4. **[12 marks]** *(APP SAT:CA06)*

A fish farm operates a fish breeding pond in which the population of a particular fish increases by 3% per month.

(a) Give the recurrence formula to calculate the fish population at the end of each month, assuming the rate does not change and the initial population is 1000 fish. [2]

(b) Calculate the population numbers at the end of the first six months of its operation, given the initial population is 1000 fish. [2]

(c) After the initial six months, 40 fish per month are removed at the end of each month. Assuming the population growth is maintained at 3%, how many fish are expected to be in the tank at the end of 12 months? [3]

(d) Describe what is happening to the population. [1]

(e) Estimate, to the nearest whole number, the maximum number of fish that may be removed from the tank per month without the numbers of fish decreasing. [4]

6. **[12 marks]** *(APP 2016:CF4)*

(a) Given the sequence 256, 128, 64, 32, …

(i) Write a recursive rule for the sequence [2]

(ii) Deduce a rule for the nth term of this sequence. Hence, calculate the 15th term, leaving your answer as a fraction. [3]

(b) Use the recursive definitions given to state the first **three** terms of each of the following sequences.

(i) $T_{n+1} = T_n + 7,\ T_1 = 11$ [2]

(ii) $T_{n+1} = 1.5T_n,\ T_2 = 7.5$ [2]

(c) Consider the sequence 12, 7, 2, –3, …

By deducing a rule for the nth term, or otherwise, determine which term of the sequence is –168. [3]

(APP 2017:CA16)

7. [8 marks]

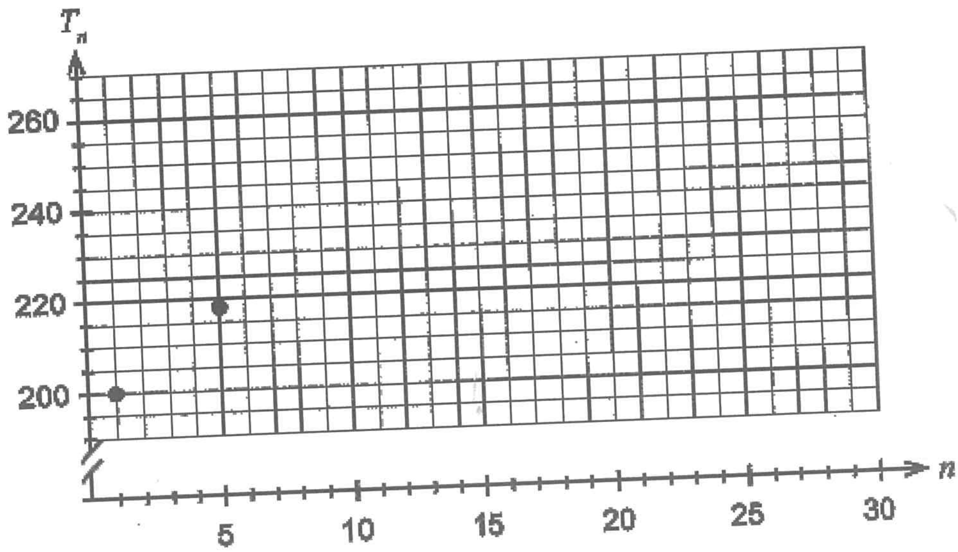
In a Northern Territory river, the crocodile population is dropping by 7.5% each year. The current population is 200. A scheme is being trialled under which 20 crocodiles are introduced to the river each year.

The population of crocodiles in the river can be modelled by the first-order linear recurrence relation $T_{n+1} = 0.925T_n + b$, $T_1 = 200$ where T_n is the number of crocodiles in the river at the beginning of the nth year.

(a) (i) Interpret the coefficient 0.925 in the context of the question. [1]

(ii) State the value of b. [1]

(b) Graph the number of crocodiles in the river for every five year period (commencing at $n = 5$), up to the 30th year on the axes below. [2]



(c) Using your graph, comment on how the population of crocodiles is changing over time. [2]

(d) To the nearest whole number, what is the long-term effect on the crocodile population? [2]

8. **[8 marks]** *(APP 2018:CA9)*

Deborah is purchasing mealworms for her pet lizard, Lizzy, to eat.

Deborah starts by buying 50 mealworms. She then buys another 15 at the start of each subsequent week. She feeds 12 mealworms to Lizzy each week, and each week a certain percentage of mealworms dies.

Deborah has found that the approximate number of mealworms at the start of the nth week can be modelled by M_n, where $M_{n+1} = 0.9(M_n - 12) + 15$, $M_1 = 50$.

(a) What percentage of the mealworms dies each week? [1]

(b) Determine the approximate number of mealworms Deborah has at the start of the fifth week. [1]

(c) Deborah claims that she will never run out of mealworms using this model. Justify her claim. [2]

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8. (cont)

After 10 weeks, hot weather results in a larger percentage of the mealworms dying, so Deborah alters the model to:

$$N_{n+1} = 0.8(N_n - 12) + 15, N_1 = c$$

(d) (i) Determine the value of c. [1]

(ii) Determine the approximate number of mealworms Deborah has at the start of the thirtieth week. [1]

Deborah's vet recommends feeding Lizzy 10 mealworms a week. She would also like to maintain a constant number of 30 mealworms at the start of each week, so she changes the model above to:

$$P_{n+1} = 0.8(P_n - 10) + k$$

(e) Determine the value of k, the number of mealworms she must buy each week, to ensure that this occurs. [2]

(APP 2019:CA7)

10. **[6 marks]**

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. [2]

(b) How many litres run out in the seventh minute? [1]

(c) How many litres have run out after eight minutes? [1]

(d) What is the capacity of the tank? [2]

(2CDMAT 2012:CF01f)

2. **[1 mark]**

For the recursive rule $T_{n+1} = T_n - 4$ with $T_3 = 15$, determine T_1.

(FM2 2013:M103)

3. **[4 marks]**

A farmer grows pear trees in one of his orchards. At the end of each year, new pear trees are always planted. It is known that 250 new pear trees are planted at the end of the third year. The total number of new pear trees, P_n, planted at the end of the nth year, is modelled by the recursive rule

$$P_{n+1} = P_n + 50 \qquad P_1 = c$$

(a) (i) Determine the value of c. [1]

(ii) The recursive rule above generates a sequence. What is the mathematical name given to this type of sequence? [1]

The total number of new pear trees planted at the end of the nth year can also be found from the rule $P_n = a + b \times n$, where a and b are constants.

(b) Determine the values of a and b. [2]

6. **[5 marks]** *(FM2 2015:M101)*

A crop of capsicums is being harvested from a field on a large farm.

In week 1 of the harvest, 2000 kg of capsicums were picked.

In week 2 of the harvest, 2150 kg of capsicums were picked.

In week 3 of the harvest, 2300 kg of capsicums were picked.

The weight of the capsicums, in kilograms, picked each week continues in this pattern for the first eight weeks of the harvest.

The weight of the capsicums, in kilograms, picked each week forms the terms of an arithmetic sequence, as shown below.

2000, 2150, 2300 ...

(a) Write down a calculation that shows that the common difference for this sequence is 150. [1]

(b) How many kilograms of capsicums will be picked in week 5? [1]

(c) How many kilograms of capsicums in total will have been picked in the first eight weeks of the harvest? [1]

(d) In which week will the total weight of the harvest first exceed 14 000 kg? [1]

(e) The weight of the capsicums, Cn, picked in week n, is modelled by a difference equation. Write down the rule for this difference equation in the box provided below. [1]

$$C_{n+1} = \boxed{}\ ; C_1 = 2000$$

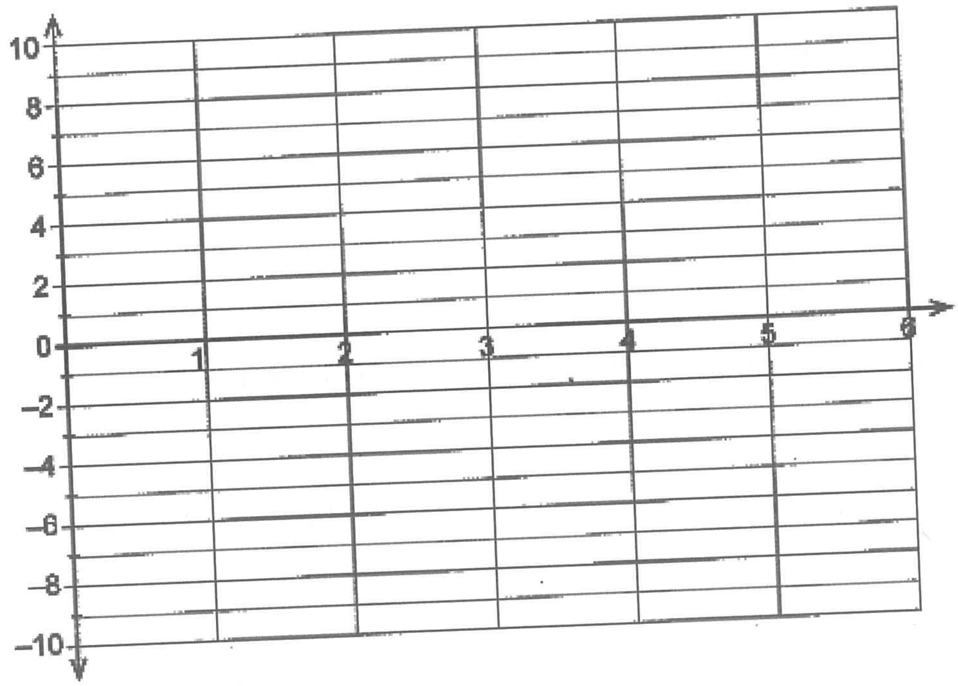
(APP 2017:CF1)

7. **[8 marks]**

Consider the following recurrence relation

$T_{n+1} = T_n - 3,\ T_3 = 2$

(a) Display the first six terms of this sequence on the axes below. Label the axes clearly. [3]



(b) (i) Deduce a rule for the n^{th} term of this sequence. [2]

(ii) Hence, determine the first term in the sequence which is less than –500. [3]

3. **[6 marks]** *(2ABMAT 2014:CA16d)*

At the beginning of 2014, Mahesh deposited \$2500 into a bank account earning interest at the rate of 7% per year. He does not make any further deposits or withdrawals. The amount of money (\$$A_n$) in his account at the beginning of each year is given by the recursive rule:

$$A_{n+1} = A_n \times 1.07, A_1 = 2500$$

(i) Complete the table below to show the amount of money in Mahesh's account at the beginning of 2017 and 2018. [2]

Year	2014	2015	2016	2017	2018
n	1	2	3	4	5
Amount of Money (\$$A_n$)	2500	2675	2862.25		

(ii) State the recursive rule in words. [2]

(iii) Determine the amount of money in Mahesh's account at the beginning of 2025. [2]

5. **[9 marks]** *(APP 2016:CA7)*

Julie buys a car with a purchase price of $13 000. However, she has been told to expect the car to depreciate in value. The value of the car after n years can be determined by using the recursive rule.

$$T_{n+1} = 0.85T_n,\ T_0 = 13\ 000$$

(a) Complete the table below to show the value of the car at the end of each year, to the nearest dollar. [2]

n	0	1	2	3
Value of car after n years ($)	13 000			

(b) Use the information above to determine the rate of depreciation of Julie's car per year [1]

(c) Determine a rule for the nth term of the sequence of value found in part (a). [2]

(d) Determine the value of Julie's car after eight years, correct to the nearest dollar. [2]

(e) Julie decided that she will sell her car at the end of the year in which its value drops to half of the purchase price. After how many years should she sell the car? [2]

7. **[7 marks]**

(APP 2019:CA10)

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $T_n = 1.34T_{n-1}$, $T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34. [1]

(b) Write the rule for the n^{th} term of this sequence. [1]

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. [2]

Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, \$114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. [3]

Finance

Chapter 8

1. **[7 marks]** *(APP 2016S:CA16)*

Karen borrowed \$20 000 to purchase a new car. Interest on the loan was set at 8.5% per annum. If the interest of the loan was compounded n times per year, the amount owed (\$$V$) after one year is given by $V = A\left(1 + \frac{r}{100n}\right)^n$.

(a) State the value of A. [1]

20 000

(b) State the value of r. [1]

0.085

(c) Calculate V when $n = 4$. [2]

$$20\,000\left(1 + \frac{8.4}{100 \times 4}\right)^4$$

$= 1754.96 + 20\,000$

$= 21754.96$

(d) Calculate the effective annual rate of interest when interest is compounded monthly. Give your answer as a percentage correct to **two** decimal places. [3]

$$i = \left(1 + \frac{0.085}{12}\right)^{12} - 1$$

$= \cancel{\text{[illegible]}}\ 8.847$

(APP 2016S:CA17)

2. **[6 marks]**

The recursive formula $T_{n+1} = 1.08T_n$, $T_1 = 2100$ can be used to calculate the value of an investment compounded annually for n years in the Farmers Bank of Western Australia.

(a) What is the annual interest rate? [1]

8%.

(b) Calculate the value of the investment after seven years. [2]

$T_{n+1} = 1.08\,T_n$ $\quad T_1 = 2100$

$T_7 = 3332.44$

(c) Determine the interest rate that would produce the same value for the investment above after a time of three years. [3]

N: 3

I =

PV = 2100

FV: 3332.44

(*APP 2018:CA14*)

8. [12 marks]

Marco is a plumber. Three years ago, he purchased a vehicle costing $48 000 for his business. He paid a deposit of $5000 and acquired a personal loan for the remainder from a financial institution, at a reducible interest rate of 22.5% per annum, compounded monthly. He agreed to make repayments of $1000 at the end of each month.

(a) (i) Use a recurrence relation to determine the amount Marco currently owes on the loan. [3]

(ii) Determine how much longer it will take to completely pay off the loan. [2]

(b) After three years, Marco finds that his vehicle is only worth $27 150. Determine the average rate of depreciation of his vehicle, expressed as a percentage. [2]

(APP 2018:CA16)

9. [11 marks]

Natalia inherits a sum of money from her grandfather. She wishes to place it in a high-interest savings account.

She is considering the following two options:

Account A: interest rate 4.40% per annum, compounded monthly
Account B: interest rate 4.30% per annum, compounded daily.

(a) The effective annual interest rate for Account A is 4.49% (correct to two decimal places). Determine the effective annual interest rate for Account B. [1]

Natalia's bank offers her another account, C, with an interest rate of 4.50% per annum.

(b) Under what circumstances will the interest rate and the effective interest rate be the same? [1]

(c) Which account (A, B or C) should Natalia choose to maximise her savings? Explain your reasoning. [2]

Natalia's sister, Elena, has inherited $25 000 from her grandfather. She decides to invest this money in a high-interest savings account, with interest compounded monthly. Elena also chooses to deposit an additional $250 into this account at the end of each month.

The table below shows Elena's account balance over the first three months.

Month	Account balance at start of month	Interest earned	Deposit	Account balance at end of month
1	$25 000.00	$125.00	$250.00	$25 375.00
2	$25 375.00	$126.88	$250.00	$25 751.88
3	$25 751.88	$128.76	$250.00	$26 130.64