**2.** **[9 marks]** *(APP SAT:CF04)*

A wedding photographer is quoting the following price for producing a wedding album for the newlyweds:

A fixed minimum cost of $150, with 80 photos in a hard-backed album. Further photos may also be added in lots of 10 photos at $0.70 per photo, up to a maximum of 200 photos.

He wants to set up a table below, showing:

- the type of album where $T_1$ is the basic album, with 80 photos at a cost of $150
- the number of photos in each of the possible album sizes
- the cost in dollars of each of the different albums.

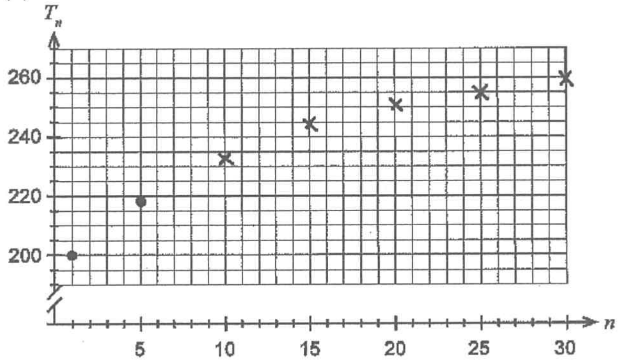
(a) Complete each of the blank cells of the table. [3]

| Type | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | | $T_n$ |
|---|---|---|---|---|---|---|---|
| Number of pictures | 80 | | | | | | 200 |
| $ cost of album | $150 | | | | | | |

(b) Write a rule that will calculate the number of pictures in album type = $T_n$. [3]

(c) Write a rule that will calculate the cost of album type = $c_n$. [3]

7. (APP 2017:CA16)
(a) (i) Decrease of 7.5% each year (ii) 20
(b)




(c) The rate of increase is slowing over time. It appears to be approaching a steady state.
(d) Stable population of 267 crocodiles

8. (APP 2018:CA9)
(a) 10%
(b) $M_1 = 50$, $M_5 = 47.249$, $\Rightarrow$ 48 mealworms
(c) Her statement is true. The long term steady state solution for this recursive formula is 42.
(d) (i) $c = 45$ (ii) 27 (steady state solution)
(e) $30 = 0.8(30 - 10) + k$
$k = 14$

9. (APP 2019:CF6)
(a) $T_{n+1} = 0.5T_n + 20$; $T_1 = 32$
(b) $T_2 = 36, T_3 = 38$, $T_4 = 39$, $T_5 = 39.5$
Turtle population approaches a steady state of 40
OR
$x = 0.5x + 20 \Rightarrow x = 40$
$0.5x = 20$
$x = 40$
Turtle population approaches a steady state of 40
(c) $80 = 0.5 \times 80 + k$
$80 = 40 + k$
$k = 40$
40 turtles are required each year.

10. (APP 2019:CA7)
(a) $a = 84$, $d = -6$
$T_n = 84 + (n - 1)(-6)$
$T_n = 90 - 6n$
(b) $T_7 = 90 - 6(7) \Rightarrow T_7 = 48$
48 L
(c) $S_8 = 84 + 78 + 72 + 66 + 60 + 54 + 48 + 72$
$S_8 = 504$
504 L
(d) $T_{15} = 0$, $S_{15} = 630$
Capacity of the tank is 630 L

## Chapter 5: The Arithmetic Sequence

1. (2CDMAT 2010S:CA16)
(a) $T_{n+1} = T_n + 4$, $T_1 = 18$
(b) Linear
(c) $n = 10 \Rightarrow T_{10} = 54$
(d) $T_{17} = 82 \Rightarrow$ Friday of week 3

2. (2CDMAT 2012:CF01f)
$T_1 = T_2 + 4 = T_3 + 4 + 4 = 23$

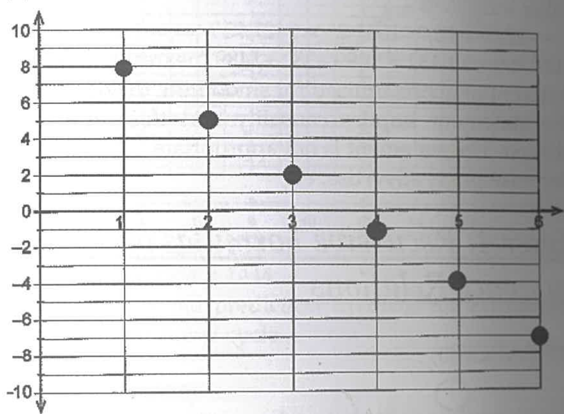
3. (FM2 2013:M103)
(a) (i) 150 (ii) Arithmetic
(b) $a = 100$, $b = 50$

4. (2CDMAT 2014:CA17)
(a) 20 decreases totalling 80 mm. Hence, each rung decreases by 4 mm.
(b) $T_{n+1} = T_n - 4$, $T_1 = 400$
(c) $L = 400 - 4(n - 1)$

5. (APP 2016S:CF01)
(a) (i) $T_4 = 43 + 12 = 55$ (ii) $T_1 = 43 - 12 - 12 = 19$
(b) (i) $u_5 = 2u_4 - u_3 = 58 - 22 = 36$ (ii) $u_n = u_{n-1} + 7$, $u_1 = 8$

6. (FM2 2015:M101)
(a) 2150 – 200 = 150
(b) 2600 kg
(c) 20 200 kg
(d) Week 6
(e) $C_n + 150$

7. (APP 2017:CF1)
(a)




(b) (i) $T_n = 8 - 3(n - 1) = -3n + 11$
(ii) $-500 = 8 - 3(n - 1)$
$n = 170.\dot{3} \Rightarrow$ 171st term

## Chapter 6: The Geometric Sequence

1. (FM2 2013:M102)
(a) $8 \times 1.2^{(5-1)} = 16.6$
(b) 9th Year ($T_8 = 28.7$, $T_9 = 34.4$, $T_{10} = 41.3$)
(c) $S_{10} \approx 208$

2. (2ABMAT 2013:CA19)
(a)

| | | |
|---|---|---|
| 1980 | 1981 | 1982 |
| 2 | 3 | 4 |
| 11 342 | 12 079 | 12 684 |

(b) $n = 36$
$P = 96\ 513$ (accept 96 514, 96 510, 96 500)
(c) $n = 12$, $P = 21\ 291$
Year = 1978 + 12 = 1990

3. (2ABMAT 2014:CA16d)
(i)

| | |
|---|---|
| **2017** | **2018** |
| 4 | 5 |
| **3062.61** | **3276.99** |

(ii) Start with 2500 and multiply each term by 1.07 to get the next term.
(iii) $n = 12$, $Value = \$5262.13$

4. (APP 2016S:CA09)
(a) $T_{n+1} = \frac{2}{3}T_n$, $T_1 = 60$
(b) $T_4 = 17.78$ cm
(c) Sum of the first seven downward movements (169.47)
Plus the sum of the last six upward movements (169.74 – 60 = 109.47)
Total distance travelled is 278.94 cm (278.93 cm if using full capacity)

5. (APP 2016:CA7)
(a) \$11 050, \$9393, \$7984
(b) 15% per annum
(c) $T_n = 13\,000(0.85)^n$
(d) $T_8 = 13\,000(0.85)^8 = \$3542$
(e) $6500 = 13\,000(0.85)^n$
$n = 4.265 \Rightarrow$ *end of the 5th year*

6. (APP 2018:CF7)
(a) $\frac{54}{36} = \frac{2}{3}$

$T_n = \frac{2}{3}T_{n-1};\ T_0 = 54$

(b) $T_n = 54\left(\frac{2}{3}\right)^n$

(c) $T_5 = 54\left(\frac{2}{3}\right)^5$

$T_5 = 2(3^3) \times \frac{32}{3^5}$

$T_5 = 2 \times \frac{32}{9}$

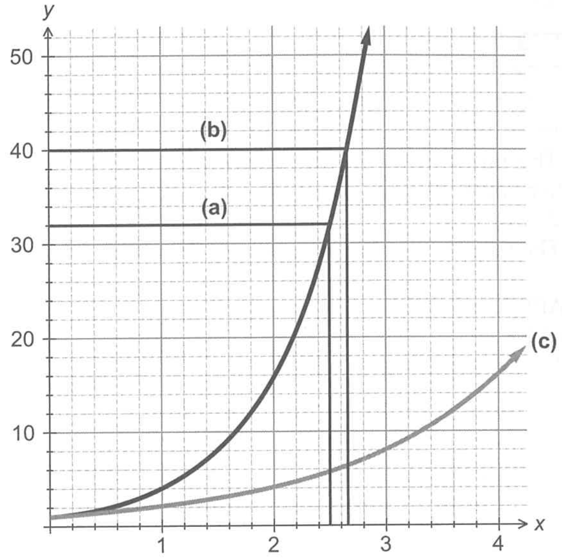
$T_5 = \frac{64}{9}$

7. (APP 2019:CA10)
(a) $r = \frac{30\,256}{22\,579} \approx 1.34$
(b) $a = 15$, $r = 1.34$
$T_n = 15(1.34)^{n-1}$
OR
$T_n = 11.19(1.34)^n$
(c) After 33 years $(T_{34}) = 234\,719$
$\therefore$ In 2025 there will be over 200 000 shops.
(d) $T_{20} = 3900$
Daily wages = 3900 × 12 × 114.80
Daily wages = \$5 372 640

## Chapter 7: Exponential Equations

1. (2ABMAT 2011:CF06)
(a) 32 (refer to axes below)
(b) $x = 2.66$ (refer to axes below)

(c)

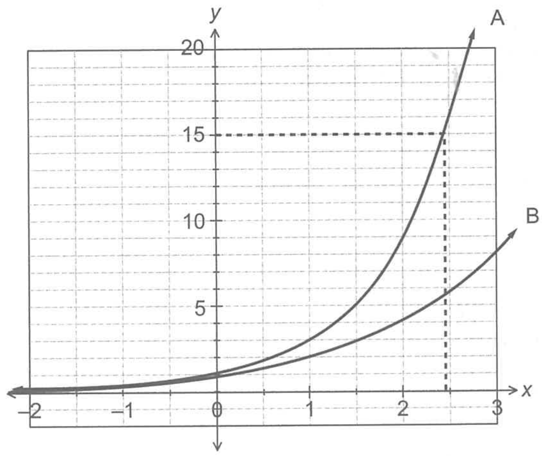



2. (2CDMAT 2012:CF02c,d)
(a) (2, $q$) belong to $y = 3^x$
$q = 3^2 = 9$
(b) $2^6 = 64 \quad 2^7 = 128 \therefore x = 6$

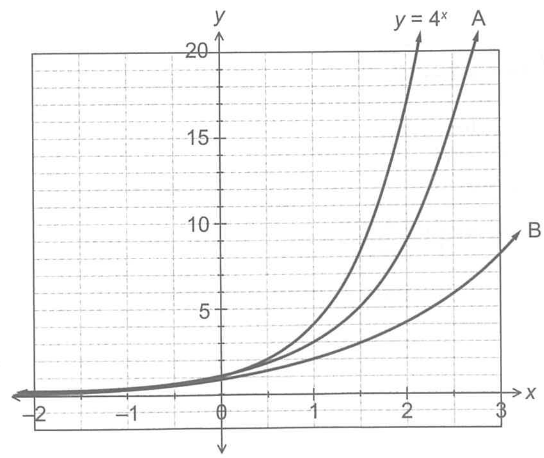
3. (2ABMAT 2014:CF04)
(a)

| Equation | Graph |
|---|---|
| $y = 2^x$ | B |
| $y = 3^x$ | A |

(b) $x \approx 2.5$




(c)

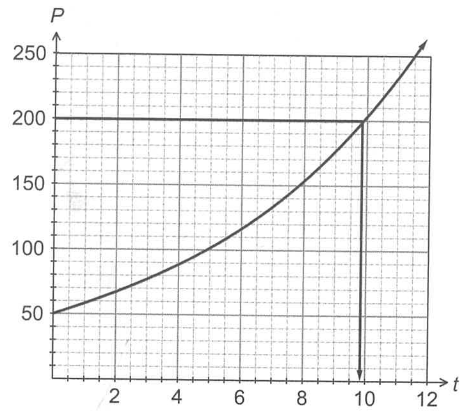


4. (2ABMAT 2014:CA19)
(a)

| | | |
|---|---|---|
| **4** | $2^0 + 2^1 + 2^2 + 2^3$ | **15** |
| **5** | $2^0 + 2^1 + 2^2 + 2^3 + 2^4$ | **31** |
| **6** | $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$ | **63** |

(b) The numbers in the last column increase by the next higher power of 2, i.e. +2, +4, +8, +16, …
(c) $2^n - 1$
(d) The value of $n$ is 10, so the sum is $2^{10} - 1 = 1023$

5. (APP 2016S:CF04)
(a) (i) 50
(ii) First exceeds in the 10th month




(b)

| | | |
|---|---|---|
| 3 | $50 \times 1.15 \times 1.15 \times 1.15$ | 76 |
| 4 | $50 \times 1.15 \times 1.15 \times 1.15 \times 1.15$ | 87 |

c) 15% per month
d) All other aspects being fair (size of lake, food supply) he population will increase exponentially.
The rule has a positive ratio ($r$ value) **or** the graph has an ncreasingly positive slope.

. (APP 2017:CA10)
a) $P = 400(1.35)^t$ and $P = 540(1.35)^{t-1}$
b) 984
c) $400(1.35)^t = 1800$
$t = 5.012 \Rightarrow$ *6th week*
d) $400(1.35)^8 = 4413$
$413(1.2)^7 = 15\,812$
. 15 weeks
e) $400(r)^{15} = 15\,812$
$= 1.2778 \Rightarrow 27.78\%$
f) $15\,812 - 250d = 0$
$= 63.248 \Rightarrow 9$ weeks

## Chapter 8: Finance

. (APP 2016S:CA16)
a) $A = \$20\,000$
b) $r = 8.5\%$
c) $V = 20\,000\left(1 + \frac{8.5}{400}\right)^4 = \$21\,754.96$
d) $i_e = (1 + 8.5 \div 100 \div 12)^{12} - 1 = 0.08839$
$i_e = 8.84\%$

(APP 2016S:CA17)
) 8% per annum
) $T_7 = (1.08)^7\, T_0 = 1.713824$
$T_7 = \$3599.03$ (Accept \$3599)

(c) $r^3 \times 2100 = 3599.03 \Rightarrow r = 1.1967$
Interest Rate = 19.67% per annum

3. (APP 2016:CA12)
(a) 1.5%
(b) $A = \$219.91$  $B = \$14\,380.32$
(c) 44 months
(d) $43 \times 500 + 460.89 = \$21\,960.89$
(e) $\frac{1.5}{100} \times 16\,000 = \$240$. Thomas will never repay the loan as his repayments only cover the interest, not the principle.
(f) \$798.79

4. (APP 2016:CA13)
(a) (i) \$6039.75
(ii) $5000\left(1 - \frac{0.065}{12}\right)^x = 10\,000$
$x = 128.31 \Rightarrow 129$ *months*
(b) $i_{effective(B)} = \left(1 - \frac{5.5}{365}\right)^{365} - 1$
$i_{effective(B)} = 5.55\%$ (2 d.p.)
Option A will pay more interest due to the higher effective interest rate.

5. (APP 2017:CF4)
(a) Quarterly. Lowest rate to minimise interest
(b) \$4.06
(c) $\frac{5.127}{100} \times 3000 + 3000 = 51.27 \times 3 + 3000 = \$3153.81$
$\Rightarrow \$3153.81$

6. (APP 2017:CA14)
(a) (i) $T_{n+1} = 1.0015T_n - 420$; $T_u = 14\,999$  (ii) $T_{12} = \$10\,189.43$
(b) $T_n = 1.0027T_{n-1} - 420$; $T_0 = 10\,109.43$
Value after two years = \$5408.99
(c) $12 + 25.13 = 37.13 \Rightarrow 38$ months
(d) $420 - 367.236 = \$52.77$
(e) $37(420) - 52.76 + 1200 = \$16\,792.77$

7. (APP 2018:CA8)
(a) (i) \$4573.20; \$4610.24
(ii) $A_{n+1} = \left(1 + \frac{0.0324}{4}\right)A_n$; $A_0 = 45\,000$
(b) $A_{16} = \$5120$
$5120 = 4500\left(1 + \frac{R}{365}\right)^{(4 \times 365)}$
$R = 0.0323$ (4 d.p.) $\Rightarrow$ annual interest rate of 3.23%
Anthony $i_{effective} = \left(1 + \frac{0.0324}{4}\right)^4 = 1.03279579$
Bryan $i_{effective} = \left(1 + \frac{0.0323}{365}\right)^{365} = 1.03282583$
Due to the increase in the compounding period, the required interest rate reduces but Bryan has a higher effective interest rate.

8. (APP 2018:CA14)
(a) (i) $A_n = \left(1 + \frac{0.225}{12}\right)A_{n-1} - 1000$; $A_0 = 43\,000$
$A_{36} = \$33\,164.78$
(ii) $89 - 36 = 53$ additional months
(b) $48\,000r^3 = 27\,150$
$r = 0.827 \Rightarrow 17.3\%$ average rate of depreciation

(c) Option A – 60 mor
is \$76 985.76
Option B – 72 months
\$76 078.56
Option A will pay of
cost an extra \$907.20
Option B will cost less, b

9. (APP 2018:CA16)
(a) Account B $i_{effective}$ =
(b) If it is compounde
(c) Option C. Highest
(d) $\frac{125}{25\,000} \times 100 \times 12$ =
(e) $c = 1.005$, $d = 250$
(f) (i) $A_{24} = 34\,536.98$
$35\,000 - 34\,536.98 = \$4$
(ii) \$268.21 required to
increase of \$18.21 eacl

10. (APP 2019:CA9)
(a) $T_{n+1} = T_n\left(1 + \frac{0.026}{12}\right.$
$T_{24} = 27\,059.30$
The amount at the
(b) (i) $\frac{20}{100} \times (280\,000$
(ii) 62 months
(c) \$868.22

11. (APP 2019:CA13)
(a) $T_{n-1} = T_n + \frac{0.0365}{12}$
(b) $T_{12} = 6784.32$
$3600 \times 2 - 6784.32$
He does not double
shortfall)
(c) $T_{36} = 13\,511.92$
Total payments =
Total interest = \$1
(d) $T_{24} = 10\,086.83$
$T_{n-1} = T_n + \frac{0.0365}{12}$
$T_{12} = 11\,925.56$
Reduction = \$13 5

## Chapter 9: Ani

1. (FM2 2014:M402)
(a) 3.75%
(b) \$20 000
(c) (i) $750 \times 1.03 = \$7$
(ii) $x \times 1.03^{10} = \$750$
$x = 558.07 \approx \$558$

2. (FM2 2015:M403)
(a) (i) $460 = 0.0368x$
$x = \$12\,500$
(ii) It will last forever
(b) 34 scholarships o

(c) Option A – 60 months to repay, total paid for the vehicle is $76 985.76
Option B – 72 months to repay, total paid for the vehicle is $76 078.56
Option A will pay off the loan 12 months faster, but will cost an extra $907.20
Option B will cost less, but will take an extra 12 months to repay.

9. (APP 2018:CA16)
(a) Account B $i_{effective} = \left(1 + \frac{0.0430}{365}\right)^{365} = 1.043935251 \therefore 4.39\%$
(b) If it is compounded annually.
(c) Option C. Highest effective interest rate.
(d) $\frac{125}{25\,000} \times 100 \times 12 = 6\%$
(e) $c = 1.005$, $d = 250$
(f) (i) $A_{24} = 34\,536.98$
35 000 – 34 536.98 = $463.02
(ii) $268.21 required to reach savings goal
increase of $18.21 each month

10. (APP 2019:CA9)
(a) $T_{n+1} = T_n\left(1 + \frac{0.026}{12}\right) + 800;\ T_0 = 7000$
$T_{24} = 27\,059.30$
The amount at the end of two years is $27 059.30
(b) (i) $\frac{20}{100} \times (280\,000 + 22\,000) = 60\,400$
(ii) 62 months
(c) $868.22

11. (APP 2019:CA13)
(a) $T_{n-1} = T_n + \frac{0.0365}{12} T_n + 250;\ T_0 = 3600$
(b) $T_{12} = 6784.32$
3600 × 2 – 6784.32 = 415.68
He does not double his investments in one year ($415.68 shortfall)
(c) $T_{36} = 13\,511.92$
Total payments = 3600 + 250(36) = 12 600
Total interest = $13 511.92 – $12 600 = $911.92
(d) $T_{24} = 10\,086.83$
$T_{n-1} = T_n + \frac{0.0365}{12} T_n + 120;\ T_0 = 10\,086.83$
$T_{12} = 11\,925.56$
Reduction = $13 511.92 – $11 925.56 = $1 586.36

## Chapter 9: Annuities and Perpetuities

1. (FM2 2014:M402)
(a) 3.75%
(b) $20 000
(c) (i) 750 × 1.03 = $772.50
(ii) $x \times 1.03^{10} = \$750$
$x = 558.07 \approx \$558$ (nearest dollar)

2. (FM2 2015:M403)
(a) (i) $460 = 0.0368x$
$x = \$12\,500$
(ii) It will last forever.
(b) 34 scholarships of $650

3. (APP 2016:CA16)
(a) (i) $A_{n+1} = 1.06A_n - 40\,000;\ A_0 = \$500\,000$ (ii) 23 years
(iii) The balance would remain at $500 000 after each withdrawal.
(b) (i) $809 531.47 (ii) $815 197.73
(iii) $75 900 (nearest dollar)

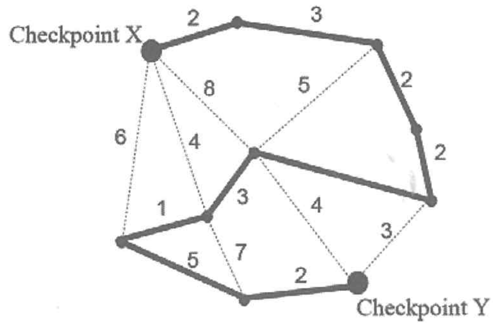
4. (APP 2017:CA8)
(a) $i_{effective} = 6.06222\%$
$\frac{6.0622}{100} \times \$98\,974 = \$6000$
(b) $4283.77

5. (APP 2018:CA11)
(a) $1921.80
(b) (i) $N = 12.92 \Rightarrow 12$ years (ii) 7.03% per annum

6. (APP 2019:CA17)
(a) $A = 0.075 \times \frac{101\,000}{12} + 355 = \$986.25$
(b) $655 539.45
(c) $5082.39
(d) $4097.12

## Chapter 10: Graphs and Networks

1. (FM2 2010:M502)
(a) 11
(b) (i) Hamiltonian path
(ii)




2. (FM2 2012:M501a)
(a) (i) 70 + 90 = 160 m (ii) 2
(iii) 1180 + 70 = 1250 m (Edge between the odd vertices must be travelled twice)

3. (FM1 2013:M502)
Edges $= \frac{4(4-1)}{2} = 6$

4. (FM2 2013:M501)
(a) 3
(b) 1000 m
(c) (i) P4
(ii) Euler Path
(d) E – P5 – P4 – P6 – P3 – P2 – P1

5. (FM1 2014:M501)
A Hamiltonian path.

6. (FM1 2014:M506&07)
(a) 1 (second from the left)
(b) All 4