h) The residual plot shows that the residuals are randomly scattered about the age axis indicating that a linear regression model is suitable for prediction purposes as the given data is linear.

ANSWERS CHAPTER FOUR

EXERCISE 4A

1. (a) 17 (b) 28 (c) -9 (d) 28 (e) 64 (f) 32 (g) 12 (h) 23
2. (a) 39 (b) 56 (c) 23 (d) -24 (e) 90 (f) 44 (g) -8 (h) 67
3. (a) 11 (b) -6 (c) 22 (d) 29 (e) -11 (f) 10 (g) 20 (h) 8 (i) 55 (j) 123
4. (a) 3, 5, 7, 9, 11 (b) -2, -1, 0, 1, 2 (c) 5, 2, -3, -10, -19 (d) 3, 9, 27, 81, 243
(e) $0, -\frac{1}{3}, -\frac{1}{2}, -\frac{3}{5}, -\frac{2}{3}$ (f) 1, 2, 4, 8, 16 (g) 2, 6, 12, 20, 30
(h) $1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{1}{3}$
5. (a) 104 (b) (i) to (iv) answer is 3 (c) For this sequence the difference between any two consecutive terms is constant.
(d) No. e.g. (i) 1, 4, 9, 16, . . . e.g. (ii) 6, 12, 24, 48, . . .
6. (a) 3, 6, 12, 24, 48, (b) (i) 3 (ii) 6, (iii) 96 (c) Difference is not constant. (d) all equal to 2 (e) For this sequence the ratio between any two consecutive terms is constant.
(f) No, e.g. (i) 1, 3, 5, . . . e.g. (ii) 10, 5, 0, . . .

EXERCISE 4B

1. 6, 15, 33, 69 **2.** 10, 28, 82, 244 **3.** 8, 33, 133, 533
4. (a) 9, 14, 19, 24 (b) $T_{n+1} - T_n = 5$ (c) The difference equation tell us the difference between any term and the preceding term in the sequence.
5. (a) 21, 19, 17, 15 (b) $T_{n+1} - T_n = -2$ (c) The difference equation tell us the difference between any term and the preceding term in the sequence.
6. 10, 11, 23, 70 **7.** (a) 14, 10, 6, 2, -2 (b) -102
(c) $T_n - T_{n-1} = -4$
8. (a) -5, -2, 1, 4, 7 (b) 20, 36, 68, 132, 260
9. (a) $T_{n+1} = T_n + 2,\ T_1 = 3$ (b) $T_{n+1} = 5T_n,\ T_1 = 2$
(c) $T_{n+1} = T_n + 5,\ T_1 = 1$ (d) $T_{n+1} = 3T_n,\ T_1 = 2$
(e) $T_{n+1} = T_n - 7,\ T_1 = 22$ (f) $T_{n+1} = 2T_n + 1,\ T_1 = 10$
10. (a) 5, 8, 11 (b) 302 (c) $T_{n+1} = T_n + 3,\ T_1 = 5$
11. (a) Straight line $T_{n+1} = T_n - 4,\ T_1 = 20$
(b) Increasing exponential function $T_{n+1} = 2T_n,\ T_1 = 1$
(c) Straight line $T_{n+1} = T_n + 2,\ T_1 = 5,$
(d) Decreasing exponential function $T_{n+1} = 0.5T_n,\ T_1 = 16$
12. (a) $T_{n+1} = T_n - 4,\ T_1 = 6\quad T_6 = -14\quad T_{10} = -30$
(b) $T_{n+1} = T_n + 3,\ T_1 = -4\quad T_6 = 11\quad n = 17$
(c) $T_{n+1} = 3T_n,\ T_1 = 2\quad T_6 = 486\quad n = 10$
(d) $T_{n+1} = 2T_n - 1,\ T_1 = 3\quad T_6 = 65\quad n = 13$
(e) $T_{n+1} = 3T_n,\ T_1 = 1\quad T_6 = 243\quad n = 10$
(f) $T_{n+1} = 2T_n + 3,\ T_1 = 2\quad T_6 = 157\quad n = 11$

EXERCISE 4C
1. (a) 16, 64 (b) 1 (c) 262144 (d) 349525
2. (a) 8 (b) 6651 (c) 13249
3. 32, 25, 18, 11, 4 **4.** 2, 5, 14, 41, 122
5. 10, 12, 14, 16, 18 **6.** $u_1 = -0.5$
7. First term is 2
8. $T_{n+1} = T_n + 2$, $T_4 = 9$ (b) $T_1 = 3$
9. $T_{n+1} = 2T_n - 1$, $T_3 = 2$ (b) $T_1 = 1.25$
10. $T_1 = 10$
11. (a) 4, 14, 44 (b) (i) 44 (ii) 14
12. (a) 4, 6, 10 (b) (i) 6 (ii) 0
13. (a) 2, 3, 0, 9 (b) (i) 3 (ii) 9
14. (a) 5, -1, -7, -13, -19, -25 (b) $T_{n+1} = T_n - 6$, $T_1 = 5$

EXERCISE 4D
1. (a) (i) 1, 3, -1, 7 (ii) 4, 6, 8, 10 (iii) 6, 11, 16, 21 (iv) -2, -4, -8, -16 (b) (iii) is not a first-order linear recurrence relation as it does not describe the relationship between two consecutive terms.
2. $T_{n+1} = 3T_n - 2, T_1 = 2$
3. $T_{n+1} = 11 - 2T_n, T_1 = 4$
4. $T_{n+1} = 0.5T_n + 6.5, T_1 = -3$

CHAPTER FOUR REVIEW EXERCISE
1.(a) 3, 5, 9, 17 (b) 7, 5, 3, 1 **2.** (a) -2 (b) 4
3. (a) 11, 22, 55, 154, 451 (b) Term 14 (c) 1 461 526 (d) Every term of this sequence can be divided by 11 without any remainder.
4. (a) -1, 1, 3, 5, 7 (b) 71 (c) $T_{n+1} = T_n + 2$, $T_1 = -1$ (d) $T_{n+1} - T_n = 2$ (e) General term rule. To use the recursive rule we would need to find all terms up to term seventy before term seventy one can be found.
5. 0.8, 3.3, 5.8 **6.** $T_n = -0.5T_{n-1}$, $T_1 = 8$ **7.** a = 2.5
8. -1, 6, 13, 20 **9.** (a) 10 (b) n = 6 **10.** $U_n = 5U_{n-1}$, $U_1 = 5$
11. (a) a = 3, b = -12 (b) -21 **12.** $T_{n+1} = 2T_n - 1$, $T_1 = 3$

ANSWERS CHAPTER FIVE

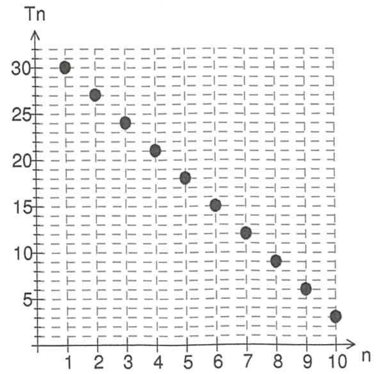
EXERCISE 5A
1. (a) 3, 8, 13, 18, 23 (b) 12, 7, 2, -3, -8 (c) -4, -2.5, -1, 0.5, 2 (d) -8, -11, -14, -17, -20
2. (a) 3 (b) 4 (c) 63 (d) $T_n = 4n - 1$
3. (a) 32 (b) -5 (c) -93 (d) $T_n = 37 - 5n$
4. (a) -20 (b) 3 (c) 70 (d) $T_n = 3n - 23$
5. (a) 16 (b) -5 (c) -44 (d) $T_n = 21 - 5n$
6. (a) -2 (b) -5 (c) -72 (d) $T_n = 3 - 5n$
7. (a) 100, 96, 92, 88, 84 (b) -296
8. 226 **9.** -2

EXERCISE 5B
1. (a) -16 (b) 187 **2.** (a) 98 (b) 38 **3.** (a) -62 (b) 63
4. (a) -2.5 (b) -28.5 **5.** (a) 7 (b) -134 **6.** (a) -7 (b) 369
7. (a) 40 (b) $T_n = 43 - 3n$ **8.** (a) Arithmetic, there is a common difference of 5 between successive terms (b) -107
9. (a) 15 pipes (b) Number = n + 2
10. (a) 88 mins (b) (6n+16)mins

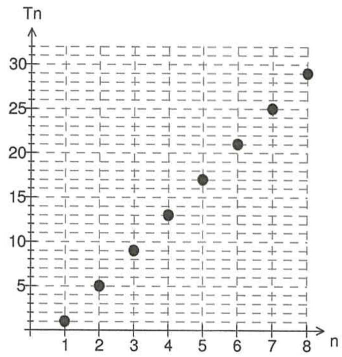
EXERCISE 5C
1. 67th term **2.** 26th term **3.** 61st term **4.** 75th term
5. (a) 164th term (b) 330th term **6.** (a) 890th term (b) 111112th term **7.** (a) 202nd term (b) 802nd term
8. (a) 668th term (b) 3002nd term
9. (a) Arithmetic, because it has a common difference of 9 between successive terms. (b) 895 (c) 111112nd term

EXERCISE 5D
1. Sequence is 5, 9, 13, . . . **2.** Sequence is 40, 47, 54, . . .
3. (a) 22 (b) -86 **4.** (a) 49 (b) -119 **5.** (a) 6 (b) 517

EXERCISE 5E
1. (a) (i) 2, 5, 8, 11, 14 (ii) $T_{n+1} = T_n + 3$, $T_1 = 2$ (iii) $T_{10} = 29$
(b) (i) 7, 9, 11, 13, 15 (ii) $T_{n+1} = T_n + 2$, $T_1 = 7$ (iii) $T_{10} = 25$
(c) (i) 0, 4, 8, 12, 16 (ii) $T_{n+1} = T_n + 4$, $T_1 = 0$ (iii) $T_{10} = 36$
(d) (i) 8, 5, 2, -1, -4 (ii) $T_{n+1} = T_n - 3$, $T_1 = 8$ (iii) $T_{10} = -19$
(e) (i) -4, 1, 6, 11, 16 (ii) $T_{n+1} = T_n + 5$, $T_1 = -4$ (iii) $T_{10} = 41$
(f) (i) 42, 41.5, 41, 40.5, 40 (ii) $T_{n+1} = T_n - 0.5$, $T_1 = 42$ (iii) $T_{10} = 37.5$
2. (a) (i) $T_{n+1} = T_n + 3$, $T_1 = 1$ (ii) $T_{20} = 58$
(b) (i) $T_{n+1} = T_n + 5$, $T_1 = 2$ (ii) $T_{20} = 97$
(c) (i) $T_{n+1} = T_n - 3$, $T_1 = 20$ (ii) $T_{20} = -37$
(d) (i) $T_{n+1} = T_n - 2$, $T_1 = -2$ (ii) $T_{20} = -40$
(e) (i) $T_{n+1} = T_n - 4$, $T_1 = 14$ (ii) $T_{20} = -62$
(f) (i) $T_{n+1} = T_n + 0.3$, $T_1 = 0.1$ (ii) $T_{20} = 5.8$
(g) (i) $T_{n+1} = T_n + 2$, $T_1 = 1.5$ (ii) $T_{20} = 39.5$
(h) (i) $T_{n+1} = T_n - 2.5$, $T_1 = 6.5$ (ii) $T_{20} = -41$
3. (a) (i) $T_1 = 3$ (ii) d = 1 (iii) $T_{30} = 32$ (iv) $T_{n+1} = T_n + 1$, $T_1 = 3$ (v) $T_{n+1} - T_n = 1$ (b) (i) $T_1 = 5$ (ii) d = 2 (iii) $T_{30} = 63$ (iv) $T_{n+1} = T_n + 2$, $T_1 = 5$ (v) $T_{n+1} - T_n = 2$
(c) (i) $T_1 = 1$ (ii) d = 2 (iii) $T_{30} = 59$ (iv) $T_{n+1} = T_n + 2$, $T_1 = 1$ (v) $T_{n+1} - T_n = 2$ (d) (i) $T_1 = 1.5$ (ii) d = 0.5 (iii) $T_{30} = 16$ (iv) $T_{n+1} = T_n + 0.5$, $T_1 = 1.5$ (v) $T_{n+1} - T_n = 0.5$
(e) (i) $T_1 = 2$ (ii) d = -1 (iii) $T_{30} = -27$ (iv) $T_{n+1} = T_n - 1$, $T_1 = 2$ (v) $T_{n+1} - T_n = -1$ (f) (i) $T_1 = -1$ (ii) d = -3 (iii) $T_{30} = -88$ (iv) $T_{n+1} = T_n - 3$, $T_1 = -1$ (v) $T_{n+1} - T_n = -3$
(g) (i) $T_1 = 3.5$ (ii) d = 0.5 (iii) $T_{30} = 18$ (iv) $T_{n+1} = T_n + 0.5$, $T_1 = 3.5$ (v) $T_{n+1} - T_n = 0.5$ (h) (i) $T_1 = 1\frac{2}{3}$ (ii) $d = -\frac{1}{3}$ (iii) $T_{30} = -8$ (iv) $T_{n+1} = T_n - \frac{1}{3}$, $T_1 = 1\frac{2}{3}$ (v) $T_{n+1} - T_n = -\frac{1}{3}$
4. 90, 84, 78, 72 $T_{n+1} = T_n - 6$, $T_1 = 90$
5. (a) 14 terms (b) 37 terms (c) 67 terms (d) 34 terms
6. 98, 96, 94, 92, 90
7. (a) 18, 13, 8, 3, -2 (b) Resulting graph would be a set of collinear points.
8. (a) 18, 31, 44, 57, 70 (b) No, 438 – 18 = 420 and 420 is not divisible by 13.
9. (a) 847, 840, 833, 826, 819
(b) Yes, 532 – 847 = -315 and -315 is divisible by -7.
10. Use the general term. $T_{87} = 9 + (87 - 1)(14) = 9 + 86 \times 14 = 1213$
11. (a) m = 9 (b) Term 114.
12. (a) Graphs (iii) and (iv) because the points are collinear.
(b) (iii) $T_{n+1} = T_n - 3$, $T_1 = 19$ (iv) $T_{n+1} = T_n + 2$, $T_1 = 3$
13. (a) 30, 27, 24, 21, 18 (b) Arithmetic because there exists a constant common difference of -3 between consecutive terms.
(c) Graph consists of a set of collinear points.



14. (a) $T_{44} = 173$ (b) Arithmetic because there exists a constant common difference of 4 between consecutive terms.
(c) $T_{n+1} = T_n + 4$, $T_1 = 1$ (d) Graph consists of a set of collinear points.



15. (a) -8, -5, -2, 1 (b) Yes, because there is a constant common difference of 3 between consecutive terms of the sequence. (c) T_{37} = -8 + (37 – 1)(3) = -8 + 36 x 3 = 100

16. (a) $T_1 = 22$ (b) 22, 19, 16, 13, 10 (c) $T_{n+1} = T_n - 3$, $T_1 = 22$

(d) T_{20} = 22 + (20 – 1)(-3) = 22 + (19) (-3) = -35

17. (a) Missing terms: 2, 5, 8, 11, 14 (b) $T_{n+1} = T_n + 3$, $T_1 = 2$

(c) Sequence is arithmetic because there is a constant common difference of 3 between consecutive terms of the sequence. (d) 610 (e) $T_{n+1} - T_n = 3$ (f) (i) 150% (ii) 60% (iii) 37.5% (g) The percentage increase between consecutive terms is decreasing. The increase between consecutive terms is constant i.e. 3 but for each consecutive term the divisor increases resulting in a smaller percentage increase.

EXERCISE 5F

1. (a) 3, 6, 12, 24, 48, (b) 2, -4, 8, -16, 32
(c) -100, -10, -1, -0.1, -0.01 (d) -64, 32, -16, 8, -4

2. (a) (i) r = 3 (ii) $T_n = 2 \times 3^{n-1}$ (iii) $T_7 = 1458$

(b) (i) r = 2 (ii) $T_n = 10 \times 2^{n-1}$ (iii) $T_7 = 640$

(c) (i) $r = \frac{1}{3}$ (ii) $T_n = (\frac{1}{3})^{n-1}$ (iii) $T_7 = \frac{1}{729}$

(d) (i) $r = \frac{1}{4}$ (ii) $T_n = \frac{4}{3}(\frac{1}{4})^{n-1}$ (iii) $T_7 = \frac{1}{3072}$

(e) (i) $r = -\frac{1}{2}$ (ii) $T_n = 8(-\frac{1}{2})^{n-1}$ (iii) $T_7 = \frac{1}{8}$

(f) (i) $r = \frac{1}{5}$ (ii) $T_n = -\frac{3}{10}(\frac{1}{5})^{n-1}$ (iii) $T_7 = -\frac{3}{156250}$

3. (a) Geometric, (i) a = 1 (ii) r = 4 (iii) $T_n = 4^{n-1}$

(b) Geometric, (i) a = 96 (ii) $r = \frac{1}{4}$ (iii) $T_n = 96 \times (\frac{1}{4})^{n-1}$

(c) Geometric, (i) a = -1 (ii) r = -3 (iii) $T_n = -1 \times (-3)^{n-1}$

(d) Not geometric, sequence does not have a common ratio

(e) Geometric, (i) a = 1 (ii) $r = -\frac{1}{2}$ (iii) $T_n = (-\frac{1}{2})^{n-1}$

(f) Geometric, (i) a = 2 (ii) $r = \frac{1}{3}$ (iii) $T_n = 2 \times (\frac{1}{3})^{n-1}$

4. (a) 12 (b) -20 (c) ±6 (d) ±15 (e) $\pm\frac{3}{4}$ (f) $\frac{b^2}{a}$

5. (a) Not geometric (b) Not geometric (c) Geometric, $r = \frac{1}{3}$

(d) Not geometric (e) Geometric, r = 3 (f) Geometric, r = 4

6. (a) (i) Geometric, r = 2 (ii) Geometric , $r = \frac{1}{2}$ (iii) Arithmetic, d = -2 (iv) Arithmetic, d = 3

(b) (i) $T_n = 2^{n-1}$ (ii) $T_n = 16 \times (\frac{1}{2})^{n-1}$ (iii) $T_n = 21 - 2n$

(iv) $T_n = 3n - 11$

7. -0.15625 **8.** $T_n = -6 \times (-6)^{n-1}$ or $(-6)^n$ T_9 = -10 077 696

9. $\frac{1.2}{0.12} = 10$ and $\frac{12}{1.2} = 10$ the sequence has a common ratio of 10, hence it is geometric. $T_n = 0.12 \times (10)^{n-1}$

10. m = -9000

11. (a) 2% (b) Geometric sequence, sequence has a common ratio of 1.02.

12. (a) Geometric, the sequence has a common ration of -0.5.

(b) $T_n = 16 \times (-0.5)^{n-1}$ (c) -0.0078125

13. Second term could be -30 or 30.

14. 1629.0125 **15.** (a) $T_n = -8 \times (-1.5)^{n-1}$ (b) -91.125

EXERCISE 5G

1. (a) 405 (b) $T_n = 5 \times 3^{n-1}$ (c) n = 10 (d) Term 13

2. (a) 1 594 323 (b) $T_n = 8192 \times (1.5)^{n-1}$ (c) n = 11 (d) Term 15

3. (a) $T_n = 1.5 \times 4^{n-1}$ (b) Term 11

4. (a) $T_9 = 512$ (b) n = 14 (c) n = 10 (d) T_{13} = 16 777 216

(e) n = 6 (f) $T_{10} = 0.0234375$ (g) n = 5 (h) Not possible, n must be a positive integer.

5. First term is 8, common ratio is 3 and $T_n = 8 \times 3^{n-1}$

6. 1024, 512, 256, 128 **7.** 18.75, 37.5, 75,
8. -6, 18, -54, 162 and 6, 18, 54, 162
9. (a) n = 13, n = 15 (b) n = 8, n = 9 (c) n = 18, n = 22
(d) n = 21, n = 24 (e) n = 68, n = 83

EXERCISE 5H

1. (a) 2, 6, 18, 54 Geometric (b) 5, 13, 29, 61 Neither
(c) 4, 6, 9, 13.5 Geometric (d) 34, 27, 20, 13 Arithmetic
(e) 9, 2, 11, 13 Neither (f) 2, 4, 8, 16 Geometric

2. (a) (i) 0.5 (ii) $T_{n+1} = 0.5T_n$, $T_1 = 8$ (iii) $T_5 = 0.5$

(b) (i) 0.25 (ii) $T_{n+1} = 0.25T_n$, $T_1 = 16$ (iii) $T_5 = 0.0625$

(c) (i) 0.1 (ii) $T_{n+1} = 0.1T_n$, $T_1 = 100$ (iii) $T_5 = 0.01$

(d) (i) -3 (ii) $T_{n+1} = -3T_n$, $T_1 = -3$ (iii) $T_5 = -243$

(e) (i) 0.5 (ii) $T_{n+1} = 0.5T_n$, $T_1 = 1$ (iii) $T_5 = 0.0625$

(f) (i) -1.5 (ii) $T_{n+1} = -1.5T_n$, $T_1 = 4$ (iii) $T_5 = 20.25$

(g) (i) -3 (ii) $T_{n+1} = -3T_n$, $T_1 = -1$ (iii) $T_5 = -81$

(h) (i) 3 (ii) $T_{n+1} = 3T_n$, $T_1 = 1.5$ (iii) $T_5 = 121.5$

(i) (i) 0.2 (ii) $T_{n+1} = 0.2T_n$, $T_1 = -0.3$ (iii) $T_5 = -0.00048$

(j) (i) -0.5 (ii) $T_{n+1} = -0.5T_n$, $T_1 = 1$ (iii) $T_5 = 0.0625$

3. (a) (i) 2 (ii) $T_{n+1} = 2T_n$, $T_1 = 2$ (iii) $T_6 = 64$

(b) (i) 3 (ii) $T_{n+1} = 3T_n$, $T_1 = 2$ (iii) $T_6 = 486$

(c) (i) 0.5 (ii) $T_{n+1} = 0.5T_n$, $T_1 = 80$ (iii) $T_6 = 2.5$

(d) (i) -0.5 (ii) $T_{n+1} = -0.5T_n$, $T_1 = 8$ (iii) $T_6 = -0.25$

4. 2, 6, 18, 54 or 2, -6, 18, -54
5. 10, 20, 40, 80 or 10, -20, 40, -80
6. -3, 6, -12, 24 or -3, -6, -12, -24
7. 3, 6, 12, 24 or 3, -6, 12, -24 **8.** 1024, 512, 256, 128

9. 7, 14, 28, 56 **10.** (a) $T_4 = \frac{3}{4}$, $T_5 = \frac{3}{16}$,

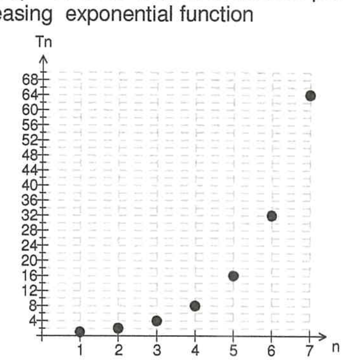
(b) $T_{n+1} = 0.25T_n$, $T_1 = 48$ (c) The graph will consist of a discrete set of points in the shape of a decreasing exponential function.

11. (a) (i) Geometric (ii) Geometric (iii) Geometric (iv) Arithmetic. Graphs of geometric sequences consist of discrete points in the characteristic shape of an exponential function. Graphs of arithmetic sequences consist of discrete points which are collinear.

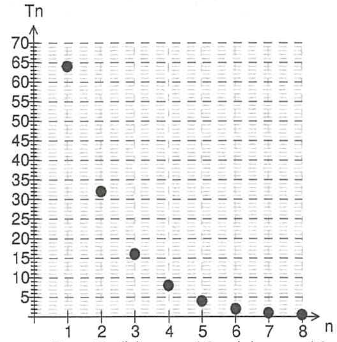
(b) (i) $T_{n+1} = 2T_n$, $T_1 = 0.5$ (ii) $T_{n+1} = 2T_n$, $T_1 = -1$

(iii) $T_{n+1} = 0.5T_n$, $T_1 = -16$ (iv) $T_{n+1} = T_n - 5$, $T_1 = 18$

12. (a) a = 1, r = 2 (b) 1, 2, 4, 8, 16 (c) Geometric. The sequence is generated using a constant ratio of 2.
(d) The graph consists of a set of discrete points in the shape of a increasing exponential function



13. (a) $a = 64$, $r = 0.5$ (b) $T_8 = 0.5$ (c) Geometric. The sequence is generated using a constant ratio of 0.5.
(d) The graph consists of a set of discrete points in the shape of a decreasing exponential function.



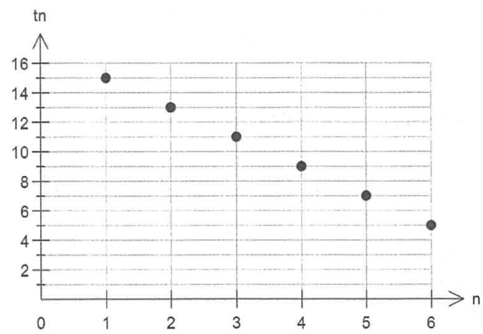
14. (a) $m = -2$ or 2 (b) $m = 12$ (c) $m = 16$ (d) $m = -6$ or 6
15. $T_4 = 24$, $T_{21} = 3\,145\,728$
16. $T_4 = 1$, $T_{21} = 1 \times 10^{-17}$
17. 10, 40, 160, 640 $T_{15} = 2\,684\,354\,560$
18. (a) 10, 11, 12.1 (b) Geometric, the sequence has a common ratio of 1.1 (c) The graph will consist of a set of discrete points in the shape of a increasing exponential function. (d) 55.60
19. (a) (i) Geometric (ii) $T_{n+1} = 0.5T_n$, $T_1 = 16$ (iii) $T_{10} = 0.03125$
(b) (i) Arithmetic (ii) $T_{n+1} = T_n - 0.5$, $T_1 = 16$ (iii) $T_{10} = 11.5$
(c) (i) Geometric (ii) $T_{n+1} = -0.5T_n$, $T_1 = -32$ (iii) $T_{10} = 0.0625$
(d) (i) Arithmetic (ii) $T_{n+1} = T_n + 0.5$, $T_1 = -32$ (iii) $T_{10} = -27.5$
20. (a) (i) $x - 3 = 12 - x \Rightarrow x = 7.5$ (ii) $T_{n+1} = T_n + 4.5$, $T_1 = -6$
(iii) -6, -1.5, 3, 7.5, 12 (b) (i) $\frac{x}{3} = \frac{12}{x} \Rightarrow x^2 = 36 \Rightarrow x = -6$ or 6
(ii) If $x = -6$ then $T_{n+1} = -2T_n$, $T_1 = 0.75$
or if $x = 6$ then $T_{n+1} = 2T_n$, $T_1 = 0.75$
(iii) If $x = -6$ then 0.75, -1.5, 3, -6, 12
or if $x = 6$ then 0.75, 1.5, 3, 6, 12
21. (a) Neither (b) Arithmetic $T_n = T_{n-1} - 5$, $T_1 = 5$
(c) Geometric $T_n = 1.5T_{n-1}$, $T_1 = -4$ (d) Neither (e) Geometric $T_n = -T_{n-1}$, $T_1 = -1$ (f) Geometric $T_n = 1.1T_{n-1}$, $T_1 = 10$
(g) Geometric $T_n = 0.25T_{n-1}$, $T_1 = 20$
22. (a) 5, 15, 45, 135 (b) $T_{n+1} = 3T_n$, $T_1 = 5$ (c) $T_n = 5(3)^{n-1}$
(d) $T_{10} = 98415$ (e) Sum = 147620
23. (a) $t_2 = 4k$ (b) $k = 1.5$ (c) 4, 6, 9, 13.5 (d) Geometric, sequence has a common ratio of 1.5. (e) $t_n = 4 \times (1.5)^{n-1}$
(f) t_{30} or the thirtieth term.
24. (a) $a_2 = 3 - k$ (b) $k = -2$ (c) 3, 5, 7, 9 (d) Arithmetic, sequence has a common difference of 2. (e) $a_n = 2n + 1$
(f) a_{499} or the 499[th] term.

EXERCISE 5I

1. (a)

n	1	2	3	4	5	6
tn	15	13	11	9	7	5

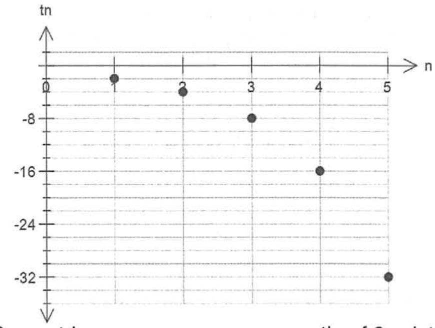
(b)



(c) Long term decreasing solution.
2. (a)

n	1	2	3	4	5
tn	-2	-4	-8	-16	-32

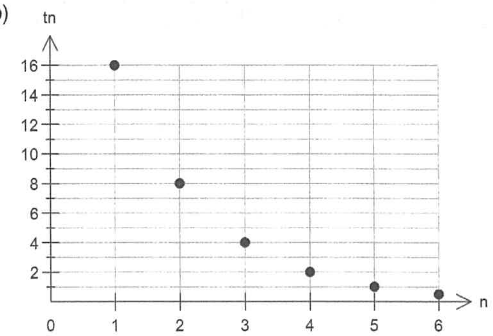
(b)



(c) Geometric sequence, a common ratio of 2 exists between successive terms of the sequence.
(d) Long term decreasing solution.
3. (a)

n	1	2	3	4	5	6
tn	16	8	4	2	1	0.5

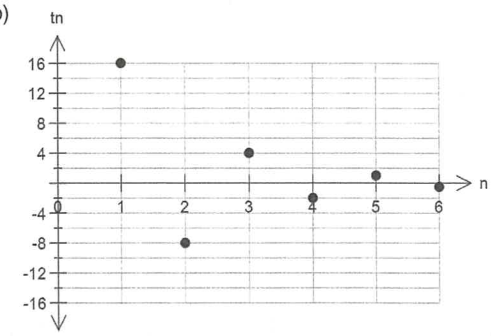
(b)



(c) Geometric sequence, a common ratio of 0.5 exists between successive terms of the sequence.
(d) Long term steady-state solution of value 0.
4. (a)

n	1	2	3	4	5	6
tn	16	-8	4	-2	1	-0.5

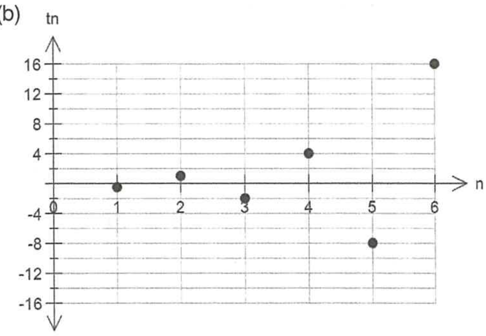
(b)



(c) Geometric sequence, a common ratio of -0.5 exists between successive terms of the sequence.
(d) Long term steady-state solution of value 0.
5. (a)

n	1	2	3	4	5	6
tn	-0.5	1	-2	4	-8	16

(b)



(c) Geometric sequence, a common ratio of -2 exists between successive terms of the sequence.
(d) None of the given solutions, i.e. no long term solution as the terms diverge with alternating sign.
6. (a) long term increasing solution (b) long term steady-state solution of 0. (c) long term decreasing solution (d) long term steady-state solution of 0. (e) long term steady-state solution of 0. (f) long term steady-state solution of 0.
7. The value of the common ratio must lie between 1 and -1, that is $-1 < r < 1$.
8. Graph A (e) Graph B (c) Graph C (a) Graph D (f) Graph E (d) Graph F (b)
9. Graph A $t_{n+1} = -0.5t_n,\ t_1 = 8$

Graph B $t_{n+1} = 0.5t_n,\ t_1 = 10$

Graph C $t_{n+1} = t_n - 1.5,\ t_1 = 10$

Graph D $t_{n+1} = 2t_n,\ t_1 = 1$

Graph E $t_{n+1} = -t_n,\ t_1 = -5$

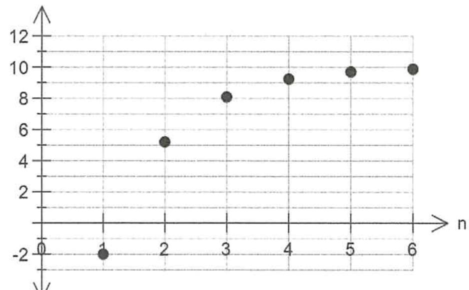
Graph F $t_{n+1} = 2t_n,\ t_1 = -0.5$

EXERCISE 5J

1. (a)

n	1	2	3	4	5	6
t_n	-2	5.2	8.08	9.232	9.6928	9.87712

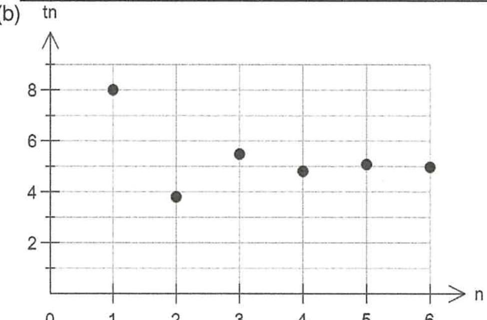
(b) tn



(c) Neither, the sequence does not have a common difference and does not have a common ratio.
(d) In the long run the terms of the sequence approach the value 10.
2. (a)

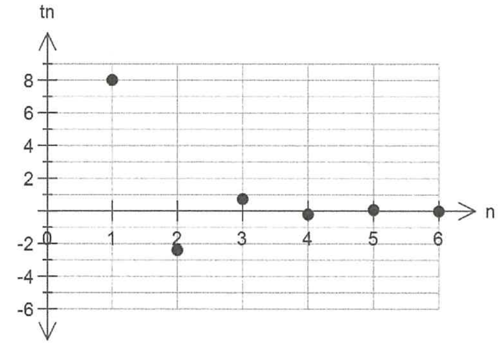
n	1	2	3	4	5	6
t_n	8	3.8	5.48	4.808	5.0768	4.96928

(b) tn



(c) Neither, the sequence does not have a common difference or a common ratio.
(d) In the long run the terms of the sequence approach the value 5.
3. (a)
(b)

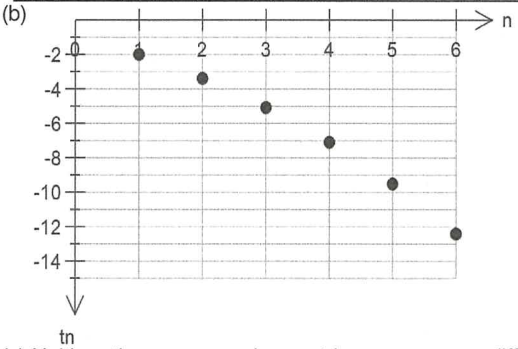
n	1	2	3	4	5	6
t_n	8	-2.4	0.72	-0.216	0.0648	-0.01944



(c) Geometric, the sequence has a common ratio of -0.3.
(d) The recurrence relation has a steady-state solution of value 0.
4. (a)

n	1	2	3	4	5	6
t_n	-2	-3.4	-5.08	-7.096	-9.5152	-12.41824

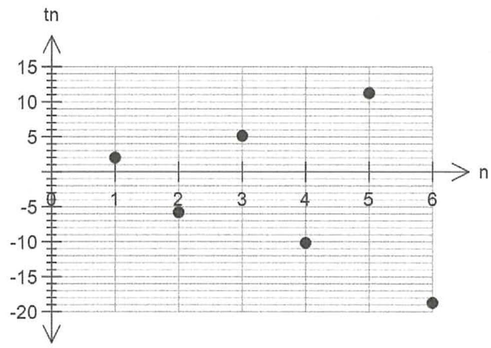
(b)



(c) Neither, the sequence does not have a common difference or a common ratio.
(d) Long term decreasing solution, the terms of the sequence decrease without bound.
5. (a)

n	1	2	3	4	5	6
t_n	2	-5.8	5.12	-10.168	11.2325	-18.72928

(b)



(c) Neither, the sequence does not have a common difference or a common ratio.
(d) In the long run the terms diverge without bound and alternate in sign.
6. (a) 60 (b) 15 (c) 50 (d) -20 (e) -175 (f) 0
7. b = 0.4 **8.** b = -0.9 **9.** k = 0.6 **10.** k = -0.25 **11.** 35
12. (a) For this sequence a long term limit to its terms exist because $-1 < 0.7 < 1$ (b) 10
13. (a) For this sequence a long term limit to its terms exist because $-1 < -\frac{3}{5} < 1$ (b) -7.5 **14.** k = -0.35
15. Sequence A's terms do not approach a limit because the coefficient of t_n is greater than 1.
Sequence B because $-1 < 0.2 < 1$. (b) 7.5
16. (a) k + 10 (b) $t_3 = 5t_2 + k = 5(k + 10) + k = 6k + 50$
(c) k = 4 (d) Terms are 2, 14, 74, 374
17. (a) 9 – k (b) 27 – 4k (c) k = -6 (d) 3, 15, 51, 159
18. From the first term of value 200 subsequent terms increase in value becoming bigger and bigger and get closer and closer to 500 as n becomes increasingly large. This sequence has a steady-state solution of 500.
19. From the first term of value 0 the second term is -140 and the third -35, the fourth is -113.75 and the fifth is -54.6875. The even terms decrease in magnitude and approach the value of -80 and the odd terms increase in magnitude and approach -80 as the number of terms becomes increasingly large. This sequence has a steady-state solution of -80.
20. (a) $t_2 = 5k$ (b) k = 6 **21.** (b) and (c)
22. (a) $-1 < 0.6 < 1$ (b) 15 (c) n = 4

CHAPTER FIVE REVIEW EXERCISE

1. (a) $T_n = 2^{n-1}$ (b) 33 554 432 **2.** (a) -96 (b) 18th term
3. -34.5, -25, -15.5 **4.** (a) -12, 24, -48 or 12 , 24, 48 (b) -3072 or 3072 **5.** (a) 13107.2 (b) 15th term (c) $T_{n+1} = 4T_n,\ T_1 = 0.05$
(d) Student has made an error as a steady-state solution does not exist because the coefficient of T_n is not within the interval from -1 to 1.

6. (a) Sequence A: -24 and 48 Sequence B: 13 and 16

(b) Sequence A: $T_n = 3x(-2)^{n-1}$ Sequence B: $T_n = 3n + 1$

(c) Sequence A: $T_{n+1} = -2T_n,\ T_1 = 3$

Sequence B: $T_{n+1} = T_n + 3,\ T_1 = 4$

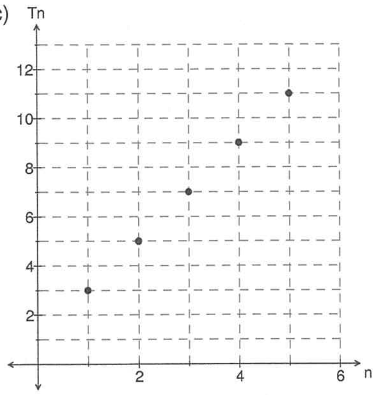
7. (a) a = 3, b = -7 (b) $U_{n+1} = 3U_n - 7,\ U_1 = 4$ (c) 125

8. (a) The sequence is geometric with common ratio 1.5. To find the next term multiply the third term, that is, 9 by 1.5 to get the fourths term which is 13.5 (b) Term 32.

9. (a) a = 2500, r = 0.2 (b) Limiting value is 0. All geometric sequences with $-1 < r < 1$ have a steady-state solution of 0.

10. (a) First five terms are 3, 5, 7, 9, 11 (b) The sequence is Arithmetic because each successive term is 2 more than the previous term, that is, the common difference between consecutive terms is constant and is 2.

(c)



(d) There is a perfect linear relationship between n and T_n because as the position of the of the term(n) increases by 1 the term value(T_n) increases by a constant value of 2.

(e) Required rule is $T_n = 2n + 1$ (f) $T_{20} = 41$

(g) The common difference in the recursive rule is the same as the gradient of the linear rule.

11. (a) $t_2 = 8k + 7$ (b) k = -0.4 (c) Neither, the sequence does not have a common difference nor a common ratio.

(d) In the long-term the terms of this sequence approach the value 5.

12. (a) Arithmetic, it has a common difference of 2.5 between successive terms. (b) $T_n = 2.5n - 0.5$

(c) $50 = 2.5n - 0.5 \Rightarrow 2.5n = 50.5 \Rightarrow n = 20.2$

50 is not a term of the sequence because n is not a counting number. (d) $T_{n+1} = T_n + 2.5,\ T_1 = 2$

(e) Arithmetic, because each successive term is found by subtracting 3 from the previous term i.e. it has a common difference of -3. (f) $T_n = 71 - 3n$

(g) $2.5n - 0.5 = 71 - 3n \Rightarrow 5.5n = 71.5 \Rightarrow n = 13$

(h) Graph the terms of both sequences. The solution will be the point of intersection of the two sets of plotted points which is the point with co-ordinates (13,32). This tells us that the 13th term for both sequences is the same i.e. 32.

13. (a) 1, 3, 6, 10, 15 (b) Graph A Neither Graph B Geometric Graph C Arithmetic Graph D Neither

(c) $T_{n+1} = T_n - 2,\ T_1 = 15 \quad T_6 = T_5 - 2 = 7 - 2 = 5$

(d) $T_n = 17 - 2n$ (e) -43

(f) Sequence B $T_{n+1} = 0.5T_n,\ T_1 = 16$

$T_6 = 0.5T_5 = 0.5(1) = 0.5$

(g) Sequence A: long-term increasing solution; Sequence B: long-term steady state solution of 0; Sequence C: long-term decreasing solution; Sequence D: long-term steady state solution of 12

14. (a) Geometric, there is a common ratio of 1.02.

(b) $T_{n+1} = 1.02T_n,\ T_1 = 5$ (c) $T_{36} = 9.9994$ (4 d.p.) and $T_{37} = 10.1994$ (4 d.p.) Hence term 37. (d) Sum of first 16 terms is 93.1964 (4 d.p.) and sum of the first 17 terms is100.0604 (4 d.p.). Hence 17 terms need to be summed.

15. (a) Sequence 1: Common difference is positive.
Sequence 2: Common difference is negative.
Sequence 3: Common difference is zero, this is a constant sequence.

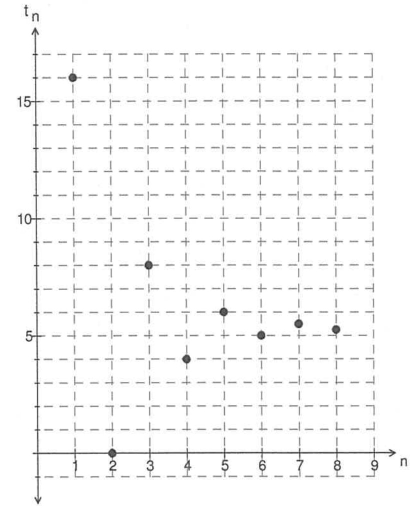
(b) Sequence 1: Common ratio is greater than one.
Sequence 2: Common ratio is between zero and one.
Sequence 3: Common ratio is negative and less than negative 1.

16. (a)

n	1	2	3	4	5	6	7	8
t_n	16	0	8	4	6	5	5.5	5.25

(b) Neither, the sequence does not have a common difference nor a common ratio.

(c)



(d) The sequence has a steady state solution of $5\frac{1}{3}$.

ANSWERS CHAPTER SIX

EXERCISE 6A

1. (a) $t_n = 60n + 2000$ (b) $2420

(c) $t_{n+1} = t_n + 60,\ T_0 = 2000$ (d) 34 years

2. (a) (i) $420 (ii) $2100 (iii) $420n (b) $(15000 + 420n)
(c) 25 years

3. (a) $13 440 (b) $t_{n+1} = t_n + 360,\ t_0 = 12\,000$

(c) During the 26th year

4. (a) $4020, $4040, $4060, $4080 (b) $t_n = 20n + 4000$

(c) (i) $4200 (ii) $4480 (d) $t_{n+1} = t_n + 20,\ t_0 = 4000$

(e) 50 months or 4 years 2 months.

5. (a) $1000, $1010, $1020, $1030 (b) Arithmetic sequence

(c) $t_{n+1} = t_n + 10,\ t_0 = 1000$ (d) (i) $1120 (ii) $1180 (iii) $1600

(e) $1210 (f) 1st July 2017 (g) 1st May 2023

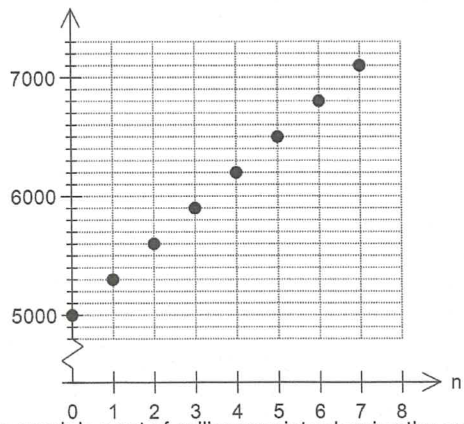
6. (a) $70, $140, $210, $280 (b) Arithmetic sequence, because there exists a common difference of $70 between the interest amounts between successive years. (c) $t_n = 70n$

(d) $1750 (e) 29 years

7. (a)

n	0	1	2	3	4	5	6	7
A_n	5000	5300	5600	5900	6200	6500	6800	7100

(b)



(c) The graph is a set of collinear points showing the amount of the investment for each of the first 7 years. The graph show that the amount invested was $5000. (d) Arithmetic, the sequence of terms has a common difference of 300.

(e) $A_n = 300n + 5000$ (f) $9800

8. (a) \$3200 (b) Simple interest. Graph show linear growth and linear growth can be modelled by an arithmetic sequence. Simple interest is modelled by arithmetic sequences hence the graph shows that simple interest is being added.
(c) $V_{n+1} = V_n + 400, \quad V_0 = 3200$ (d) 12.5% p.a.

EXERCISE 6B

1. (a)

Balance after 2 years	Balance after 3 years	Balance after 4 years
\$2000 x 1.04^2	\$2000 x 1.04^3	\$4000 x 1.04^4
\$2163.20	\$2249.728	\$2339.71712

(b) $t_{n+1} = 1.04t_n, \quad t_0 = 2000$ (c) p is the common ratio or growth factor (d) q is the opening balance. (e) Balance of the investment after 1 year. (f) \$2960.49
2. (a) \$1980, \$2178, \$2395.80. \$2635.38 (b) Geometric sequence, there exists a common ratio of 1.1 between successive terms. (c) (i) $t_{n+1} = 1.1t_n, \quad t_0 = 1800$ (ii) \$4668.74
(d) (i) $T_n = 1980 \times (1.1)^{n-1}$ (ii) \$4668.74
3. (a) \$8440, \$8904.20, \$9393.93 \$9910.60 (b) Geometric sequence, there exists a common ratio of 1.055 between successive terms. (c) (i) $t_{n+1} = 1.055t_n, \quad t_0 = 8000$
(ii) \$13665.16 (d) (i) $T_n = 8440 \times (1.055)^{n-1}$ (ii) 17859.81
4. (a) $t_{n+1} = 1.07t_n, \quad t_1 = 10\,700$ (b) \$10 000 (c) During the 11th year.
5. (a) 1.05 is the annual growth factor, it tells us that the value of the investment is increasing by a factor of 1.05 every year. (b) 5% p.a. (c) The opening balance is \$4000 or the amount invested is \$4000. (d) \$6205.31 (e) \$2205.31 (f) \$295.49 (g) During the 15th year.
6. (a) 1.007 is the monthly growth factor, it tells us that the value of the investment is increasing by a factor of 1.007 every month (b) 8.4% p.a. (c) The opening balance is \$6500 or the amount invested is \$6500. (d) \$6873.04 (e) \$8712.67 (f) \$1855.54 (g) \$49.13
7. (a) (a) $A_{n+1} = 1.06A_n, \quad A_0 = 4000$ (b) \$5352.90
(c) Towards the end of 2016 (d) $M_n = 1.005M_{n-1}, \quad M_0 = 4000$
(e) \$5395.40 (f) \$42.50
8. (a) Compounding interest as the amount of interest increases for each successive each month. For flat-rate interest the amount of interest would be the same for each month. (b) 0.6% p.m. (c) $B_n = 1.006B_{n-1}, \quad B_0 = 4000$
(d) \$4146.18 (e) \$4617.55

EXERCISE 6C

1. (a) 29 seats (b) Arithmetic as each successive row has 5 more seats than the previous row, i.e. common difference is 5.
(c) $T_{n+1} = T_n + 5, \quad T_1 = 14$ (d) 139 people (e) 1989
2. 26 years from **now** we can expect a profit of \$95 000.
3. 13 years, starts to receive a salary of \$81 500 in the 14th year 4. (a) $C_t = 1.8C_{t-1}, \quad C_1 = 100$ (b) \$3401
(c) \$7528 5. (a) 1200, 1440, 1728, 2074 (2073.6)
(b) Geometric (c) $P_t = 1.2P_{t-1}, \quad P_0 = 1000$ (d) 6200
(e) 79 500 (79 496.84) (f) 22 hours
6. (a) $T_n = 1.019T_{n-1}, \quad T_1 = 2.4$, where T_1 is the population in 2011 in millions. (b) 2.8 million (2.8430 . . .)
(c) During 2039
7. (a) 33.82% (2 d.p.) (b) During the 12th year.
8. (a) 1500 (b) 118 200 (c) $P_t = 1.22P_{t-1}, \quad P_0 = 1000$
(d) 22% per hour (e) 20 hours (f) 26 005
9. (a) \$40 900 (\$40 855.9843) (b) \$210 640.36 (c) 2014
(d) Yes. The assumption is that the increase in average earnings is 6.5% p.a. over the period under discussion.
10. (a) \$2 023 000 (b) an 7 storey car park

EXERCISE 6D

1. (a) \$12 650 (b) \$15 472.25
(c) $B_{t+1} = 1.065B_t + 2000, \quad B_0 = 10\,000$ or
$B_t = 1.065B_{t-1} + 2000, \quad B_1 = 12\,650$ (d) \$25 088.15
2. (a) \$2640 (b) \$3324.80 (c) $B_{t+1} = 1.07B_t + 500, \quad B_0 = 2000$ or $B_t = 1.07B_{t-1} + 500, \quad B_1 = 2640$ (d) \$9665.91
3. (a) \$1206 (b) $A_{n+1} = 1.006A_n + 200, \quad A_0 = 1000$ or $A_n = 1.006A_{n-1} + 200, \quad A_1 = 1206$ (c) \$3116.52 (d) \$12 419.61
4. (a) Balance end Q1 = \$5000 x 1.02 + 400 = \$5500
(b) $B_{t+1} = 1.02B_t + 400, \quad B_0 = 5000$ or $B_t = 1.02B_{t-1} + 400, \quad B_1 = 5500$ (c) \$17 148.68
5. (a) \$4648 (b) $B_{t+1} = 1.012B_t + 600, \quad B_0 = 4000$ or $B_t = 1.012B_{t-1} + 600, \quad B_1 = 4648$ (c) \$15 355.47
6. (a) \$3146.81 (b) \$2946.81
7. \$9945.25
8. (a) \$200.60, \$301.80, \$403.61 (b) $A_n = 1.006A_{n-1} + 100, \quad A_0 = 100$ (c) \$279 459.97
9. (a) 7.6% p.a. (b) $t_n = 1.076t_{n-1} + 1000, \quad t_0 = 1000$
(c) \$4479.54 (d) \$31 550.40 (e) \$15 550.40
10. (a) 79 mL (b) 78.2 mL (c) $T_{n+1} = 0.8T_n + 15, \quad T_0 = 80$
(d) 76.3 mL (e) After each injection the amount decreases and is approaching a steady state solution of 75 mL. (f) 75 mL.
11. (a) 125.92 mL (b) 118.66 mL (c) $T_{n+1} = 0.72T_n + 28, \quad T_0 = 136$
(d) 109.67 mL (e) After each injection the amount decreases and is approaching a steady state solution of 100 mL. (f) 100mL.
12. (a) 202 million (b) Population stabilises at 200 million.
13. (a) 23.2 units (b) $Q_n = 0.58Q_{n-1} + 40, \quad Q_0 = 40$ (c) Yes, the steady-state solution is 95.24 units (2 d.p.). In the long-term the amount of the antibiotic in the bloodstream will be below 100 units.
14. (a) $V_n = 0.6V_{n-1} + 1000, \quad V_1 = 3000$ (b) 2505 litres
(c) 1002 litres (d) 2500 litres (e) 1000 litres
15. (a) $P_n = 1.15P_{n-1}, \quad P_1 = 2000$ (b) 4023 (c) During 2022
(d)

2016	2017	2018	2019	2020	2021
3626	3170	2646	2043	1349	552

(e) $Q_n = 1.15Q_{n-1} - 1000, \quad Q_0 = 4023$ (f) Population is decreasing because it is being overfished. (g) 603

EXERCISE 6E

1. (a) (a) \$307.20 (b) \$112.80 (c) (i) \$2594.40 (ii) \$2481.60 (iii) \$2368.80 (d) $a_n = a_{n-1} - 112.8, \quad a_0 = 2707.2$
(e) (i) \$1692 (ii) \$1353.60 (f) At the start of the 17th month
2. (a) \$4200 (b) \$337.50 (c) $a_n = a_{n-1} - 337.5, \quad a_1 = 16\,200$
(d) (i) \$14 175 (ii) \$4050
3. (a) \$1872 (b) \$144 (c) $a_n = a_{n-1} - 144, \quad a_0 = 1872$
(d) \$1152
4. (a) \$1120 (b) $a_n = a_{n-1} - 78.75, \quad a_0 = 945$ (c) \$630
(d) 35% p.a.
5. (a) A = 8360, B = 8360 C = 7980 D = 4180 E = 380 F = 3800 (b) $a_n = a_{n-1} - 380, \quad a_1 = 8740$ (c) \$3040
(d) After the 22nd repayment. (e) 7% p.a.

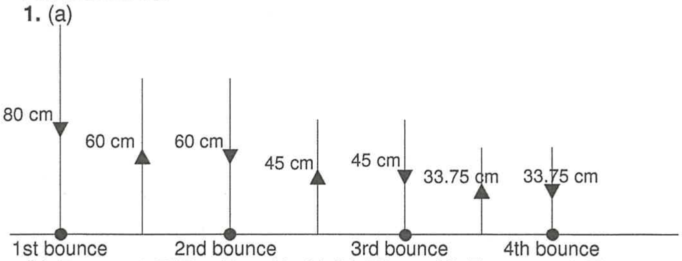
EXERCISE 6F

1. (a) $A_n = 1.024A_{n-1} - 25\,000, \quad A_0 = 180\,000$ (b) \$71 516.25
(c) 8 years
2. (a) $A_n = 1.0055A_{n-1} - 400, \quad A_0 = 20\,000$ (b) \$12 581.50
(c) 59 months (4 years 11 months) (d) \$252.41 (e) \$3452.41
3. (a) $A_n = 1.005A_{n-1} - 250, \quad A_0 = 5000$ (b) \$2224.50
(c) 22 months (1 years 10 months) (d) \$31.25 (e) \$281.25
4. (a) $A_n = 1.021A_{n-1} - 5000, \quad A_0 = 80\,000$ (b) \$35220.63
(c) 5 years (d) \$3524.58 (e) \$18 524.58
5. (a) $A_n = (1 + \frac{0.1}{12})A_{n-1} - 900, \quad A_0 = 20\,000$ (b) \$10 785.25
(c) In 2 years and 1 month (d) \$610.64 (e) \$2210.64
(f) \$9214.75 (g) During the 19th month.
6. (a) $A_n = 1.008A_{n-1} - 400, \quad A_0 = 12\,000$ (b) \$3991.68
(c) In 2 years and 11 months (d) \$177.05 (e) \$1777.05
7. (a) 242 204.04 (b) \$257 486
8. (a) 2 year 10 months (b) \$587.85 (c) \$1737.85
(d) \$17 592.68

9. (a) 3 years (b) \$4753.87 (c) 2 years 5 months
(d) Saves \$841.66
10. (a) a = 1.0125, b = -500
(b) April 6784.9375; 84.8117;500; 6369.7492
(c) 18 months (d) \$981.52

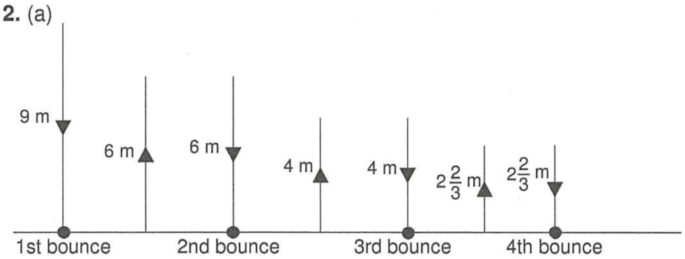
EXERCISE 6G

1. (a)



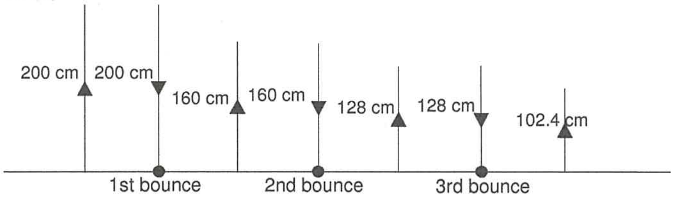
(b) $T_{n+1} = 0.75T_n$, $T_1 = 60$ (c) 33.75 cm (d) $T_{n+1} = 0.75T_n$, $T_1 = 80$ (e) 18.98 cm (f) The common ratio lies in the interval between -1 and 1 which means that the values of the terms decrease and approach the value 0 as the number of terms increase without bound. (g) $D_n = 0.75D_{n-1} + 140$ where $D_1 = 80$ (h) 560 m

2. (a)



(b) $T_{n+1} = \frac{2}{3}T_n$, $T_1 = 6$ (c)1.19 m or 119 cm (d) 16 cm
(e) $D_{n+1} = \frac{2}{3}D_n + 15$ where $D_1 = 9$ (f) $37\frac{8}{9}$m (g) 45 metres

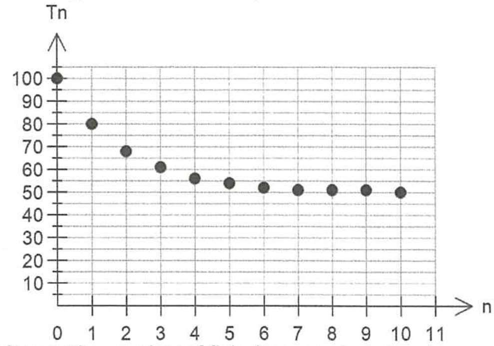
3. (a)



(b) $T_{n+1} = 0.8T_n$, $T_1 = 200$ (c) 65.54 cm
(d) $T_{n+1} = 0.8T_n + 400$ where $T_1 = 400$ (e) 1964 cm
(f) 2000 cm or 20 m
4. (a) 180 m (b) 162 m (c) $R_{t+1} = 0.9R_t$, $R_1 = 200$ (d) 77.48 m (2 d.p.) (e) Distance it rise decreases and approaches a limit of zero metres. (f) 200 m (g) 380 m, 542m, 687.8 m
(h) $A_{t+1} = 0.9A_t + 200$, $A_1 = 200$ (i) 1303 m (j) 2000 m
5. (a) $A_{t+1} = 0.95A_t$, $A_1 = 1000$ (b) \$630.25 (c) \$1950, \$2852.50, \$3709.88 (d) $S_{n+1} = 0.95S_n + 1000$, $S_1 = 1000$
(e) \$8624 (f) \$20 000
6. (a) 2000 gms (b) $A_t = 0.99A_{t-1}$, $A_0 = 2000$
(c) 1902 grams (d) 69 years (e) 299 years.
7. (a) 1400grams (b) $Q_t = 0.97Q_{t-1}$, $Q_0 = 1400$
(c) 305.3 grams (1 d.p.) (d) 23 years
8. (a) 15m, 11.25m, 8.4375m (b) $R_n = 0.75R_{n-1}$, $R_1 = 15$
(c) 2.67m (d) $H_n = 0.75H_{n-1}$, $H_1 = 20$ (e) 3.56 m
(f) $D_{n+1} = 0.75D_n + 35$, $D_1 = 20$ (g) 131 m (h) 140 m
9. (a) \$35 000, \$32 000, \$29 000 \$26 000 (b) Arithmetic, there is a common difference of -\$3000 between consecutive terms.
(c) $T_{n+1} = T_n - 3000$, $T_0 = 38\,000$ (d) \$5000
10. (a) \$14 400, \$12 960, \$11 664, \$10 497.60 M,
(b) 14 400, 12 960, 11664, . . . geometric sequence because there is a common ratio of 0.9 between the terms
(c) $T_{n+1} = 0.9T_n$, $T_0 = 16\,000$ (d) \$5021
11. (a) \$23 600 (b) Geometric, there is a common ratio of 0.9 between successive outputs. (c) $T_n = 0.9T_{n-1}$, $T_1 = 40\,000$
(d) End of 2023. **12.** 480 000 copies
13. (a) 21 000, 22050, 23153
(b) $P_n = 1.05P_{n-1}$, $P_0 = 20\,000$ (c) 25 500 (d) Population will reach 30 000 during the 9th year (e) $P_n = 0.96P_{n-1}$, $P_0 = 20\,000$
(f) 16 300 (g) During the 17th year.
14. (a) 24 250 (b) $X_{n+1} = 0.3X_n + 250$, $X_0 = 80\,000$
(c) 13 100 (d) $Y_{n+1} = 0.16Y_n + 300$, $Y_0 = 80\,000$
(e) Neither. In the long term both insecticides have the same steady-state solution of 357 aphids.

CHAPTER SIX REVIEW EXERCISE

1. (a) $V_{n+1} = V_n + 28.9$, $V_0 = 850$ (b) \$1110.10 (c) \$346.80
(d) $V_n = 28.9n + 850$ (e) \$144.50 (f) During the 30th year
2. (a) $A_n = 1.08A_{n-1}$, $A_0 = 6000$ (b) \$9521.25 (c) During the 10th year (d) $Q_n = 1.02Q_{n-1}$, $Q_0 = 6000$ (e) \$9650.62
(f) Extra \$129.37 (g) $M_n = (1+\frac{0.08}{12})M_{n-1}$, $M_0 = 6000$
(h) \$9681.01 (i) Extra \$159.76
3. (a) \$1002.33 (b) $A_n = (1+\frac{0.035}{12})A_{n-1} + 200$, $A_0 = 800$
(c) \$2850.10 (d) At the end of 43 months (3 years 7months
(e) \$11 208.45 (f) \$808.45
4. (a) $A_n = 1.0475A_{n-1} - 100$, $A_0 = 1000$ (b) \$711.35
(c) 14 years, \$88.74 (d) \$388.74
5. (a) $A_n = 1.008A_{n-1} - 500$, $A_0 = 20000$ (b) \$15 735.61
(c) 27 months (d) After 49 months, i.e. 4 years and 1 month
(e) \$200.67 (f) \$4200.67
6. (a) \$756 (b) \$146 (c) $a_n = a_{n-1} - 146$, $a_0 = 5256$ (d) \$1752
7. (a) $R_n = 0.9R_{n-1}$, $R_1 = 18$ (b) $F_n = 0.9F_{n-1}$, $F_1 = 20$
(c) 6.97 metres (d) 7.75 metres
8. (a) 51 200 (b) Approx. 4
9. (a) During the 13th year. (b) During the 13th year, because the growth rate is the same for both towns and regardless the initial population it will always treble during the 13th year.
10. (a) $P_{n+1} = 1.05P_n$, $P_0 = 1000$ (b) 1340 (c) The rate of increase of 5% per year has not changed over the 6 years.
(d) 1536 (e) Koala bear population is still increasing.
(f) 70 need to be removed.
11. (a) $V_n = 0.82V_n + 1$, $V_0 = 12$ (b) 10 L (c) No. Water volume will fall below 9 litres during week4 and the car may overheat and stop working. (d) 1.7 litres
12. (a) $V_{n+1} = 1.2V_{n-1} - 90$, $V_0 = 800$ (b) 1495 mL
(c) Balloon bursts on the 11th puff.
13. (a) $D_n = 0.85D_{n-1} + 1.65$, $D_0 = 12$ (b) 11.3 mg/L
(c) Yes, steady-state solution to the recurrence relation is 11 which tells Joshua that in the long-term this pump will maintain a steady oxygen level of 11 mg/L with the current fish stock.
14. (a) \$31 160 (b) $V_n = 0.82V_{n-1}$, $V_0 = 38\,000$ (c) \$14 100
(d) $V_n = 31160(0.82)^{n-1}$ or $V_n = 38\,000(0.82)^n$
15. (a) 6.84 cm (b) $H_n = 1.14H_{n-1}$, $H_0 = 6$ (c) 22 February 2016 (d) 2.68 metres
16. (a) $T_n = 0.6T_{n-1} + 20$, $T_0 = 100$ (b) 56 (c) Caught 99 for his table. 1 original fish still in the pool.
(d)



(e) Each year the number of fish decrease but at a slower rate and in the long-term reach a steady-state of 50 fish in the tank at the end of each year.

ANSWERS CHAPTER SEVEN

EXERCISE 7A

1. (a) (i) 3 (ii) 2 (iii) 4 (b) (i) 3 (ii) 3 (iii) 6 (c) (i) 3 (ii) 2 (iii) 4 (d) (i) 3 (ii) 3 (iii) 6 (e) (i) 4 (ii) 4 (iii) 8 (f) (i) 4 (ii) 6 (iii) 12 (g) (i) 4 (ii) 4 (iii) 8 (h) (i) 4 (ii) 4 (iii) 8 (i) (i) 2 (ii) 1 (iii) 2 (j) (i) 5 (ii) 8 (iii) 16 (k) (i) 4 (ii) 7 (iii) 14 (l) (i) 5 (ii) 8 (iii) 16 (m) (i) 3 (ii) 4 (iii) 8 (n) (i) 4 (ii) 6 (iii) 12 (o) (i) 3 (ii) 4 (iii) 8 (p) (i) 5 (ii) 4 (iii) 8(q) (i) 3 (ii) 3 (iii) 6 (r) (i) 5 (ii) 6 (iii) 12 (s) (i) 4 (ii) 7 (iii) 14 (t) (i) 8 (ii) 7 (iii) 14

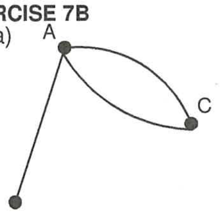
2. (a) B, C and D (b) A and D (c) A and C (d) A and D (e) C to C (f) 7 (g) 2 (h) 5 (i) 3 (j) A (k) B and D

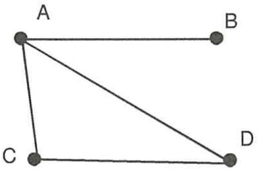
3. (a) A, B, D and D (b) AC, BC, CD, CD (c) 4 (d) C and D (e) E (f) D and F (g) 3 (h) D and F (i) F to F (j) Degree is 0. There are no edges connected to vertex E.

4. (a) 3 (b) Corrigin, Kulin, Katanning, Kojonup (c) Nyabing (d) Kulin and Katanning (e) Kojonup-Katanning-Nyabing or Kojonup-Wagin-Katanning-Nyabing or Kojonup-Wagin-Kulin-Nyabing. (f) Corrigin-Wagin-Kulin-Nyabing; Corrigin-Wagin-Katanning-Nyabing; Corrigin-Wagin-Kojonup-Katanning-Nyabing; Corigin-Kojonup-Wagin-Wagin-Kulin-Nyabing; Corigin-Kojonup-Wagin-Katanning-Nyabing; Corigin-Kojonup-Katanning-Nyabing

5. A in-degree 1, out-degree 1; B in-degree 2, out-degree 1; C in-degree 1, out-degree 3; D in-degree 3, out-degree 2; E in-degree 1, out-degree 1.

EXERCISE 7B

(a)


(b)


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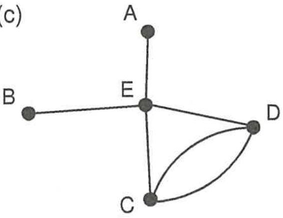
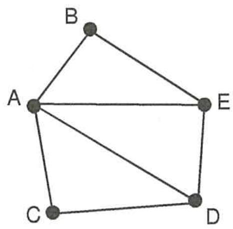
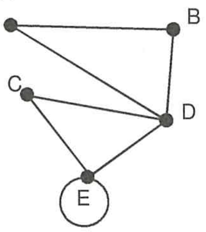
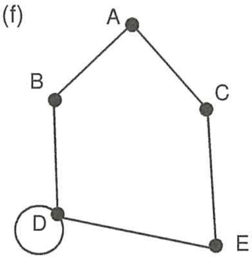
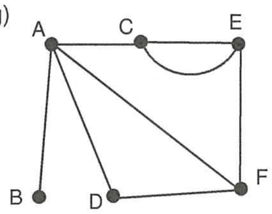
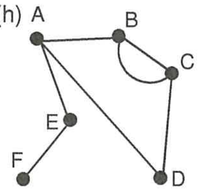


(d)


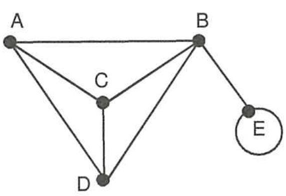
(e)


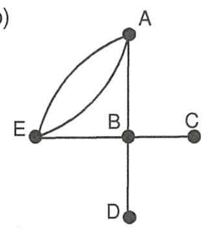
(f)


(g)


(h)


2.

(a)


(b)


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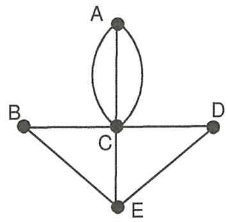
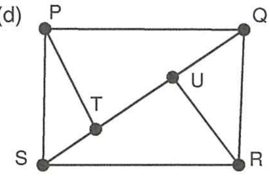
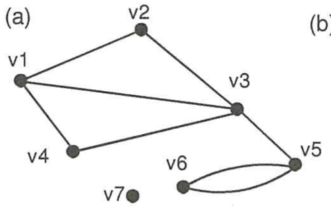
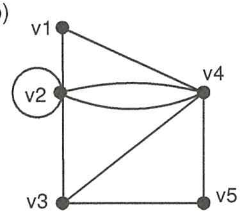


(d)


3.

(a)


(b)


4.

Vertex	Graph 3 (a)	Graph 3 (b)
v1	3	2
v2	2	6
v3	4	3
v4	2	5
v5	3	2
v6	2	
v7	0	

5. (a)
$$\begin{array}{c|c} & A\ B\ C \\ \hline \begin{array}{c}A\\B\\C\end{array} & \begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}\end{array}$$

(b)
$$\begin{array}{c|c} & A\ B\ C\ D \\ \hline \begin{array}{c}A\\B\\C\\D\end{array} & \begin{bmatrix}0&1&1&0\\1&0&1&0\\1&1&0&1\\0&0&1&0\end{bmatrix}\end{array}$$

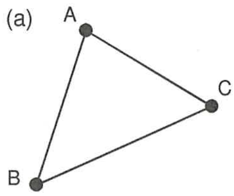
(c)
$$\begin{array}{c|c} & A\ B\ C\ D\ E \\ \hline \begin{array}{c}A\\B\\C\\D\\E\end{array} & \begin{bmatrix}0&0&1&0&0\\0&0&1&0&0\\1&1&0&1&1\\0&0&1&0&1\\0&0&1&1&0\end{bmatrix}\end{array}$$

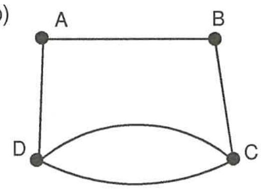
(d)
$$\begin{array}{c|c} & A\ B\ C \\ \hline \begin{array}{c}A\\B\\C\end{array} & \begin{bmatrix}0&1&1\\1&0&2\\1&2&1\end{bmatrix}\end{array}$$

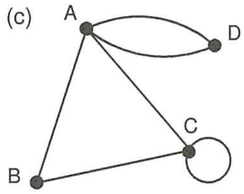
(e)
$$\begin{array}{c|c} & A\ B\ C\ D \\ \hline \begin{array}{c}A\\B\\C\\D\end{array} & \begin{bmatrix}0&1&2&0\\1&0&1&0\\2&1&1&0\\0&0&0&0\end{bmatrix}\end{array}$$

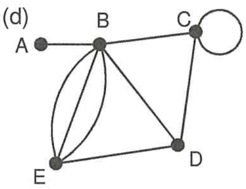
(f)
$$\begin{array}{c|c} & A\ B\ C\ D\ E \\ \hline \begin{array}{c}A\\B\\C\\D\\E\end{array} & \begin{bmatrix}0&1&0&0&0\\1&0&1&1&0\\0&1&0&2&0\\0&1&2&0&1\\0&0&0&1&1\end{bmatrix}\end{array}$$

6.

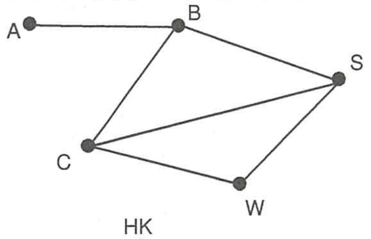
(a)


(b)


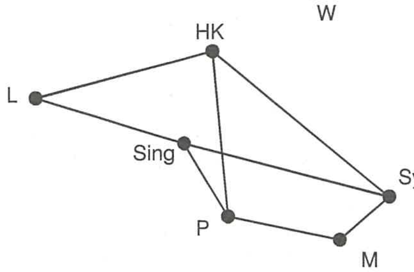
(c)


(d)


7. (a) V = {A, B, C, S, W} (b) E = {AB, BC, BS, CS, CW, WS}
(c)



8. (a)



(b) Melbourne, Sydney, Hong Kong, London
Melbourne, Sydney, Singapore, London
Melbourne, Perth, Hong Kong, London
Melbourne, Perth, Singapore, London

9. (a) Graph 1 Vertices: A, B, C, D
Edges: AB, AC, AD, BC, BD, CD
Graph 2 Vertices: A, B, C, D
Edges: AB, AC, AD, BC, BD, CD

(b) Both diagrams represent the same graph. Edges in graph 1 (planar graph) do not intersect each other whereas the edges AC and BD intersect in graph 2.

(c)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(d) (d) All the elements of the leading diagonal are zero and the matrix is symmetrical about the leading diagonal.

(e) Elements in the leading diagonal are zero as each vertex does not have an edge that connects to itself (i.e. no loops). All other elements are 1 which tells us that each vertex has one edge between it and all of the other vertices and there is symmetry about the leading diagonal.

EXERCISE 7C

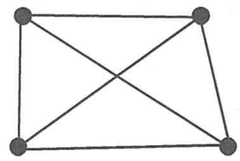
1. (a) Connected graph: Every vertex in the graph is connected to every other vertex by one or more edges.
(b) Simple graph: A graph with no loops, no multiple edges.
(c) Directed graph: A graph with directed edges which indicate the direction of travel or flow.
(d) Complete graph: A simple graph in which every vertex is connected to every other vertex by an edge.
(e) Disconnected graph: A graph with at least one pair of vertices with no path between them.
(f) Regular graph: Regular graphs have all vertices with the same degree.
(g) Subgraph: A graph which is part of another graph.
(h) Weighted graph: A graph in which each edge is labelled with a number used to represent some quantity associated with the edge.

2. (a) Connected, simple graph (b) Connected, not a simple graph as it has multiple edges. (c) Disconnected, simple graph (d) Disconnected, not a simple graph because it has a loop (e) Disconnected simple graph (f) Disconnected, not a simple graph as it has multiple edges.

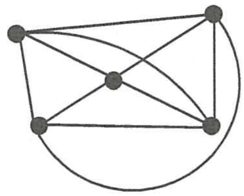
3. (a) Connected (b) Simple, connected, complete and regular (c) Simple directed and connected (d) Simple, connected and regular (e) Connected and regular (f) Simple, not connected and regular.

4. (a) Complete (b) Not complete as all vertices are not connected (c) Not complete as all vertices are not connected (d) Not a simple graph, multiple edges, hence cannot be complete (e) Complete (f) Not a simple graph, multiple edges, hence not complete.

5.
(b)

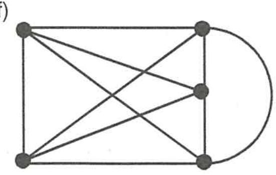


(c)



(d)

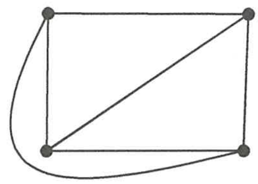
(f)



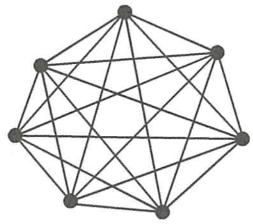
6. Degree of each vertex: 0, 1, 2, 3, 4, 5, $(n-1)$, 19

Number of edges: 0, 1, 3, 6, 10, 15, $\frac{n(n-1)}{2}$, 190

7. (a) simple graph, connected graph, regular graph
(b)



(c)



8. (a) 10 edges (b) 28 edges (c) 1485 edges

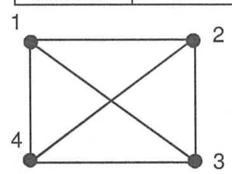
9. (a) 9 vertices (b) 25 vertices (c) 101 vertices

10. (a) Not a subgraph, it contains edge DF which is not in the original graph. (b) Not a subgraph, vertices F and G have different positions. (c) Not a subgraph, it contains edge BF which is not in the original graph. (d) Is a subgraph (e) Not a subgraph, as it contains edge BE which is not in the original graph.

11. (a)

	Round 1	Round 2	Round 3
Game 1	1 - 2	1 -3	1 - 4
Game 2	3 - 4	2 - 4	2 - 3

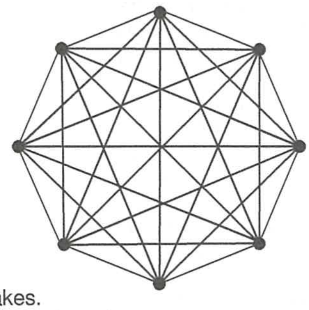
(b)



(c) Complete graph.
(d) The number of players.
(e) The number of games
(f) With 4 players each player must play the other 3 players, hence 3 rounds need to be played.
(g) If n represents the number of players, then

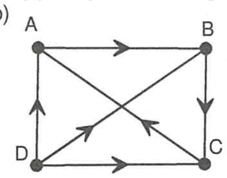
Number of games $= \frac{n(n-1)}{2}$

12. (a) Complete graph K_8. (b) 7 games (c) 28 games
(d)

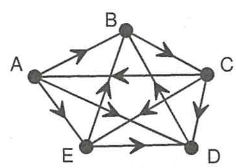


13. 190 handshakes.

14. (a) Simple directed graph
(b)



(c)

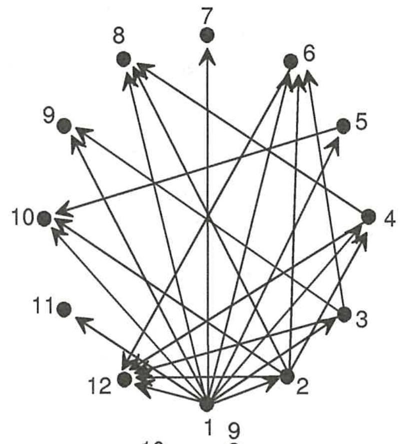


15. Each round consists of a maximum of 9 games making the total number of games played 207 (23x9) as there are 23 rounds that are played.
Using graph theory the total number of home games can be

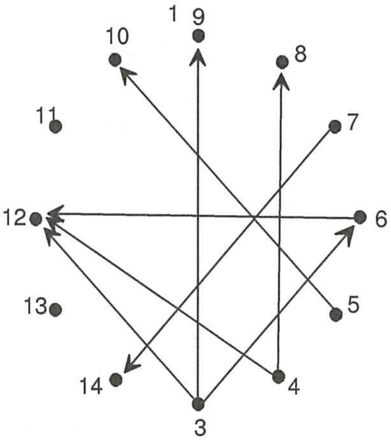
determined using the complete graph K_{18}, where each team is represented by a vertex and each edge represents the home game between the two teams.

Using the formula: No. of edges = $\frac{n(n-1)}{2}$ we find that 153 home games need to be played. With the same number of away games that need to be played, the total number of home-and-away games is 2 x 153 = 306. The total actually played is 207, which means that 99 games are not played to complete the home-and away games. Conclusion: the competition cannot be said to be fair as all teams do not play each other home-and-away.

16. (a)



(b)



17. (a) Simple directed graph (b) B and C (c) A, B and C (d) 2, A-C and A-B-C (e) 3, D-C, D-A-C, D-A-B-C

18. (a) Digraph, we must show direction. Just because person A has been to person's B wedding that does not mean that person B has been to person's A wedding.

(b) Digraph, we must show direction. Just because A likes B that does not mean that B likes A.

(c) Undirected graph, no need to show direction. If A has played golf with B then B must have played golf with A.

(d) Undirected graph, no need to show direction. If A has been to the football with B, then B must have been to the football with A

(e) Undirected graph, no need to show direction. If A is related to B then B is related to A.

(f) Digraph, we must show direction. Just because A has the phone number of B it does not mean that B has A's phone number

(g) Undirected graph, no need to show direction. If A has worked at the same school as B then B has worked at the same school as A.

(h) Digraph, need to show direction. Just because A may be eaten by B that does not mean that B will eat A. A bird may eat a fly but a fly will not be able to eat a bird.

(i) Undirected graph, no need to show direction. If A is a friend of B then automatically B is a friend of A.

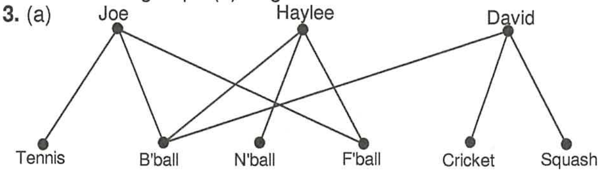
19. (a) Simple digraph or simple directed graph (b) B, C, D, F, G, I, J, K (c) E, F, I, J, K (d) a, B, H, L (e) 2 ways. G-J-K and G-F-I-J-K

EXERCISE 7D

(a) Simple, connected, regular complete graph. (b) Simple, connected graph. (c) Simple, connected, bipartite graph (d) Simple, connected, bipartite graph. (e) Connected (f) Unconnected graph. (g) Simple, connected bipartite (h) Connected, regular graph. (i) Simple, connected, bipartite.

2. (a) If the vertices are split into two groups; these being group 1 vertices A, D and G, group 2 vertices B, C, E and F, we find that all edges have a vertex in group 1 and a vertex in group 2 except for edge EF whose vertices are both in group 2. Hence the graph is not bipartite. To be bipartite all edges must have a vertex in each group. (b) Edge EF must be removed.

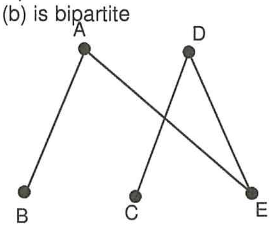
3. (a)



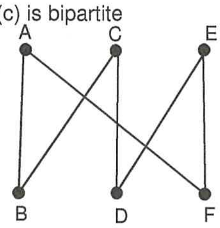
(b) Bipartite graph. The data consists of two distinct groups, people and sports played, hence is represented by a bipartite graph.

4. Graph (a) is not a bipartite graph. If we assign vertex A to group 1 then vertices B and E must belong to group 2. This leaves only vertices C and D to assign to either group 1 or group 2. If we assign vertex C to group 1 then vertex D must belong to group 2. Now vertex E is adjacent to vertex D and is also in group 2. Hence we have the situation where adjacent vertices are in the same group. Therefore the graph is not bipartite.

(b) is bipartite

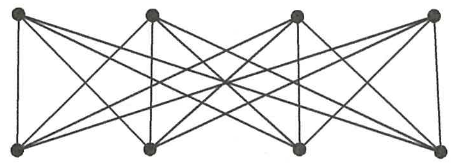


(c) is bipartite



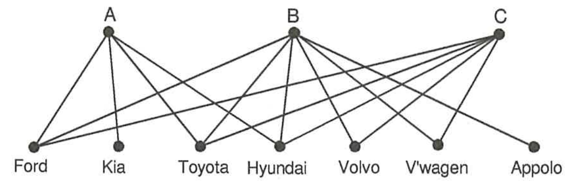
5. (a) Not bipartite. (b) Bipartite (c) Not bipartite. (d) Not bipartite. (e) Not bipartite (f) Not bipartite

6. (a)

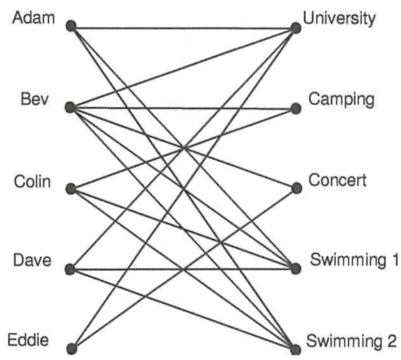


(b) A complete bipartite graph.

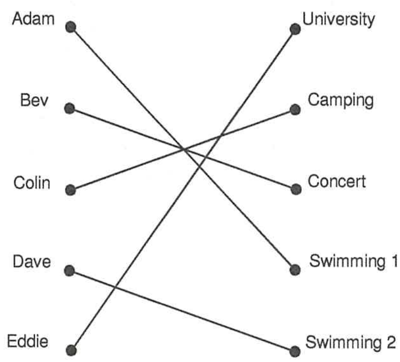
7.



8. (a)



(b)



9. Graphs 2 and 4 are bipartite. Graph 1 is not bipartite because vertices form triangles. Consider triangle BCE, if vertex B is in group 1 then vertices B and C must be in group 2 As the graph shows and edge between B and C then they are adjacent. Now vertices in the same group cannot be adjacent if the graph is bipartite, hence graph 1 is not bipartite.
Graph 3 is not bipartite because vertices form a triangle.
10. This is a classical mathematical puzzle known as the Three Utilities Problem or The Water, Gas and Electricity Problem. Research this problem on the Internet.

EXERCISE 7E

1. (a)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

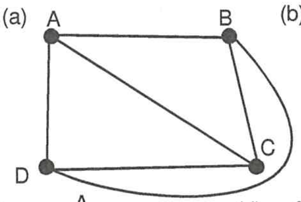
(c)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(d)
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

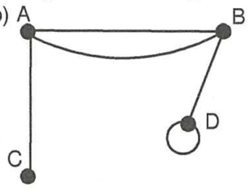
(e)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(f)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

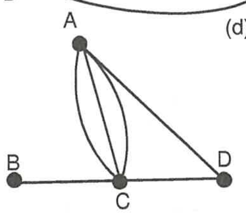
2. (a)



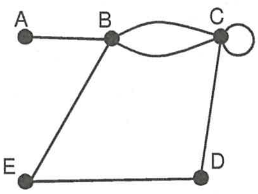
(b)



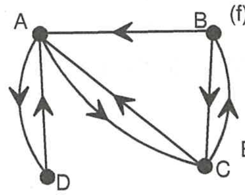
(c)



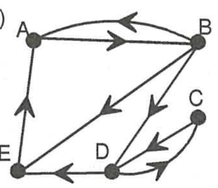
(d)



(e)



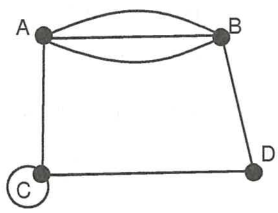
(f)



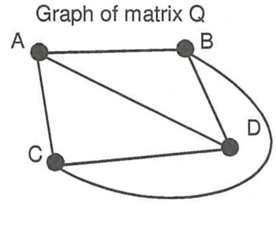
3. (a) (i) 12 (ii) 12 (iii) They are the same.
(b) (i) 10 (ii) 9 (iii) They are not the same.
(c) Answers are not the same due to the presence of a loop in graph (b) at vertex D. The loop in the matrix has a count of one as it is one edge. For the graph counting the degree of a vertex the loop has a count of two as both ends connect to the vertex.
(d) (i) In-degree sum is 7 (ii) Out-degree sum is 7 (iii) Sum is 7 (iv) Answers all the same as every edge has an in-degree and an out-degree.
(e) Adding a directed edge will increase in-degree sum, out-degree sum and sum of entries in the adjacency matrix by 1 in each case.
4. (a) Matrix T not symmetrical across the leading diagonal
(b) Matrix S two distinct vertex groups, A and B in one group, C, D and E in the other group.
(c) Matrix Q each vertex has an edge to every other vertex.
(d) Matrix P there is a entry of 1 in the leading diagonal indicating a loop at vertex C
(e) Matrix R, vertex E is not connected to any other vertex,

5.

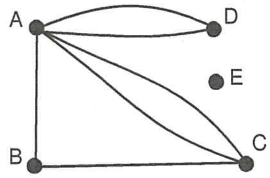
Graph of matrix P



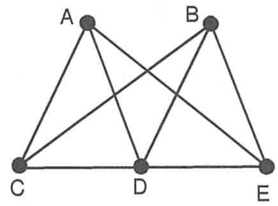
Graph of matrix Q



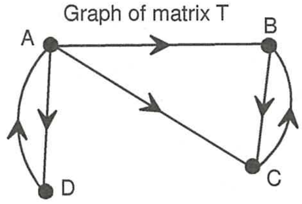
Graph of matrix R



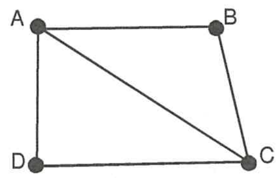
Graph of matrix S



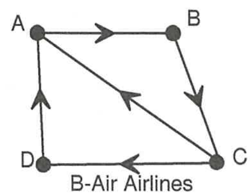
Graph of matrix T



6. (a) B-Air Airlines has 'one way only' flights as there is no symmetry across the leading diagonal.
(b)



A-Air Airlines



B-Air Airlines

7. (a) 2 routes, ABC and ADC (b) 3 routes, BAB, BCB, BDB.
(c) 3 routes, ABDC, ADBC, ADEC (d) 3 routes, ECBA, ECDA and EDBA. (e) not possible
(f)
$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(g)
$$A^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 3 & 1 & 2 & 2 & 0 \\ 2 & 1 & 3 & 2 & 1 & 1 \\ 1 & 2 & 2 & 4 & 1 & 1 \\ 1 & 2 & 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Matrix A gives the number of direct routes between the six towns.

A^2 gives the number of routes between these towns with just one stop-over.

(h) A^3 gives the number of 3 step routes between the towns or the number of routes between the towns with two stop-overs.
(i) The numerical answers to parts (c), (d) and (e) can be verified.
(j) It is not possible to travel from F to F with two stop-overs.
(k) There are 3 distinct two stop-over routes between towns B and E.
(l)
$$S = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 2 & 2 & 2 & 2 & 1 & 0 \\ 2 & 3 & 2 & 3 & 2 & 0 \\ 2 & 2 & 3 & 3 & 2 & 1 \\ 2 & 3 & 3 & 4 & 2 & 1 \\ 1 & 2 & 2 & 2 & 3 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

S tells us the number of routes between these towns with at most one stop-over.
(m) $s_{16} = 0$. There is no direct route between A and F and no one stop-over route between A and F or With at most one stop-over you cannot travel from A to F.

(n)

$$T = \begin{array}{cc} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 4 & 7 & 5 & 8 & 4 & 1 \\ 7 & 7 & 9 & 10 & 5 & 2 \\ 5 & 9 & 7 & 10 & 8 & 2 \\ 8 & 10 & 10 & 10 & 9 & 2 \\ 4 & 5 & 8 & 9 & 5 & 4 \\ 1 & 2 & 2 & 2 & 4 & 1 \end{bmatrix} \end{array}$$

T tells us the number of routes between these towns with at most two stop-overs.

(o) With at most two stop-overs travel is possible between all of these towns.

8. (a) BAD, BED (b) EBE, EDE, EFE (c) ABEF, ADEF
(d) ABED, ABAD, ADAD, ADED

(e)

$$M = \begin{array}{cc} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(f)

$$M^3 = \begin{array}{cc} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 1 & 5 & 0 & 4 & 1 & 2 \\ 5 & 5 & 0 & 2 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 5 & 0 \\ 1 & 6 & 0 & 5 & 1 & 3 \\ 2 & 1 & 0 & 0 & 3 & 0 \end{bmatrix} \end{array}$$

(g) There is 1 three-step sequence of vertices from E to E. The sequence is EBBE.
(h) 6 sequences, BADE, BABE, BBBE, BEBE, BEFE, BEBE
(i) Disconnected graph. (j) Both have only zero's because vertex C is an isolated vertex.

9. (a) Construct, A, the adjacency matrix. Determine A^3, the entry a^3_{24} will give you the required number.
(b) BDBD, BCBD and BDED

10. (a)

$$Q = \begin{array}{cc} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

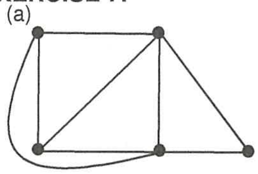
(b) Reject claim.

$$Q+Q^2 = \begin{array}{cc} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 2 & 2 & 1 & 2 & 2 & 0 \\ 2 & 3 & 2 & 3 & 2 & 1 \\ 1 & 2 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 4 & 3 & 1 \\ 2 & 2 & 2 & 3 & 4 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

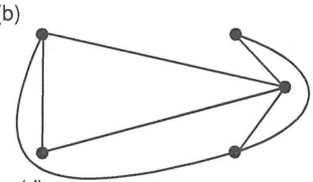
Matrix $Q + Q^2$ shows the number of flights with at most one stop-over. The zero's in this matrix indicate that it is not possible to fly from A to F or from F to A with at most one stop-over. Claim is false.

EXERCISE 7F

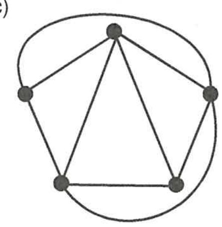
1. (a)



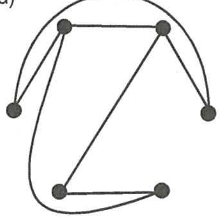
(b)



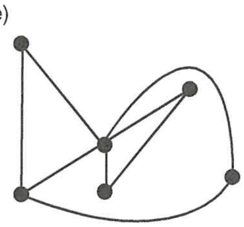
(c)



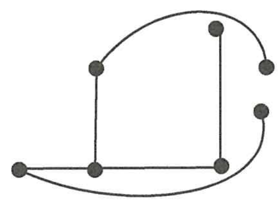
(d)



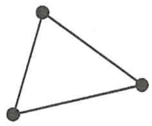
(e)



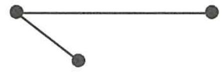
(f)



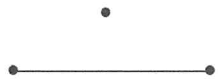
2. (a)



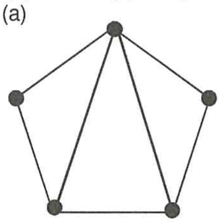
(b)



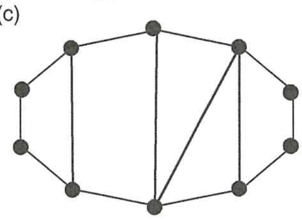
(c) A graph with 3 vertices and 4 edges will have a two edges between two of the vertices and hence not simple. Hence not possible
(d)



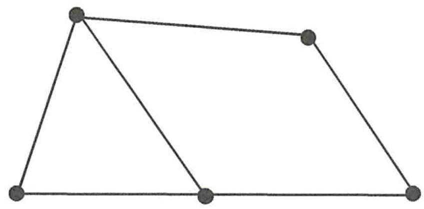
3. (a) Simple planar (b) Not simple planar, not a simple graph, multiple edges between vertices A and C (c) Simple planar (d) Not simple planar, not a simple graph, multiple edges between vertices A and C.
4. (a) Planar (b) Not planar (c) Planar (d) Planar
5. (a)



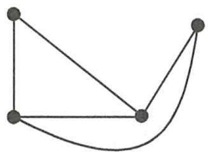
(c)



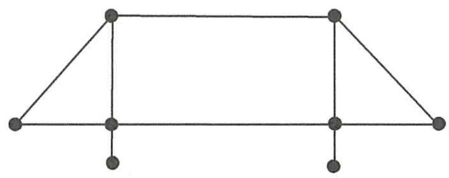
(d)



6. 8 edges **7.** 6 faces **8.** 8 vertices
9. (a) 5 edges (b) 3 faces
(c)

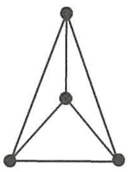


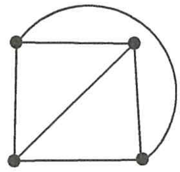
10. (a) 10 edges (b) 4 faces
(c)

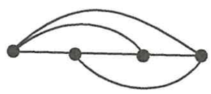


CHAPTER SEVEN REVIEW EXERCISE

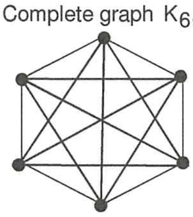
1. 4 vertices. Three





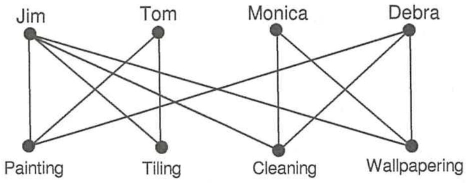


2. (a) 15 matches (b) Complete graph K_6.



3. (a) Yes. Graph B is a part of graph A with no new edges or vertices (b) Yes. Every graph is a subgraph of itself. (c) No. Graph C has no 'triangles'. (d) No. Graph C has no 'triangles'
4. (a) and (c) are connected. (b) is not connected as there are no edges from either E or F to vertices A, B, C or D (d) is not connected because there is no edge from F or G or H to E.

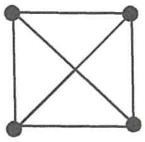
5. (a)



(b) A bipartite graph. There are two distinct groups in this situation, people and tasks. Hence the vertices of the graph need to be split into two groups, the only appropriate graph is bipartite.

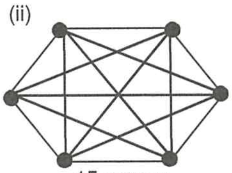
6. (a) A complete graph where every vertex is connected to every other vertex. Vertices represent the players and the edges the games that need to be played.

(b) (i)



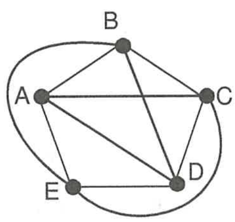
6 games

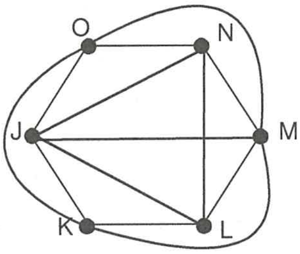
(ii)



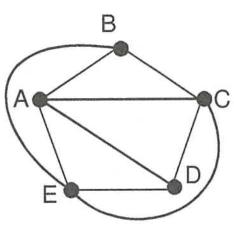
15 games

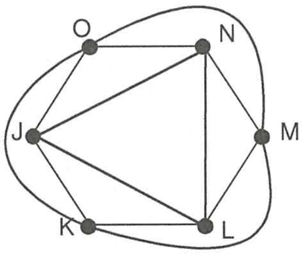
7. (a) Remove BD or AC (b) Remove JM or NL



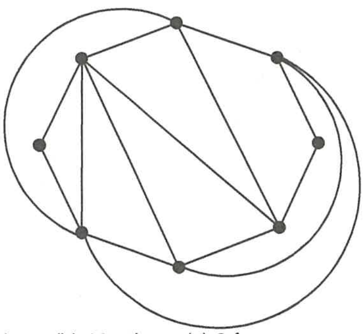


(a) After removing BD (b) After removing JM

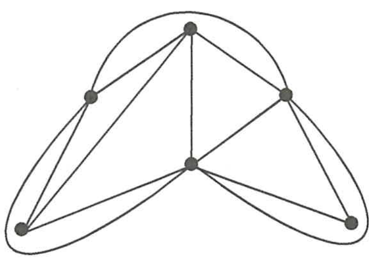




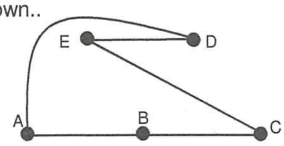
8. (a) Graph 2
(b)



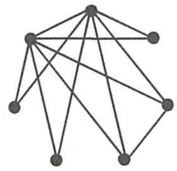
9. (a) 6 vertices (b) 12 edges (c) 8 faces
(d)



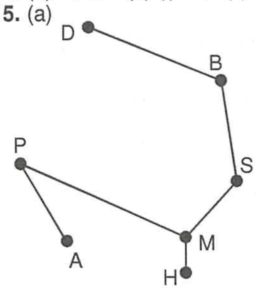
10. (a) V = {A, B, C, D, E} E = AB, AD, BC, CE, DE
(b) Yes it is planar as it can be drawn with no edges intersecting as shown..



11. 10 edges
12. 12 edges.



13. Does not apply to (c) because the graph is not planar.
14. (a) Adam (b) (i) PG (ii) PF (iii) SF
15. (a)



(b)

$$X = \begin{array}{c} \\ P \\ D \\ B \\ S \\ M \\ H \\ A \end{array} \begin{array}{c} \begin{array}{ccccccc} P & D & B & S & M & H & A \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

(c) Darwin, Perth and Hobart. (d) Adelaide and Brisbane
(e) The matrix $X + X^2 + X^3$ contains zeros indicating that there are cities which cannot be connected to each other by taking one, two or three flights. Statement is false.
(f)... at most, five flights.\

ANSWERS CHAPTER EIGHT

EXERCISE 8A

1.

Walk	Open/ closed	Trail, Path or Cycle
ABCHA	Closed	Cycle
HCDEFG	Open	Path
CHGFEHA	Open	Trail
EHCDECHE	Closed	Walk
BCEDCHAB	Closed	Trial
FGHCDEF	Closed	Cycle

2. (a) (i) Walk (ii) Not a walk (iii) Not a walk (iv) Walk (v) Walk (vi) Walk
(b) (i) Trail (iv) Trail (v)Path (vi) Cycle
3. (a) ABCDE, ABDE, ABE, AFBCDE, AFBDE, AFBE, AFE
(b) ABC, ABDC, ABEDC, ABFEDC, AFBC, AFBDC, AFBEDC, AFEDC, AFEBC, AFEBDC, AFEDBC.
4. (a) Path, no repeat of vertices or edges. (b) Trail (closed), vertices D and A have been repeated but no edge has been repeated.
(c) Cycle, starts and ends at A with no repeat of vertices or edges. (d) Walk (closed), sequence of vertices has repeated vertices and edges. (e) length 7 (f) Sequence of vertices has repeated vertices and edges. (g) length 6 (h) Sequence of vertices has no repeated vertices and no repeated edges.
(i) length 7 (j) Sequence of vertices has repeated vertices but not edges. (k) Length 6 (l) Sequence of vertices is called a cycle because the sequence starts and ends at the same vertex and has no repeat of vertices (except A, start and end vertex) and no repeat of edges. (m) Length 5
(n) Sequence is called a closed path because it starts at ends at the same vertex and has no repeat vertices or repeat edges.
(o) Cycle (p) Two bridges. (q) Edges GH and HI.
5. (a) walk, trail (b) walk, closed walk (c) walk, closed walk, trail, closed trail, path, cycle
6. 9 paths; ADEF, ADEGF, ADEHGF, ABDEF, ABDEGF, ABDEHGF, ABCDEF, ABCDEGF, ABCDEHGF

EXERCISE 8B

1. (a) Eulerian (b) Not Eulerian, has odd vertices.
(c) Not Eulerian, has odd vertices. (d) Eulerian
(e) Not Eulerian, has odd vertices. (f) Eulerian
(g) Not Eulerian, has odd vertices. (h) Eulerian
(i) Eulerian

2. (a) Semi-Eulerian (b) Neither, graph has more than two odd vertices (c) Semi-Eulerian (d) Eulerian (e) Semi-Eulerian (f) Eulerian (g) Neither, graph has more than two odd vertices. (h) Eulerian (i) Semi-Eulerian

3. (a) (c) (d) (e) (f) (h) (i)

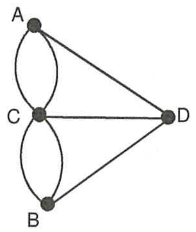
4. (a) Eulerian (b) Semi-Eulerian (c) Eulerian (d) Semi-Eulerian (e) Insufficient information (f) Neither
(g) Insufficient information (h) Insufficient information
(i) Insufficient information (j) Semi-Eulerian

5. (a) The map has two odd vertices so it is impossible to travel all roads starting and finishing at G. (b) Finish at J. (c) Yes, the map will have all even vertices making is Eulerian graph and hence a Eulerian trail is possible.
(d) GDABCFBDEFTHJKIJGHEG

6. (a) The route map has only two odd vertices and hence is a semi-Eulerian graph. He should start at one of the odd vertices and he will finish at the other odd vertex. That is start a B finish at D or start at D and finish at B.
(b) Add a direct flight between B and D. With this added flight all map vertices are of even degree and hence we have a Eulerian graph. Hence a trainee pilot can start his flight from any city travel all routes once only and return to his starting point.

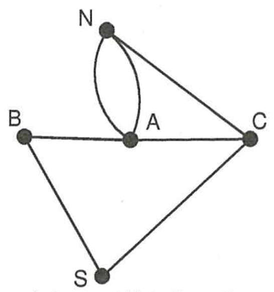
7. (a) Graph 1 and graph 3. Both graphs have exactly two odd vertices.
(b) Graph 1 ABCEADC Graph 2 BACBFDBEAFEDCE

8. (a)



(b) Not possible as the map is not traversable as it has more than 2 odd vertices.
(c) Adding an extra bridge would make the graph semi-Eulerian and the problem will be solved. As all vertices of the graph are odd it does not matter where the extra bridge is built.
(d) Removing a bridge would make the graph semi-Eulerian and the problem would be solved. As all of the vertices of the graph are odd it does not matter which bridge is removed.

9. (a) Not possible as the graph has two odd vertices and there is no Eulerian trail.
(b)



(c) Yes, the graph is semi-Eulerian. Start at either N or C and finish at C or N.

10. (a) All are connected graphs. (b) All are planar (c) Graph 2 (d) Graph 3 (e) Graphs 1, 2 and 4 (f) All graphs have an Eulerian trail (g) Graph 4 (h) None, they all have at least one cycle.

11. Graph 1: Has two odd vertices so there is a semi-Eulerian trail.
Graph 2: Graph has more than two odd vertices, not Eulerian or semi-Eulerian.
Graph 3: Graph has all vertices of even degree hence it has an Eulerian trail and a Eulerian circuit.
Graph 4: K_{10} is a complete graph with 10 vertices and 9 edges at each vertex. As the graph has all odd vertices it is not Eulerian or semi-Eulerian.

EXERCISE 8C

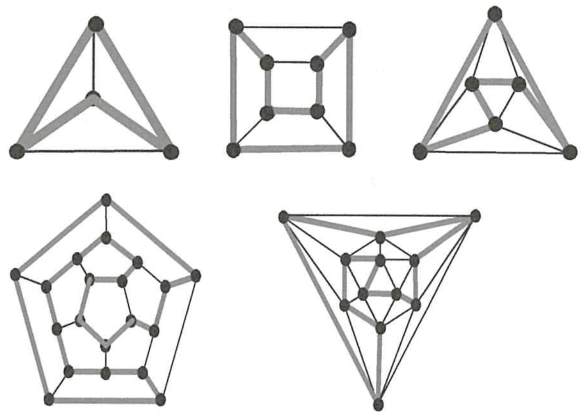
1. A Hamiltonian cycle is a path in a graph that passes through every vertex of the graph exactly once, starting and finishing at the same vertex.

2. (a) Hamiltonian path (b) Neither (c) Neither (d) Hamiltonian path (e) Hamiltonian path (f) Hamiltonian cycle (g) Neither (h) Neither

3. (a) ABCDEFA (b) FBDCEAF (c) ABCDGEFA
(d) ABCFEDA (e) BADCHEFGB (f) ABCDEHFGA
(g) AEFGHBCDA (h) ACEBGHIFDA

4. (a) No Hamiltonian cycle. Once we travel to vertex F from Vertex E there is no way of leaving F without returning to E so there is no possibility of a Hamiltonian cycle.
(b) Yes, one is ABDCEF, there are others.

5.



6. The octahedron, it is the only platonic solid whose graph has even vertices. The graphs of all of the other platonic solids have more than two odd vertices hence not Eulerian.

7. (a) n must be a counting number (b) n must be an odd

8. (a) No, town D will be visited twice. (b) The graph has a Hamiltonian path but not a Hamiltonian cycle. (c) Graph is semi-Eulerian, it has two odd vertices.

9. (a) Undirected, weighted simple graph. (b) 169 km, AEDCBFA (or AFBCDEA) (c) Directed, weighted simple graph. (d) AEDBCDFA, 162 km (e) Shortest circuit is not Hamiltonian because the circuit pass through town D twice.

10. (a) At either vertex C or G (b) 2900 metres
(c) ABCEDFGA or AGFDECBA

11. (a) m = n (b) m and n must be even numbers

12. (a) 926 km (b) 6 ways. Istanbul-Varna-Yalta;
Istanbul-Varna-Samsun-Sochi-Yalta;
Istanbul-Varna-Samsun-Poti-Sochi-Yalta;
Istanbul-Samsun-Varna-Yalta;
Istanbul-Samsun-Sochi-Yalta;
Istanbul-Samsun-Poti-Sochi-Yalta;
(c) Start at Varna and finish at Sochi or start at Sochi and finish at Varna. (d) 1333 km shorter.

13. (a) Start at A finish at D or start at D finish at A.
(b) ABCDEACEBD (c) 265 km (d) 95 km (ABCDE)
(e) Road connecting A and E. The network will be a Eulerian as all vertices will be even.

EXERCISE 8D

1. (i) 53 units (ii) ABCZ
2. (i) 80 units (ii) ACBZ
3. (i) 42 units (ii) ADCZ
4. (i) 57 units (ii) ABCDZ
5. (i) 60 units (ii) ACDFGZ
6. (i) 40 units (ii) ACDGFZ
7. (i) 58 units (ii) ABCDFHZ
8. (i) 39 units (ii) AEFZ
9. (i) 27 units (ii) ACDEIZ
10. (i) 205 units (ii) ACDFHIZ

11. (a) AGEF (b) 140metres (c) New route AGECDF 130 metres (d) Routes ABDF, ABCDF, ABCF,AGHF and AGF (e) 150 metres, routes ABDF and AGF.

12. (a) 135m; Admin – Engineering – Mathematics – Physics - Music
(b) 181m; Admin – Engineering – Mathematics – Physics – Languages - Music
(c) 161m; Admin – Law – Library – History - Music

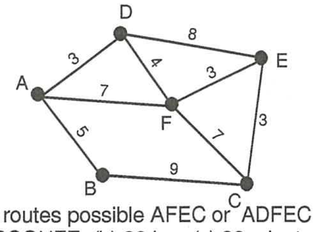
13. (a) AGCDH 412 km (b) AFEDH 494 km

14. (a) BACDGF (b) 45 km (c) New shortest path BACEGF of length 46 km.

15. 118 km (b) AEFIGHD (c) Shortest distance affected, new route is ABCD and this route is 7 km longer i.e. new shortest distance is 125 km.

16. (a) Routes ABC; AHGFEDC; AHGFEC (b) KBC; KC; KEC; KEDC (c) 19 routes (d) AHKBC 19 mins (e) AHGFKEDC 40 mins

17. (a)



(b) Two routes possible AFEC or ADFEC both 13 minutes

18. (a) BCGHFE (b) 39 km (c) 39 minutes (d) BCGHIDE 41 km

19. (a) 35 mins route is AFHJCD (b) AFHED 46 mins (c) No as GH does not lie on the shortest route. (d) Yes, because FH is in the shortest route (e) ABGHJCD (f) 37 mins

20. (a) Bunbury – Collie – Darkan – Arthur River - Wagin – Katanning Distance is 234 km.

(b) Round trip required: Bunbury – Collie – Darkan – Arthur River – Wagin – Katanning – Kojonup – Boyup Brook – Mumballup – Donnybrook – Boyanup – Bunbury.(or in reverse order)

(c) 234 + 248 = 482 km.

CHAPTER EIGHT REVIEW EXERCISE

1. (a) C (b) 3, AD, ABD, ABED (c) 4, CD, CAD, CABD, CABED (d) They only have incoming edges or outgoing edges and not both.

2. (a) Traversable (i), (iii), (iv), (v), (vi), (vii) Not traversable (ii) and (viii)

(b) Eulerian: (vi) and (v) semi-Eulerian: (i), (iii), (vi), (vii) neither: (ii) and (viii)

(c) (vii)

3. Additional edge is BD. This will result in a graph with all even vertices hence Eulerian.

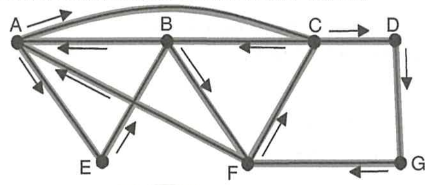
4. (a) Graph 1 is semi-Eulerian, has two odd vertices. Graph 2 is neither, has more than two odd vertices.

(b) Graph 1 semi-Hamiltonian Graph 2 Neither

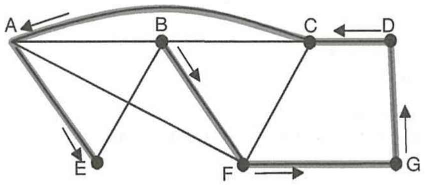
5. (a) Eulerian because all vertices are even.

(b) The graph has a cycle which includes each vertex once only and the cycle starts and finishes at the same vertex.

(c)



(d)



6. (a) Hamiltonian path (b) Semi-Eulerian trail (c) Hamiltonian cycle (d) Hamiltonian path.

7. (a) Complete bipartite graph of order $K_{3,3}$

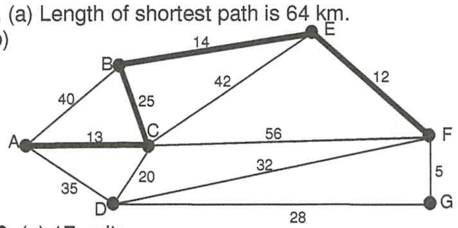
(b) Complete bipartite graph of order $K_{3,4}$

(c) Complete bipartite graphs of order $K_{m,n}$ have a Hamiltonian cycle if m = n. Hence graph 1 has a Hamiltonian cycle and graph 2 does not.

8. (a) 4 odd vertices hence neither (b) all vertices are even hence has both (c) Has 2 odd vertices has Eulerian trail but not Eulerian circuit (d) Each vertex has degree (n-1) which is odd hence neither.

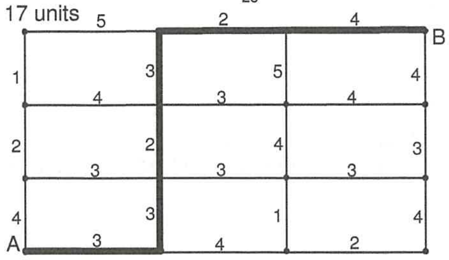
9. (a) Length of shortest path is 64 km.

(b)



10. (a) 17 units

(b)

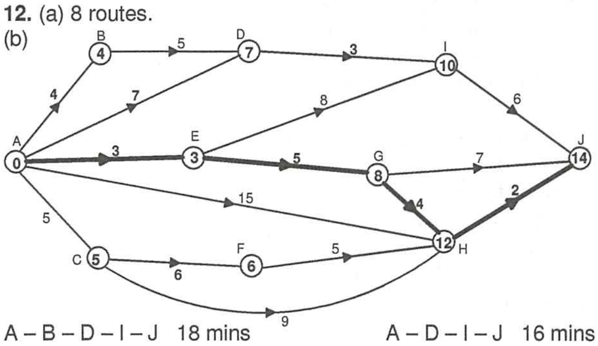


(c) 18 units

11. (a) 13 + 4 + 4 + 3 + 5 + 4 + 12 + 12 + 4 + 14 = 75 km.
or 14 + 4 + 12 + 12 + 4 + 5 + 3 + 4 + 4 + 13 = 75 km.
(b) 13 + 4 +4 + 3 + 3 + 13 + 14 +4 + 12 + 12 + 4 + 14=100km
or 14 + 4 + 12 + 12 + 4 + 14 + 13 + 3 + 3 + 4 + 4 + 13 = 100km

12. (a) 8 routes.

(b)



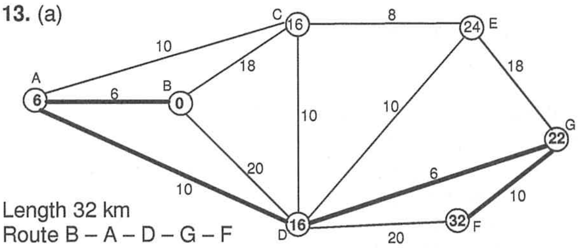
A – B – D – I – J 18 mins	A – D – I – J 16 mins
A – E – I – J 17 mins	A – E – G – J 15 mins
A – E – G – H – J 14 mins	A – F – J 17 mins
A – C – F – H – J 18 mins	A – C – H – J 16 mins

Minimum time 14 minutes, ✓ route A – E – G – H – J

(c) Time taken not affected as H is on the shortest path. ✓

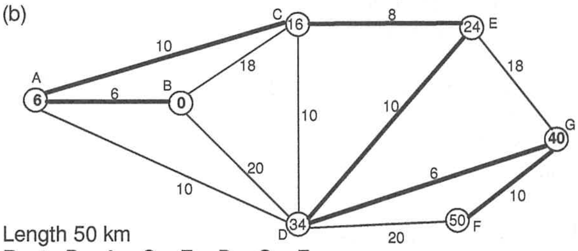
(d) Suburb I is not on the shortest path. New route A – D – I – J which takes 16 minutes.

13. (a)



Length 32 km
Route B – A – D – G – F

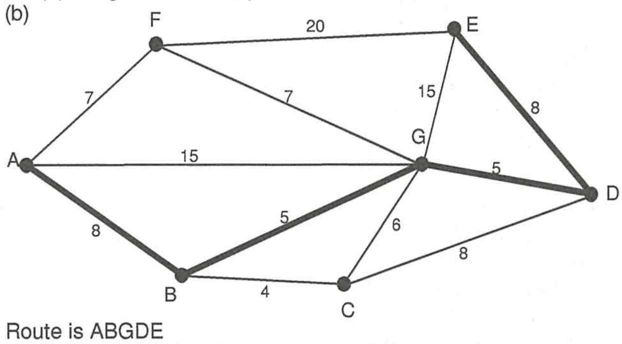
(b)



Length 50 km
Route B – A – C – E – D – G – F

14. (a) Length of shortest path is 26 metres.

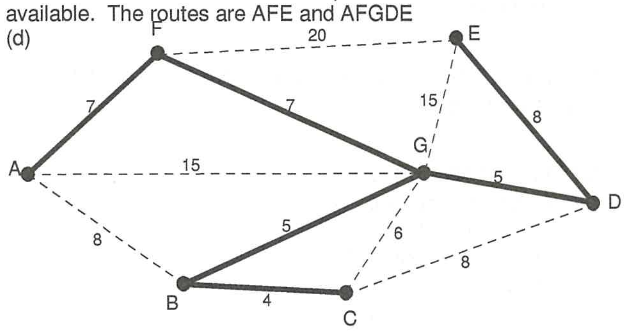
(b)



Route is ABGDE

(c) The length of the shortest path would increase by 1 metre to 27 metres and would not be unique as two routes will be available. The routes are AFE and AFGDE

(d)



(e) Length of the connection is 36 metres.

INDEX