Chapter

4

Recursive Formulas and Recurrence Relations

Essential Syllabus	
3.2	Growth and decay in sequences
	• The arithmetic sequence
3.2.1	o use recursion to generate an arithmetic sequence
3.2.5	o use recursion to generate a geometric sequence
3.2.9	o use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form
3.2.10	o generate a sequence defined by a first-order linear recurrence relation that gives long term increasing, decreasing or steady-state solutions
3.2.11	o use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems

1. **[8 marks]** *(APP 2017:CA16)*

In a Northern Territory river, the crocodile population is dropping by 7.5% each year. The current population is 200. A scheme is being trialled under which 20 crocodiles are introduced to the river each year.

The population of crocodiles in the river can be modelled by the first-order linear recurrence relation $T_{n+1} = 0.925T_n + b$, $T_1 = 200$ where T_n is the number of crocodiles in the river at the beginning of the nth year.

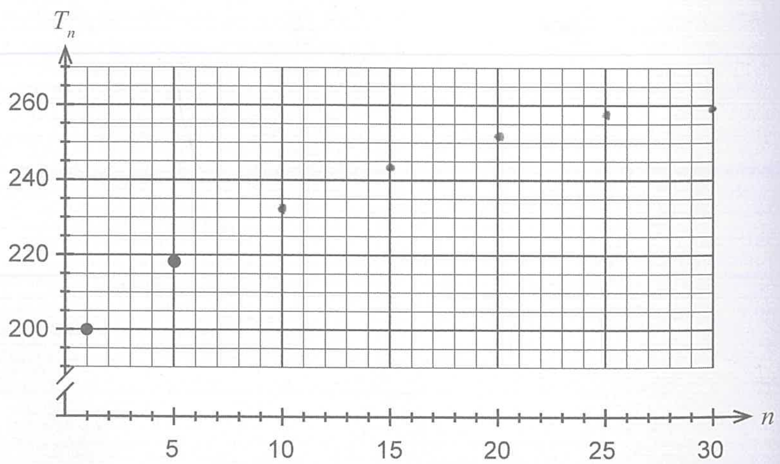
(a) (i) Interpret the coefficient 0.925 in the context of the question. [1]

The pop of croc decreases by 7.5% each year

(ii) State the value of b. [1]

$b = 20$

(b) Graph the number of crocodiles in the river for every five year period (commencing at $n = 5$), up to the 30th year on the axes below. [2]



1. (cont)

(c) Using your graph, comment on how the population of crocodiles is changing over time. [2]

Increasing at a decreasing rate

approaching steady state

(d) To the nearest whole number, what is the long-term effect on the crocodile population? [2]

Remain stable at 267 croc

2. **[8 marks]** *(APP 2018:CA09)*

Deborah is purchasing mealworms for her pet lizard, Lizzy, to eat.

Deborah starts by buying 50 mealworms. She then buys another 15 at the start of each subsequent week. She feeds 12 mealworms to Lizzy each week, and each week a certain percentage of mealworms dies.

Deborah has found that the approximate number of mealworms at the start of the nth week can be modelled by M_n, where $M_{n+1} = 0.9(M_n - 12) + 15$, $M_1 = 50$.

(a) What percentage of the mealworms dies each week? [1]

10 %

(b) Determine the approximate number of mealworms Deborah has at the start of the fifth week. [1]

$M_5 = 47.249$, 48 mealworms (round up)

(c) Deborah claims that she will never run out of mealworms using this model. Justify her claim. [2]

Her claims are true because the model approaches a steady state of 42 meal worms

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2. (cont)

After 10 weeks, hot weather results in a larger percentage of the mealworms dying, so Deborah alters the model to:

$$N_{n+1} = 0.8(N_n - 12) + 15, N_1 = c$$

(d) (i) Determine the value of c. [1]

45

$0.9(M_n - 12) + 5$

$M_{10} = 45$

(ii) Determine the approximate number of mealworms Deborah has at the start of the thirtieth week. [1]

$N_{30} = 27$

Deborah's vet recommends feeding Lizzy 10 mealworms a week. She would also like to maintain a constant number of 30 mealworms at the start of each week, so she changes the model above to:

$$P_{n+1} = 0.8(P_n - 10) + k$$

(e) Determine the value of k, the number of mealworms she must buy each week, to ensure that this occurs. [2]

$30 = 0.8(20) + k$

$14 = k$

14 meal worms

3. **[6 marks]** *(APP 2019:CF06)*

The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.

(a) Determine a recursive rule → Recurrence relations for the turtle population. [2]

$T_{n+1} = 0.5T_n + 20, \ T_1 = 32$

(b) Determine the long-term steady state of the turtle population. [2]

$T_{n+1} = 0.9T_n + 20$

$x = 0.5x + 20$

$0.5x = 20$

$x = 40$

040
5√200
-0
0
20
-20
000
000

2
2√40
4
0
40

Turtle pop approaches ss of 40 turtles

(c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. [2]

$80 = 0.5(80) + x$

$80 = 40 + x$

$x = 40$

800
× 5
4000

40 turtles are required each year

4. **[6 marks]** *(APP 2019:CA07)*

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. [2]

$T_n = 84 - 6(n-1)$

(b) How many litres run out in the seventh minute? [1]

$T_7 = 84 - 6(7-1) = 48$ liters

(c) How many litres have run out after eight minutes? [1]

$T_9 = 84 - 6(9-1) = 36$ liters

(d) What is the capacity of the tank? [2]

90 liters

84 + 6

5. **[6 marks]** *(APP 2020:CA07)*

The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing.

(a) Calculate his annual growth (in cm) in his fourth and fifth years. [1]

(b) Deduce the rule for his annual growth in the n^{th} year, until he stopped growing. [2]

(c) In which year did he first not grow any taller? [1]

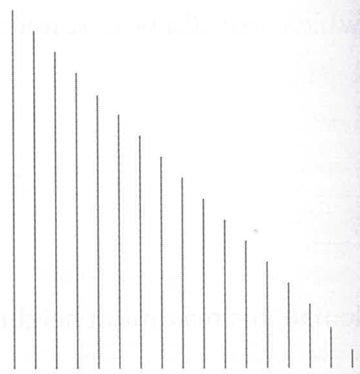
(d) Calculate his maximum height. [2]

Chapter 5 The Arithmetic Sequence

Essential Syllabus	
3.2	Growth and decay in sequences
	• The arithmetic sequence
3.2.1	o use recursion to generate an arithmetic sequence
3.2.2	o display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations
3.2.3	o deduce a rule for the n^{th} term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions
3.2.4	o use arithmetic sequences to model and analyse practical situations involving linear growth or decay

1. [6 marks] *(Projected:CF)*

An art installation has 17 posts lined up in a row. The first post is 240 cm and each post to the right of it is shorter than the one before it by a constant amount. The final post on the far right is 48 cm long.



(a) How much shorter is each subsequent post compared to the one immediately to its left? [2]

(b) Write a recurse rule for determining the length of (T_n) of the nth post using the length of the post to the left of it. [2]

(c) Deduce a rule calculating the length of the nth post. [2]

2. [7 marks] *(Projected:CA)*

On the 1st of November, Andrea was given a new novel for her birthday and she reads the first 9 pages before going to bed. The next night she read 12 pages, and the night after that she read 15 pages. Andrea continues to increase the number of pages she reads each night in this manner.

(a) Write a rule to determine the number of pages Andrea reads on any given night. [2]

(b) How many pages will Andrea read on the 6th of November? [1]

(c) If there are 600 pages in the book, how many more will Andrea need to read after the 6th of November to complete the novel? [2]

(d) On what date will she start reading more than 55 pages a night? [2]

3. **[6 marks]** *(Projected:CA)*

A large field contains a crop of cucumbers that is currently being harvested.

In the first week the weight of the cucumbers harvested was 1600 kg

In the second week the weight of the cucumbers harvested was 1740 kg

In the third week the weight of the cucumbers harvested was 1880 kg.

The weight, in kilograms, of cucumbers harvested each week, continues to increase in this manner for the first 7 weeks of the harvesting season.

The weight, in kilograms, of cucumbers harvested each week, forms the terms of an arithmetic sequence, as shown below.

1600, 1740, 1880 ...

(a) Show how a common difference of 140 can be determined from the values in the sequence. [1]

(b) How many kilograms of cucumbers will be harvested in Week 5? [1]

(c) What is the total number of kilograms of cucumbers picked over the first 7 weeks of the season? [1]

(d) In which week will the total weight of the cucumbers harvested first total above 10 000 kg? [1]

(e) The weight of cucumbers, C_n, harvested in a week, n, is modelled by a recursive definition. Write down this rule for the recursive equation in the box below. [1]

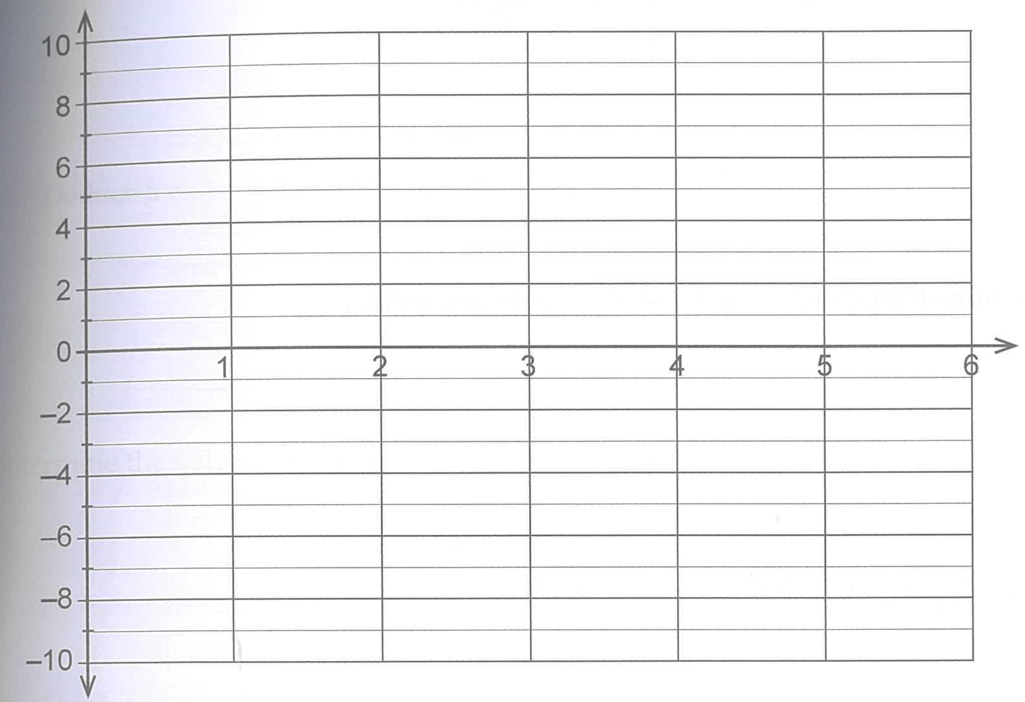
$C_{n+1} = \boxed{}\ ;\ C_1 = 1600$

4. **[8 marks]** *(APP 2017:CF01)*

Consider the following recurrence relation

$T_{n+1} = T_n - 3,\ T_3 = 2$

(a) Display the first six terms of this sequence on the axes below. Label the axes clearly. [3]



(b) (i) Deduce a rule for the n^{th} term of this sequence. [2]

(ii) Hence, determine the first term in the sequence which is less than –500. [3]

5. **[5 marks]** *(APP 2021:CF01)*

Hanai is a successful college basketball player. His coach has warned him that he will lose his scholarship if he scores 54% or below on a weekly assessment. On his first three weekly assessments he scored 84%, 81% and 78% respectively.

Assume Hanai's weekly assessments continue to follow this pattern.

(a) Deduce a rule for the n^{th} term of this sequence. [2]

(b) Determine Hanai's score on his sixth weekly assessment. [1]

(c) Predict when Hanai will lose his scholarship. [2]

6. **[7 marks]** *(APP 2022:CF02)*

A gardener purchases a new lawnmower valued at $4800. The lawnmower depreciates at a constant rate of $250 per year.

(a) (i) Determine a recursive rule for the value of the lawnmower after n years. [2]

(ii) Deduce a rule for the nth term of this sequence. [1]

(b) Determine the value of the lawnmower after 4 years. [1]

As part of his business plan, the gardener will sell his lawnmower when its value drops below $1300.

(c) Calculate when the lawnmower will be sold. [3]

Chapter 6

The Geometric Sequence

Essential Syllabus	
3.2	Growth and decay in sequences
	• The geometric sequence
3.2.5	o use recursion to generate a geometric sequence
3.2.6	o display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations
3.2.7	o deduce a rule for the n^{th} term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions
3.2.8	o use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay

1. **[12 marks]**

(APP 2017:CA10)

In a laboratory experiment, the population of a particular bacteria began with 400 present. The bacteria grew at a rate of 35% each week, where P is the number of bacteria and t is the number of weeks from the start of the experiment.

(a) Four possible equations were produced to model this experiment.

$P = 400(1.35)^t$

$P = 400(0.35)^t$

$P = 540(1.35)^{t-1}$

$P = 540(1.35)^{t+1}$

Circle the correct equation(s). [2]

(b) Calculate the population of bacteria after three weeks. [1]

1. (cont)

(c) During which week did the population of bacteria first reach 1800? [2]

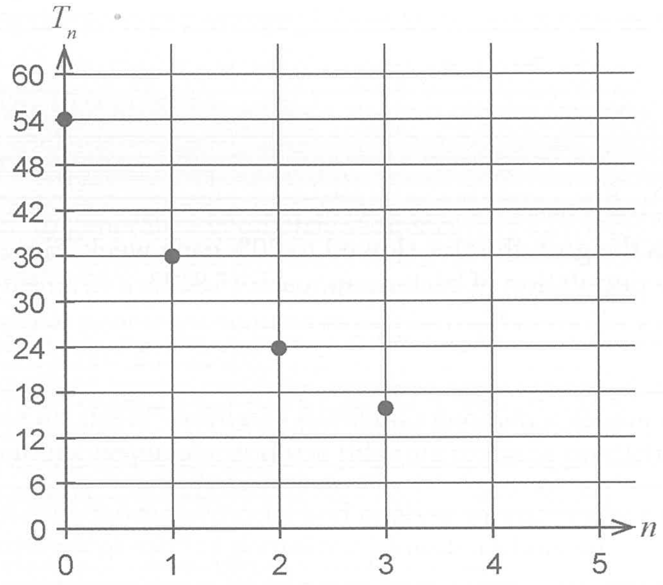
(d) After eight weeks the growth rates slowed to 20% each week. How many weeks in total did it take for the population of bacteria to reach 15 812? [3]

(e) What constant weekly growth rate would produce the same change in population from 400 to 15 812 in the same time as found in part (d)? [2]

(f) Once the bacteria population reached 15 812 it began to die out at a rate of 250 each day. Approximately how many weeks did it take for the bacteria to die out completely? [2]

2. **[6 marks]** *(APP 2018:CF07)*

A researcher compared the performance of various golf balls. The graph below shows the height reached above the ground by a particular golf ball after each of the first three bounces. It was initially dropped from a height of 54 cm.



(a) Write a recursive rule for this sequence. [3]

(b) Write a rule for the nth term of this sequence. [1]

(c) Show that the height reached by the gold ball above the ground after the fifth bounce is $\frac{64}{9}$ cm. [2]

(APP 2019:CA10)

3. **[7 marks]**

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $T_n = 1.34T_{n-1}$, $T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34. [1]

(b) Write the rule for the n^{th} term of this sequence. [1]

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. [2]

Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, \$114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. [3]

4. **[9 marks]** *(APP 2020:CA08)*

A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish.

(a) Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. [2]

(b) Deduce a rule for the n^{th} term of this sequence. [2]

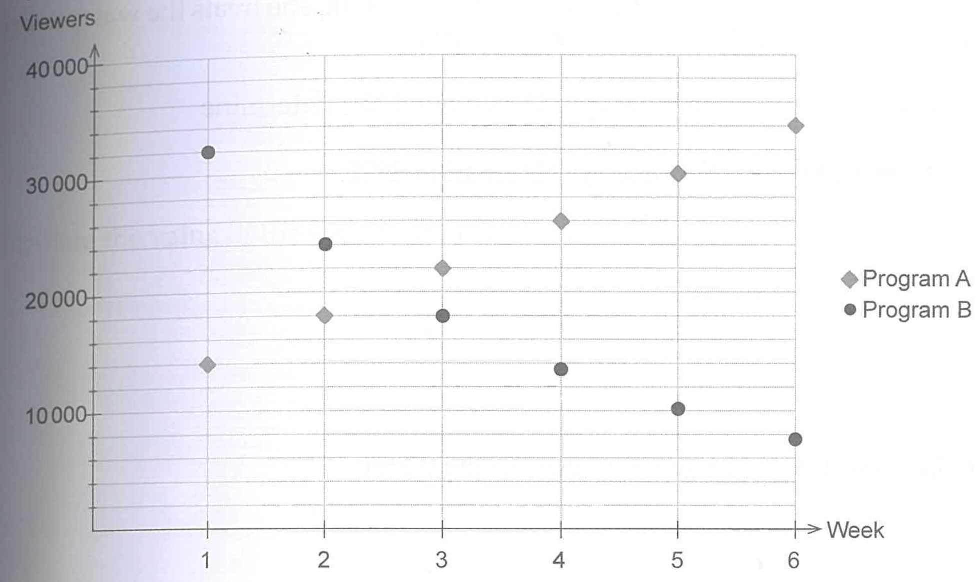
The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000.

(c) Determine how many months after the beginning of 2020 fishing will commence. [2]

(d) The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month. [3]

5. **[7 marks]** *(APP 2021:CF06)*

A television network programmer was analysing the number of viewers for two children's programs over a period of several weeks, to decide which program should be given the better time slot. The viewing numbers, displayed on the graph below, formed an arithmetic sequence and a geometric sequence.



(a) Write a recursive rule for the arithmetic sequence. [2]

(b) Using the first two data points, deduce a rule for the n^{th} term of the geometric sequence. [2]

(c) Explain which program should be given the better time slot. [2]

(d) Determine the number of viewers for the more successful program in Week 8. [1]

6. **[8 marks]**

(APP 2021:CA11)

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule $C_{n+1} = 1.025C_n$, where $C_1 = 200$ units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L.

(a) If Judith treated the water on Sunday, 6 December 2020, determine

(i) the concentration on Wednesday, 9 December 2020. [2]

(ii) the day and date when she next treated the water. [2]

(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

Day	1	2	3	4	5	6	7
Concentration (C)	200	206	212.18	218.55	225.10	231.85	238.81

(i) Determine the revised recursive rule. [2]

(ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used. [2]

(APP 2021:CA12)

7. **[9 marks]**

Virat purchases a new motor vehicle for $24 500. For the first two years the vehicle depreciates at a rate of 13% per year and for the third year it depreciates at a lower rate of 9.5% per year.

(a) Calculate the value of the vehicle after one year. [1]

(b) Calculate the value of the vehicle after the first three years. [2]

For the next three years the rate of depreciation is constant at r% per year. The average rate of depreciation for the first six years is 11% per year.

(c) Calculate the value of r as a percentage. [3]

8. **[7 marks]** (APP 2022:CA0

A study of a penguin colony on an island was conducted and it found the initial popula[tion] size of 1200 was dropping by 14% each year due to the introduction of non-native predators.

(a) Explain why the population after n weeks is $1200 \times (0.86)^n$ penguins. [2]

After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by 6%.

(b) State the recursive formula that models the new population growth. [2]

(c) How many weeks will it take to get the population back up to the initial size? [1]

Once the population returns to the initial size, it is further helped by the introduction of penguins from a breeding program at the zoo.

The new population growth model can be represented by

$P_{n+1} = -0.25P_n + 3000,\ P_0 = 1200.$

(d) Discuss the long-term behaviour of the penguin population, now that it is being supported by the breeding program. [2]

Investments

Chapter **7**

Essential Syllabus	
4.2	Loans, investments and annuities
	• Compound interest loans and investments
4.2.1	o use a recurrence relation to model a compound interest loan or investment and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment
4.2.2	o calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly
4.2.3	o with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans, investments and depreciating assets

1. **[7 marks]** (APP 2018:CA08)

Anthony and Bryan each invest $4500 in accounts earning compound interest for a period of four years.

(a) Anthony places his money in an account earning interest at the rate of 3.24% per annum, compounded quarterly.

(i) Complete the table below, showing the value of Anthony's investment at the end of the second and third quarters. [2]

Number of quarters money is invested	1	2	3	…	16
Value of investment ($)	4536.45			…	5120.00

(ii) State the recursive rule for Anthony's investment, which gives the values shown in the table above. [2]

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Chapter 8 Reducing Balance Loans

Essential Syllabus	
4.2	Loans, investments and annuities
	• Reducing balance loans (compound interest loans with periodic repayments)
4.2.4	o use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan
4.2.5	o with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans

1. [5 marks] *(APP 2017:CF04)*

Ryan was keen to compare interest rates offered by different banks, so he decided to construct a table showing the effective annual rates of interest (%). Part of this table is shown below.

	Rate of interest (p.a.)				
Compounding period	**4%**	**4.5%**	**5%**	**5.5%**	**6%**
Quarterly	4.060	4.577	5.095	5.614	6.136
Monthly	4.074	4.594	5.116	5.641	6.168
Daily	4.081	4.602	5.127	5.654	6.183

(a) Ryan wants to borrow $5000 to purchase a second-hand car. A bank offers to lend him the money at the rate of 6% p.a. for one year. He plans to pay off the entire loan (including the interest) at the end of the year. Which compounding period should he sign up for? Justify your decision. [2]

(b) Ryan is curious to know how much interest he would earn by investing $100 for a year earning 4% p.a. with interest compounded quarterly. Determine the interest he would earn. [1]

(c) Ryan's sister has $3000 to invest for a year. She has been offered a rate of 5% p.a., with Interest compounded daily. Determine the value of her investment at the end of the year. [2]

2. [13 marks] *(APP 2017:CA14)*

Andrew takes out a $14 999 loan to purchase his first car after paying a $1200 deposit. The car dealer offered the loan at an introductory rate of 1.80% p.a. for the first year and then the rate becomes 3.24% p.a. for the remaining time of the loan. Interest is added monthly and Andrew has calculated that he can afford to make monthly repayments of $420.

(a) (i) Express the loan repayment process for the first year as a recursive formula. [2]

(ii) How much does Andrew still owe after one year? [1]

(b) How much does Andrew owe after two years? [3]

(c) How long does it take for Andrew to repay the loan? [2]

(d) Determine the amount of the final repayment. [2]

(e) Calculate the total cost of the car. [3]

3. **[12 marks]**

(APP 2018:CA14)

Marco is a plumber. Three years ago, he purchased a vehicle costing $48 000 for his business. He paid a deposit of $5000 and acquired a personal loan for the remainder from a financial institution, at a reducible interest rate of 22.5% per annum, compounded monthly. He agreed to make repayments of $1000 at the end of each month.

(a) (i) Use a recurrence relation to determine the amount Marco currently owes on the loan. [3]

(ii) Determine how much longer it will take to completely pay off the loan. [2]

(b) After three years, Marco finds that his vehicle is only worth $27 150. Determine the average rate of depreciation of his vehicle, expressed as a percentage. [2]

3. (cont)

(c) When Marco originally took out a personal loan to purchase his vehicle, he was given two options from the financial institution. These were:

- Increasing his monthly repayment by $200, or
- Taking an option of reducing the interest rate to 18.5% and maintaining repayments of $1000 per month.

In terms of time taken to pay off the loan and the total paid for his vehicle, which should he have chosen and why? [5]

4. [9 marks] (APP 2020:CA11)

Shari is offered a choice of two loan options for the first three years, both of which have interest calculated daily.

- Option 1 An introductory compound interest rate of 2.55% per annum for the first year which changes to 2.99% per annum for the next two years.
- Option 2 A compound interest rate of 2.85% per annum fixed for the first three years.

(a) Describe briefly the benefit of making two repayments of $700 each month instead of one repayment of $1400 at the end of each month. [1]

(b) For Option 1, calculate

(i) the loan balance at the end of the first year. [3]

(ii) the loan balance at the end of the third year. [2]

(c) Determine which option gives the best result for Shari after three years and by how much. [3]

5. [10 marks] (APP 2020:CA12)

Jessica wants to borrow $15 000 from her parents to purchase a car. They will be charging her compound interest at the rate of 4% per annum, with interest added yearly.

(a) Jessica is currently studying so she will not want to be making any regular repayments.

(i) Complete the table below to show the amount she will owe her parents at the end of each year. [2]

Number of years (n)	0	1	2	3
Amount owing ($)	15 000			

(ii) Write a recursive rule to calculate the amount owing at the end of each year. [2]

Jessica's parents are encouraging her to get a part-time job so that she can make repayments along the way. Jessica estimates that she will be able to earn enough money to pay off $2400 each year.

(b) If interest is charged yearly and she repays the $2400 at the end of each year, write a recursive rule to calculate the amount owing at the end of each year. [1]

(c) If interest is charged monthly and she makes equal monthly repayments,

(i) write a recursive rule to calculate the amount owing at the end of each month. [1]

(ii) calculate how many months it will take to repay the loan. [1]

(iii) calculate the total amount Jessica would pay over the duration of the loan. [3]

6. **[14 marks]** *(APP 2021:CA10)*

Wendy moved into an apartment and organised a loan of $16 000 to purchase new furniture. To pay off the loan Wendy makes repayments of $600 at the end of each month. The spreadsheet below shows the progress of her loan.

Month	Opening balance	Interest	Repayment	Closing balance
1	16 000.00	98.67	600.00	15 498.67
2	15 498.67	95.58	600.00	14 994.24
3	14 994.24	92.46	600.00	14 486.71
4				

(a) Write a calculation to show that the yearly interest rate is approximately 7.4%. [2]

(b) Complete the fourth row of the spreadsheet. [3]

(c) Write a recursive rule to determine the closing balance of the loan at the end of each month. [2]

(d) Determine how many months it will take Wendy to pay off the loan. [1]

(e) Calculate how much interest is paid over the duration of the loan. [3]

6. (cont)

On reflection, Wendy realised she could have repaid $800 each month.

(f) Determine the maximum amount Wendy would have been able to borrow, if all other details of the loan and repayment time remained the same. [3]

7. **[7 marks]** *(APP 2022:CA…*

After paying a deposit for his new apartment, Declan obtains a bank loan for the remain… amount of \$112 000 at 3.26% per annum compounded monthly. He can currently afford… repay \$970 per month at the end of every month.

(a) Calculate how much he would owe after the 40th repayment.

(b) Declan decided to deposit a one-off extra amount of \$1600, after the 16th repayment. Calculate the new amount he would owe after the 40th repayment. [4]

Annuities and Perpetuities

Chapter 9

Essential Syllabus	
4.2	Loans, investments and annuities
	• Annuities and perpetuities (compound interest investments with periodic payments made from the investment)
4.2.6	o use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity
4.2.7	o with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including perpetuities as a special case)

1. **[6 marks]** *(APP 2017:CA08)*

Ming, a former high school student and now a successful business owner, wishes to set a perpetuity of \$6000 per year to be paid to a deserving student for her school. The perpetuity is to be paid at the start of the year in one single payment.

(a) A financial institution has agreed to maintain an account for this perpetuity paying a fixed rate of 5.9% p.a. compounded monthly.

Show that an amount of \$98 974, to the nearest dollar, is required to maintain this perpetuity. [3]

(b) Ming allows herself five years to accumulate the required \$98 974 by making regular quarterly payments into an account paying 5.4% p.a. compounded monthly.

Determine the quarterly payment needed to reach the required amount after five years if Ming starts the account with an initial deposit of \$1000. [3]

9. **[8 marks]**

(APP 2020:CF06)

A small business office has five separate areas connected by doorways shown as gaps in this diagram:

This adjacency matrix below represents the number of doorways directly between each area:

$$\begin{array}{c} \\ B \\ K \\ M \\ R \\ W \end{array} \begin{array}{c} \begin{array}{ccccc} B & K & M & R & W \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & Y \\ 0 & X & 1 & 2 & 0 \end{bmatrix} \end{array}$$

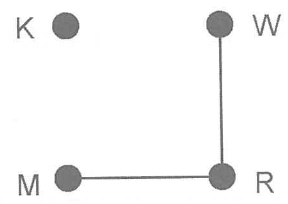
(a) State the meaning of the zero entries in the matrix. [1]

(b) Determine the value of X and Y. [2]

(c) Describe how the total number of doorways for each area can be found from the adjacency matrix. [1]

9. (cont)

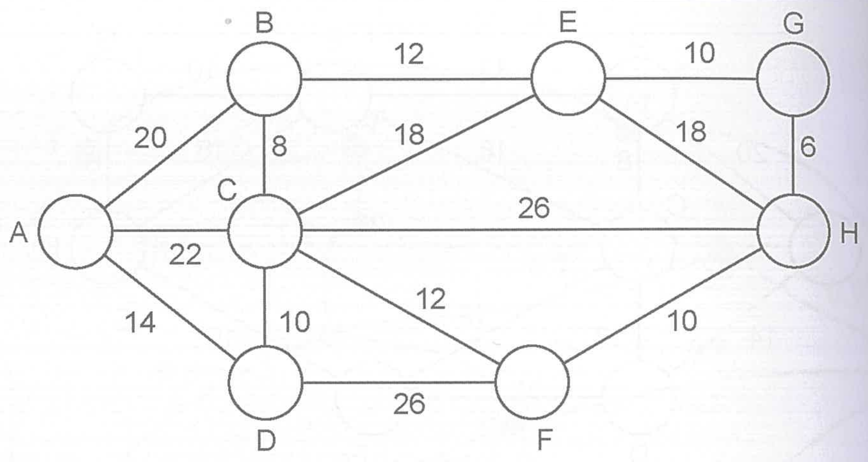
(d) Complete this network with vertices corresponding to the office areas and the edges representing the doorways. [3]



(e) Determine how many different routes there are between the meeting room and the workspace that pass through exactly two doorways. [1]

3. (cont)

Road CH presently goes around what is now a dry salt lake. It is proposed that a direct road be constructed that will reduce the distance between retail outlets C and H.



(c) By how much can the direct road between C and H be reduced, so that the shortest path from the warehouse to H includes the direct road CH? [3]

(FM2 2021:M402)

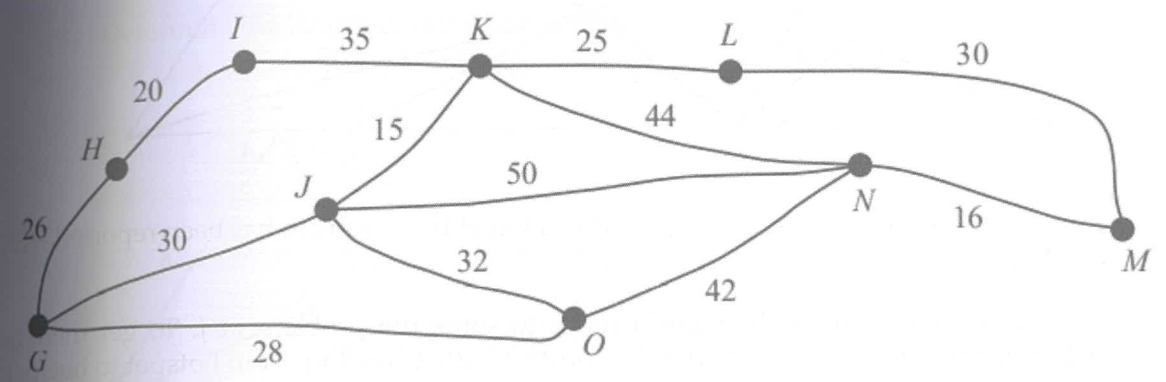
4. **[3 marks]**

George lives in Town G and Maggie lives in Town M.

The diagram below shows the network of main roads between Town G and Town M.

The vertices G, H, I, J, K, L, M, N and O represent towns.

The edges represent the main roads. The numbers on the edges indicate the distances, in kilometres, between adjacent towns.

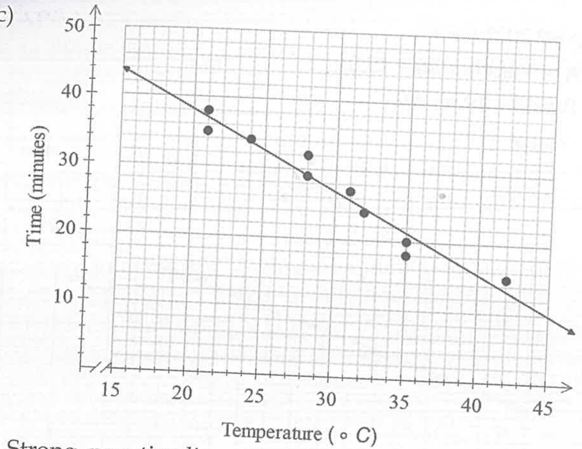


(a) What is the shortest distance, in kilometres, between Town G and Town M? [2]

(b) George plans to travel to Maggie's house by taking a Hamiltonian Cycle through the towns shown above. George plans to take the shortest route possible. State his route. [1]

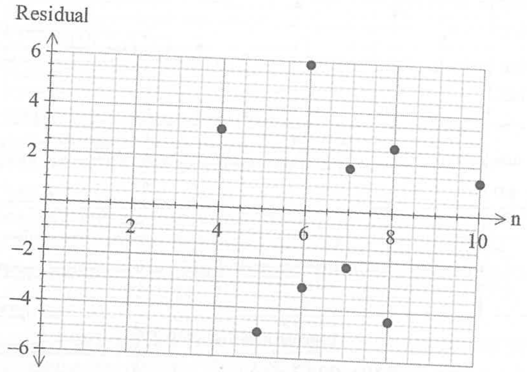
(b) $y = -1.129x + 60.744$
$r = -0.972$

(c)



(d) Strong, negative linear correlation
(e) 94.5%
(f) wind, humidity
(g) (i) $-1.129(17) + 60.744 \approx 41.53$ minutes
(ii) The prediction involves an extrapolation.
The predicted is not reliable.

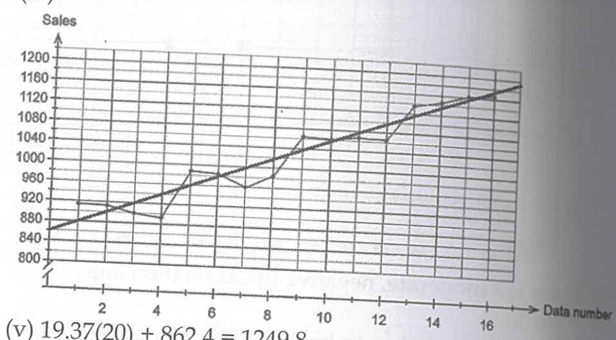
12. (APP 2022:CA13)
(a) For every additional mobile phone tower in the suburb there is an additional 5 births over the 12-month period.
(b) There is a strong, positive correlation between number of phone towers and number of births.
(c) $5.13(9) + 1.31 = 47.48 \approx 48$ births.
(d) Predicated value is valid. High correlation ($r = 0.92$) and an interpolation.
(e)



(f) There is no pattern to the residuals.
The regression model is suitable.
(g The suburb has a high density of elderly residents.
(h) Correlation does not imply causality.
Suburbs with higher number of mobile phone towers have a higher number of births.
There are other factors that contribute to the number of births.

Chapter 3: Periodic Time Series Analysis

1. (APP 2017:CA15)
(a) (i) 21
(ii) $\frac{0.5(14)+17+18+24+21+19+16(0.5)}{6} = 19$
(b) (i) $A = 884 \quad B = 1153.5 \quad C = 87.66$
(ii) 1.1554
(iii) 1062
(iv)



(v) $19.37(20) + 862.4 = 1249.8$
$1249.8 \times 1.1554 = 1444$
(vi) Reliable as the prediction involves an extrapolation within one additional cycle.

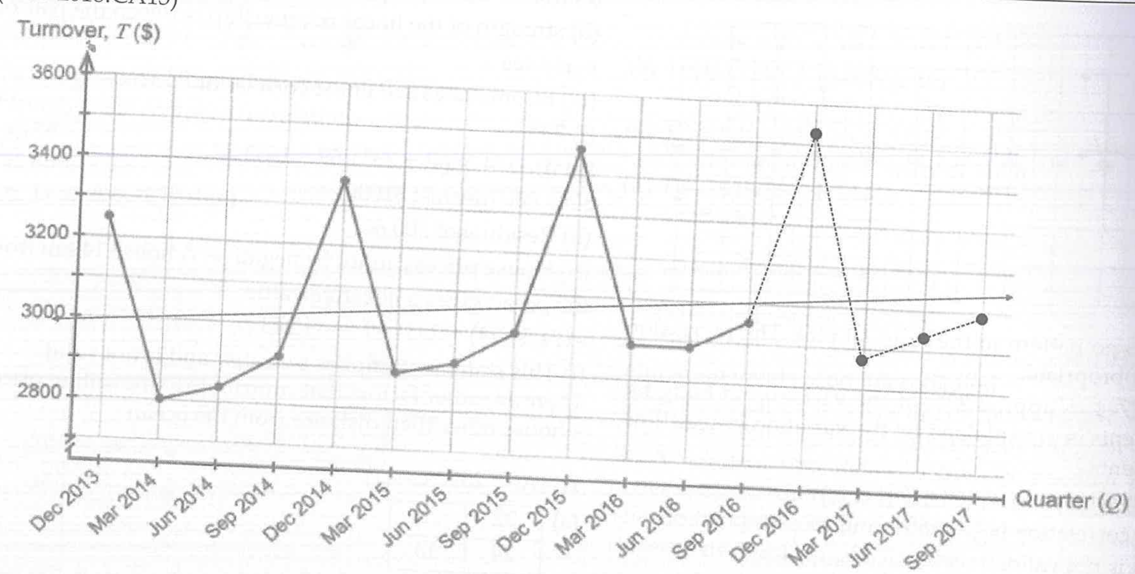
2. (APP 2018:CA12)
(a) To smooth the data and make the overall trend easier to see
(b) $\frac{864 + 834 + A}{3} = 838 \Rightarrow A = 816$

$B = \frac{828 + 918 + 927 + 879 + 852}{5} = 880.8$

$C = \frac{0.5(840) + 927 + 936 + 894 + 867 + 828 + 918(0.5)}{6} = 888.5$

(c) 5-point moving average.
When graphed, the data shows a period of 5.

3. (APP 2018:CA13)
(a)



(b) (i) Key points include (1, 2996) & (12, 3102)
(ii) Increasing trend, period of 4 with peaks in turnover (in $s) December and troughs in turnovers in March.
(c) $\frac{0.5x + 3521.40 + 2980.10 + 3045.00 + 3075.30(0.5)}{4} = 3152.78$
$x = \$3053.94$
(d) (i) SI June = 400 − 110.76 − 95 − 98.20 = 96.04%
(ii) $\frac{3521.40}{1.1076} = \$3179.31$
(iii) $3142.42 \times 0.95 = \$2985.30$

4. (APP 2019:CA11)
(a) $\frac{1577 + A + 1463 + 1274}{4} = 1524 \Rightarrow A = 1782$

$B = \frac{1904}{1660.5} \times 100 \Rightarrow B = 114.66$

$C = \frac{0.5(1463) + 1274 + 1600 + 1745 + 1504(0.5)}{4} = 1525.625$

(b) Season Index for Autumn is 0.9755
(c) $\frac{1274}{0.8276} = 1540.8805 \approx 1541$ (nearest whole)
(d) Value increased
(e) $R = -12.071(12) + 1681.25 = 1427.759$
$1427.759 \times 1.0432 = 1489.438 \approx 1489$ (nearest whole)
(f) Different methods utilised for smoothing the data
(g) The statement implies causality and this cannot be established. The statement is not appropriate.
(h) Decrease price, advertise, etc.

5. (APP 2020:CA14)
(a) $\frac{A + 18 + 11 + 17}{4} = 14 \Rightarrow \mathbf{A} = 10$

$B = \frac{18}{14} \times 100 = 128.6$ (1 d.p.)

$C = \frac{15 + 16 + 11 + 15}{4} = 14.25$

(b)

Season	Summer	Autumn	Winter	Spring
Seasonal Index	95.8	114.3	74.6	115.3

(c) $\frac{13}{95.8} \times 100 = 13.6$ (1 d.p.)
(d) Winter – Seasonal index of 74.6
(e) $\hat{y} = -0.240x + 14.876$ (3 d.p.)
(f) $\hat{y} = -0.240(17) + 14.876$
$y = 10.8$ (1 d.p.)
$10.796 \times \frac{95.8}{100} = 10.34$
10 sprinkler systems will be installed in Summer 2021
(g) Negative gradient (−0.240) indicates a decline in sales.

6. (APP 2021:CA08)
(a) 5 point moving average
(b) $A = \frac{12}{17.6} \times 100 \Rightarrow A = 68.18$ (2 dp)

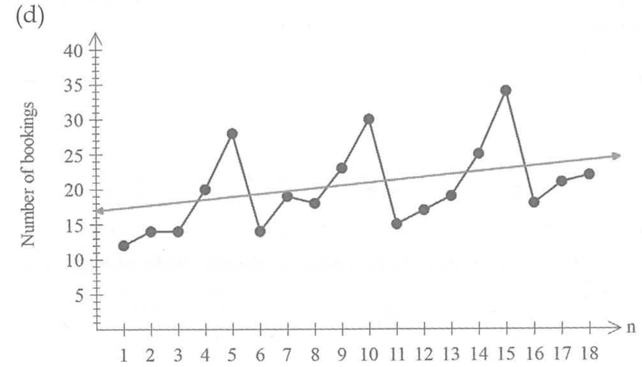
$\frac{14}{B} \times 100 = 67.31 \Rightarrow B = 20.8$ (1 dp)

$\frac{C}{22} \times 100 = 68.18 \Rightarrow C = 15$ (nearest whole)

(c) $\frac{159.09 + 144.23 + 154.55}{3} = 152.6233333$

Season Index for Saturday is 1.526 (3 dp)

(d)



(e) (i) $y = 0.40(25) + 16.94$
$y = 26.94$
$26.94 \times 1.526 \approx 41$ bookings
(ii) This predicated value is not valid.
Large extrapolation beyond one cycle

7. (APP 2022:CF05)
(a) $\mathbf{5.33} = \frac{9+4+3}{3}$

$\mathbf{120} = \frac{6}{5} \times 100$

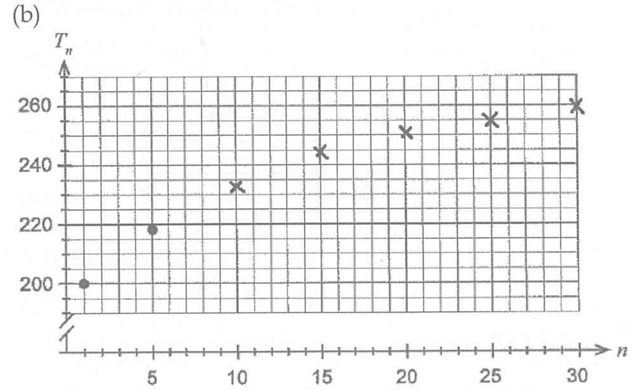
$\mathbf{6} = \frac{3+6+9}{3}$

(b) $\frac{40+60+50}{3} = 50$

Over the three week period, Friday sales were on average 50% of the daily mean
(c) (i) To reveal the overall trend
(ii) $3 \div 0.5 = 6$
(d) The gradient (0.2) is positive
(e) $y = 0.2(10) + 4.3$
$y = 6.3$
$6.3 \times 0.5 = 3.15 \approx 3$
⇒ forecasted that 3 handbags will be sold on Friday of Week 4

Chapter 4: Recursive Formulas and Recurrence Relations

1. (APP 2017:CA16)
(a) (i) Decrease of 7.5% each year (ii) 20
(b)



(c) The rate of increase is slowing over time. It appears to be approaching a steady state.
(d) Stable population of 267 crocodiles

2. (APP 2018:CA09)
(a) 10%
(b) $M_1 = 50$, $M_5 = 47.249$, $\Rightarrow$ 48 mealworms
(c) Her statement is true. The long term steady state solution for this recursive formula is 42.
(d) (i) $c = 45$ (ii) 27 (steady state solution)
(e) $30 = 0.8(30 - 10) + k$
$k = 14$

3. (APP 2019:CF06)
(a) $T_{n+1} = 0.5T_n + 20$; $T_1 = 32$
(b) $T_2 = 36, T_3 = 38, T_4 = 39, T_5 = 39.5$
Turtle population approaches a steady state of 40
OR
$x = 0.5x + 20 \Rightarrow x = 40$
$0.5x = 20$
$x = 40$
Turtle population approaches a steady state of 40
(c) $80 = 0.5 \times 80 + k$
$80 = 40 + k$
$k = 40$
40 turtles are required each year.

4. (APP 2019:CA07)
(a) $a = 84$, $d = -6$
$T_n = 84 + (n - 1)(-6)$
$T_n = 90 - 6n$
(b) $T_7 = 90 - 6(7) \Rightarrow T_7 = 48$
48 L
(c) $S_8 = 84 + 78 + 72 + 66 + 60 + 54 + 48 + 72$
$S_8 = 504$
504 L
(d) $T_{15} = 0$, $S_{15} = 630$
Capacity of the tank is 630 L

5. (APP 2020:CA07)
(a) 4th year – 22 cm, 5th year – 20 cm
(b) $T_n = 28 - 2(n - 1)$
$T_n = 30 - 2n$
(c) 15th year
(d) 270 cm

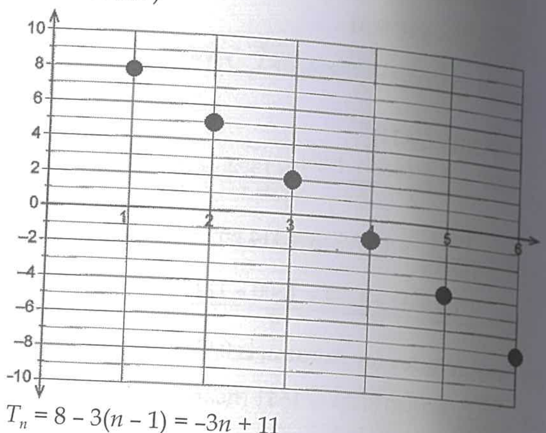
Chapter 5: The Arithmetic Sequence

1. (Projected:CF)
(a) $\frac{240-48}{17-1} = \frac{192}{16} = 12$ cm
(b) $T_{n+1} = T_n - 12$; $T_1 = 240$
(c) $T_n = 240 - (n - 1)12$
$T_n = 240 - 12n + 12$
$T_n = 252 - 12n$

2. (Projected:CA)
(a) $T_n = 9 + (n - 1)3$
$T_n = 6 + 3n$
(b) $T_6 = 24$
(c) $S_6 = 99$
$600 - 99 = 501$
(d) $55 = 6 + 3n$
$49 = 3n$
$n = 16\frac{1}{3} \Rightarrow$ 17th Nov

3. (Projected CA)
(a) $1740 - 1600 = 140$
(b) $1600 + 140(4) = 2160$ kg
(c) 14 140 kg
(d) 6th Week
(e) $C_n + 140$

4. (APP 2017:CF01)
(a)



(b) (i) $T_n = 8 - 3(n - 1) = -3n + 11$
(ii) $-500 = 8 - 3(n - 1)$
$n = 170.\dot{3} \Rightarrow$ 171st term

5. (APP 2021:CF01)
(a) $T_n = 84 - 3(n - 1)$
$T_n = 87 - 3n$
(b) $T_6 = 87 - 3(6) = 69\%$
(c) $54 = 87 - 3n$
$n = 11$
11th week

6. (APP 2022:CF02)
(a) (i) $T_{n+1} = T_n - 250$; $T_0 = 4800$ (ii) $T_n = 4800 - 250n$
(b) $T_4 = \$3800$
(c) $1300 = 4800 - 250n$
$-3500 = -250n$
$n = 14$
$\Rightarrow$ after 14 years

Chapter 6: The Geometric Sequence

1. (APP 2017:CA10)
(a) $P = 400(1.35)^t$ and $P = 540(1.35)^{t-1}$
(b) 984
(c) $400(1.35)^t = 1800$
$t = 5.012 \Rightarrow$ *6th week*
(d) $400(1.35)^8 = 4413$
$4413(1.2)^7 = 15\,812$
$\therefore$ 15 weeks
(e) $400(r)^{15} = 15\,812$
$r = 1.2778 \Rightarrow 27.78\%$
(f) $15\,812 - 250d = 0$
$d = 63.248 \Rightarrow$ 9 weeks

2. (APP 2018:CF07)
(a) $\frac{54}{36} = \frac{2}{3}$

$T_n = \frac{2}{3}T_{n-1}$; $T_0 = 54$

(b) $T_n = 54\left(\frac{2}{3}\right)^n$

(c) $T_5 = 54\left(\frac{2}{3}\right)^5$

$T_5 = 2(3^3) \times \frac{32}{3^5}$

$T_5 = 2 \times \frac{32}{9}$

$T_5 = \frac{64}{9}$

3. (APP 2019:CA10)
(a) $r = \frac{30\,256}{22\,579} \approx 1.34$
(b) $a = 15$, $r = 1.34$
$T_n = 15(1.34)^{n-1}$
OR
$T_n = 11.19(1.34)^n$
(c) After 33 years $(T_{34}) = 234\,719$
$\therefore$ In 2025 there will be over 200 000 shops.
(d) $T_{20} = 3900$
Daily wages = 3900 × 12 × 114.80
Daily wages = \$5 372 640

4. (APP 2020:CA08)
(a) $A_{n+1} = 1.09A_n$; $A_0 = 660$
(b) $A_n = 660(1.09)^n$
(c) $A_{21} \approx 4031 \Rightarrow$ 21st month
(d) $4031 \times 0.09 = 362.79$
$\Rightarrow$ 362 fish per month

5. (APP 2021:CF06)
(a) $A_{n+1} = A_n + 4000$; $A_1 = 14\,000$
(b) $T_n = 32\,000(0.75)^{n-1}$
(c) Program A
From the third week onwards, the number of viewers for A surpasses the number of viewers for B.
(d) 42 000

6. (APP 2021:CA11)

$c_{n+1}=1.025\cdot c_n$

n	c_n	Σc_n
1	200	200
2	205	405
3	210.13	615.13
4	215.38	830.50
5	220.76	1051.3
6	226.28	1277.5
7	231.94	1509.5
8	237.74	1747.2
9	243.68	1990.9
10	249.77	2240.7
11	256.02	2496.7

(a) (i) 215 units per 100 L (nearest whole)
(ii) $C_{11} = 256.02$
Wednesday 16th December
(b) (i) $C_{n+1} = 1.03C_n$; $C_1 = 200$
(ii) $C_{11} = 269$ units per 100 L (nearest whole)

7. (APP 2021:CA12)
(a) 24 500 × 0.87 = \$21 315.00
(b) $24\,500 \times 0.87^2 \times 0.905 = \$16\,782.37$
(c) $24\,500 \times 0.89^6 = \$12\,176.04$
$\$12\,176.04 = 24\,500 \times 0.87^2 \times 0.905 \times x^3$
$x = 0.898568036079063$
$\therefore r = 10.14\%$

8. (APP 2022:CA09)
(a) 1200 represents the initial population
$\frac{100-14}{100} = 0.86$ represents the 14% decline.
(b) $T_8 = 1200(0.86)^8 \approx 359$ (nearest whole)
$T_{n+1} = 1.06T_n$; $T_0 = 359$
(c)

$a_{n+1}=1.06\cdot a_n$

n	a_n
19	1086.2
20	1151.4
21	1220.4
22	1293.7
23	1371.3
24	1453.6

1220.4433325959

21 weeks
(d)

$a_{n+1}=-0.25\cdot a_n+3000$

n	a_n
483	2400
484	2400
485	2400
486	2400
487	2400
488	2400

2400

The population of penguins will reach a steady state of 2400 penguins.

Chapter 7: Investments

1. (APP 2018:CA08)
(a) (i) \$4573.20; \$4610.24
(ii) $A_{n+1} = \left(1 + \frac{0.0324}{4}\right)A_n$; $A_0 = 45\,000$
(b) $A_{16} = \$5120$
$5120 = 4500\left(1 + \frac{R}{365}\right)^{(4 \times 365)}$
$R = 0.0323$ (4 d.p.) $\Rightarrow$ annual interest rate of 3.23%
Anthony $i_{effective} = \left(1 + \frac{0.0324}{4}\right)^4 = 1.03279579$

Bryan $i_{effective} = \left(1 + \frac{0.0323}{365}\right)^{365} = 1.03282583$

Due to the increase in the compounding period, the required interest rate reduces but Bryan has a higher effective interest rate.

2. (APP 2018:CA16)
(a) Account B $i_{effective} = \left(1 + \frac{0.0430}{365}\right)^{365} = 1.043935251 \therefore 4.39\%$
(b) If it is compounded annually.
(c) Option C. Highest effective interest rate.
(d) $\frac{125}{25\,000} \times 100 \times 12 = 6\%$ (e) $c = 1.005$, $d = 250$
(f) (i) $A_{24} = 34\,536.98$
35 000 – 34 536.98 = \$463.02
(ii) \$268.21 required to reach savings goal
increase of \$18.21 each month

3. (APP 2019:CA09)
(a) $T_{n+1} = T_n\left(1 + \frac{0.026}{12}\right) + 800$; $T_0 = 7000$
$T_{24} = 27\,059.30$
The amount at the end of two years is \$27 059.30
(b) (i) $\frac{20}{100} \times (280\,000 + 22\,000) = 60\,400$
(ii) 62 months
(c) \$868.22

4. (APP 2019:CA13)

(a) $T_{n+1} = T_n + \frac{0.0365}{12}T_n + 250;\ T_0 = 3600$

(b) $T_{12} = 6784.32$

$3600 \times 2 - 6784.32 = 415.68$

He does not double his investments in one year ($415.68 shortfall)

(c) $T_{36} = 13\,511.92$

Total payments = 3600 + 250(36) = 12 600

Total interest = $13 511.92 – $12 600 = $911.92

(d) $T_{24} = 10\,086.83$

$T_{n+1} = T_n + \frac{0.0365}{12}T_n + 120;\ T_0 = 10\,086.83$

$T_{12} = 11\,925.56$

Reduction = $13 511.92 – $11 925.56 = $1 586.36

5. (APP 2022:CA08)

(a) $E_{Option\,1} = \left(1+\frac{0.0305}{12}\right)^{12} - 1 = 0.0309$

$E_{Option\,2} = \left(1+\frac{0.0301}{365}\right)^{365} - 1 = 0.0306$

Option 1 represents the better rate of return with an effective interest rate of 3.09%.

(b) (i)

Compound Interest	
N	52
I%	3.5
PV	-12300
PMT	-300
FV	28608.95862
P/Y	52
C/Y	365

30 000 – 28 608.95862 = $1391.04

After 12 months, the investment will have grown to $28 608.96

This has fallen short of the required amount by $1391.04

(ii)

Compound Interest	
N	52
I%	3.5
PV	-12300
PMT	-326.2942575
FV	30000
P/Y	52
C/Y	365

$326.29

Chapter 8: Reducing Balance Loans

1. (APP 2017:CF04)

(a) Quarterly. Lowest rate to minimise interest

(b) $4.06

(c) $\frac{5.127}{100} \times 3000 + 3000 = 51.27 \times 3 + 3000 = \$3153.81$

$\Rightarrow$ $3153.81

2. (APP 2017:CA14)

(a) (i) $T_{n+1} = 1.0015T_n - 420;\ T_0 = 14\,999$ (ii) $T_{12} = \$10\,189.43$

(b) $T_n = 1.0027T_{n-1} - 420;\ T_0 = 10\,109.43$

Value after two years = $5408.99

(c) 12 + 25.13 = 37.13 $\Rightarrow$ 38 months

(d) 420 – 367.236 = $52.77

(e) 37(420) – 52.76 + 1200 = $16 792.77

3. (APP 2018:CA14)

(a) (i) $A_n = \left(1+\frac{0.225}{12}\right)A_{n-1} - 1000;\ A_0 = 43\,000$

$A_{36} = \$33\,164.78$

(ii) 89 – 36 = 53 additional months

(b) $48\,000r^3 = 27\,150$

$r = 0.827 \Rightarrow 17.3\%$ average rate of depreciation

(c) Option A – 60 months to repay, total paid for the vehicle is $76 985.76

Option B – 72 months to repay, total paid for the vehicle is $76 078.56

Option A will pay off the loan 12 months faster, but will cost an extra $907.20

Option B will cost less, but will take an extra 12 months to repay.

4. (APP 2020:CA11)

(a) Each payment reduces the balance and less interest is accrued the following period.

(b) (i) $316 386.79

(ii) $301 279.69

Compound Interest	
N	24
I%	2.55
PV	325000
PMT	-700
FV	-316386.7857
P/Y	24
C/Y	365

Compound Interest	
N	48
I%	2.99
PV	316386.7857
PMT	-700
FV	-301279.6928
P/Y	24
C/Y	365

(c) Option 2 balance after 3 years is $301 422.81

Option 1 gives the best result by $143.12

Compound Interest	
N	72
I%	2.85
PV	325000
PMT	-700
FV	-301422.8075
P/Y	24
C/Y	365

5. (APP 2020:CA12)

(a) (i)

Number of years (n)	0	1	2	3
Amount owing ($)	15 000	15 600	16 224	16 872.96

(ii) $A_{n+1} = 1.04A_n;\ A_0 = \$15\,000$

(b) $A_{n+1} = 1.04A_n - 2400;\ A_0 = \$15\,000$

(c) (i) $\frac{0.04}{12} = 0.003$

$A_{n+1} = 1 + 1.003A_n - 200;\ A_0 = \$15\,000$

(ii) 87 months

(iii) Total repayment: (86 × 200) + $89.76 = $17 289.76

6. (APP 2021:CA10)

(a) $I = \frac{98.67}{16\,000} \times 100 \times 12 \Rightarrow I = 7.40025 \approx 7.4\%$ (1 dp)

(b)

4	14 486.71	89.33	600.00	13 976.04

(c) $A_{n+1} = \left(1+\frac{0.074}{12}\right)A_n - 600;\ A_0 = 16\,000$

(d) 30 months

Final Repayment $134.44

Total Interest (29 × 600) + 134.44 – 16 000 = $1534.44

$21 849.53

Compound Interest	
N	30
I%	7.4
PV	21849.52817
PMT	-800
FV	0
P/Y	12
C/Y	12

7. (APP 2022:CA16)

(a)

Compound Interest	
N	40
I%	3.26
PV	112000
PMT	-970
FV	-83910.19026
P/Y	12
C/Y	12

(b) $A_{16} = \$101\,128.4617$

$101 128.4617 – 1600 = $99 528.46

Compound Interest	
N	24
I%	3.26
PV	99528.4617
PMT	-970
FV	-82202.54526
P/Y	12
C/Y	12

Chapter 9: Annuities and Perpetuities

1. (APP 2017:CA08)

(a) $i_{effective} = 6.06222\%$

$\frac{6.0622}{100} \times \$98\,974 = \$6000$

(b) $4283.77

2. (APP 2018:CA11)

(a) $1921.80

(b) (i) $N = 12.92 \Rightarrow 12$ years (ii) 7.03% per annum

3. (APP 2019:CA17)

(a) $A = 0.075 \times \frac{101\,000}{12} + 355 = \$986.25$

(b) $655 539.45

(c) $5082.39

(d) $4097.12

4. (APP 2020:CA09)

(a)

P	Q	X	Y
$15 000	$648 000	$645 312	$12 260.93

(b) 0.019 × 4 = 0.076 $\Rightarrow$ 7.6% p.a.

(c) $T_{66} = \$299\,501.07$

$\Rightarrow$ 16.5 years

Compound Interest	
N	65.94587102
I%	7.6
PV	-648000
PMT	15000
FV	300000
P/Y	4
C/Y	4

(d) 0.019 × 299 501.07 = $5690.52

(e) $\left(1+\frac{0.076}{4}\right)^4 - 1 = 0.07819356632 \Rightarrow 7.82\%$

Giuseppe is incorrect.

5. (APP 2021:CA14)

Compound Interest	
N	12
I%	5.7
PV	-717850
PMT	4500
FV	704420.1986
P/Y	12
C/Y	12

(a) (i) annuity

(ii) $704 420.20 – 4000 = $700 420.20

(iii) $685 970.53 – 4000 = $681 970.53

Compound Interest	
N	12
I%	5.7
PV	-700420.1986
PMT	4500
FV	685970.5279
P/Y	12
C/Y	12

Compound Interest	
N	12
I%	5.7
PV	-717850
PMT	10278.02891
FV	717850
P/Y	4
C/Y	12

(b) $P = 717\,850 \times \left(1+\frac{0.057}{12}\right)^3 - 717\,850$

$P = \$10\,278.03$

6. (APP 2022:CA15)

(a) 69 444.44 × 0.072 = 4999.99968 ≈ $5000

(b) (i) $\frac{0.061}{12} = 0.00508$ (5 dp)

$a = 775\,320 - 69\,444.44 = \$705\,875.56$

(ii) 141 months

Compound Interest	
N	141.7388076
I%	6.1
PV	-705875.56
PMT	7000
FV	0
P/Y	12
C/Y	12

2. (APP 2018:CA09)
(a) 10%
(b) $M_1 = 50$, $M_5 = 47.249$, $\Rightarrow$ 48 mealworms
(c) Her statement is true. The long term steady state solution for this recursive formula is 42.
(d) (i) $c = 45$ (ii) 27 (steady state solution)
(e) $30 = 0.8(30 - 10) + k$
$k = 14$

3. (APP 2019:CF06)
(a) $T_{n+1} = 0.5T_n + 20$; $T_1 = 32$
(b) $T_2 = 36, T_3 = 38, T_4 = 39, T_5 = 39.5$
Turtle population approaches a steady state of 40
OR
$x = 0.5x + 20 \Rightarrow x = 40$
$0.5x = 20$
$x = 40$
Turtle population approaches a steady state of 40
(c) $80 = 0.5 \times 80 + k$
$80 = 40 + k$
$k = 40$
40 turtles are required each year.

4. (APP 2019:CA07)
(a) $a = 84, d = -6$
$T_n = 84 + (n - 1)(- 6)$
$T_n = 90 - 6n$
(b) $T_7 = 90 - 6(7) \Rightarrow T_7 = 48$
48 L
(c) $S_8 = 84 + 78 + 72 + 66 + 60 + 54 + 48 + 72$
$S_8 = 504$
504 L
(d) $T_{15} = 0$, $S_{15} = 630$
Capacity of the tank is 630 L

5. (APP 2020:CA07)
(a) 4th year – 22 cm, 5th year – 20 cm
(b) $T_n = 28 - 2(n - 1)$
$T_n = 30 - 2n$
(c) 15th year
(d) 270 cm

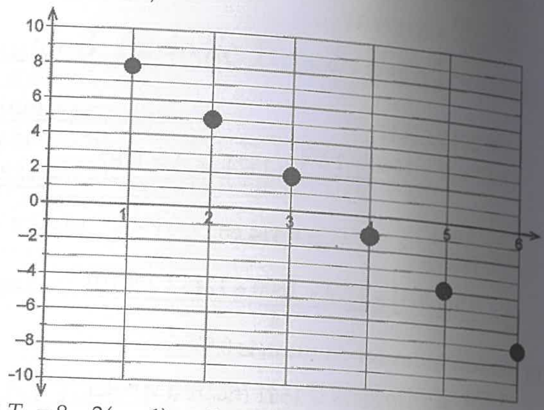
Chapter 5: The Arithmetic Sequence

1. (Projected:CF)
(a) $\frac{240-48}{17-1} = \frac{192}{16} = 12$ cm
(b) $T_{n+1} = T_n - 12$; $T_1 = 240$
(c) $T_n = 240 - (n - 1)12$
$T_n = 240 - 12n + 12$
$T_n = 252 - 12n$

2. (Projected:CA)
(a) $T_n = 9 + (n - 1)3$
$T_n = 6 + 3n$
(b) $T_6 = 24$
(c) $S_6 = 99$
$600 - 99 = 501$
(d) $55 = 6 + 3n$
$49 = 3n$
$n = 16\frac{1}{3} \Rightarrow$ 17th Nov

3. (Projected CA)
(a) $1740 - 1600 = 140$
(b) $1600 + 140(4) = 2160$ kg
(c) 14 140 kg
(d) 6th Week
(e) $C_n + 140$

4. (APP 2017:CF01)
(a)



(b) (i) $T_n = 8 - 3(n - 1) = -3n + 11$
(ii) $-500 = 8 - 3(n - 1)$
$n = 170.\dot{3} \Rightarrow$ 171st term

5. (APP 2021:CF01)
(a) $T_n = 84 - 3(n - 1)$
$T_n = 87 - 3n$
(b) $T_6 = 87 - 3(6) = 69\%$
(c) $54 = 87 - 3n$
$n = 11$
11th week

6. (APP 2022:CF02)
(a) (i) $T_{n+1} = T_n - 250$; $T_0 = 4800$ (ii) $T_n = 4800 - 250n$
(b) $T_4 = \$3800$
(c) $1300 = 4800 - 250n$
$-3500 = -250n$
$n = 14$
$\Rightarrow$ after 14 years

Chapter 6: The Geometric Sequence

1. (APP 2017:CA10)
(a) $P = 400(1.35)^t$ and $P = 540(1.35)^{t-1}$
(b) 984
(c) $400(1.35)^t = 1800$
$t = 5.012 \Rightarrow$ *6th week*
(d) $400(1.35)^8 = 4413$
$4413(1.2)^7 = 15\,812$
$\therefore$ 15 weeks
(e) $400(r)^{15} = 15\,812$
$r = 1.2778 \Rightarrow 27.78\%$
(f) $15\,812 - 250d = 0$
$d = 63.248 \Rightarrow$ 9 weeks

2. (APP 2018:CF07)
(a) $\frac{54}{36} = \frac{2}{3}$
$T_n = \frac{2}{3}T_{n-1}$; $T_0 = 54$

(b) $T_n = 54\left(\frac{2}{3}\right)^n$
(c) $T_5 = 54\left(\frac{2}{3}\right)^5$
$T_5 = 2(3^3) \times \frac{32}{3^5}$
$T_5 = 2 \times \frac{32}{9}$
$T_5 = \frac{64}{9}$

3. (APP 2019:CA10)
(a) $r = \frac{30\,256}{22\,579} \approx 1.34$
(b) $a = 15, r = 1.34$
$T_n = 15(1.34)^{n-1}$
OR
$T_n = 11.19(1.34)^n$
(c) After 33 years $(T_{34}) = 234\,719$
$\therefore$ In 2025 there will be over 200 000 shops.
(d) $T_{20} = 3900$
Daily wages = 3900 × 12 × 114.80
Daily wages = \$5 372 640

4. (APP 2020:CA08)
(a) $A_{n+1} = 1.09A_n$; $A_0 = 660$
(b) $A_n = 660(1.09)^n$
(c) $A_{21} \approx 4031 \Rightarrow$ 21st month
(d) $4031 \times 0.09 = 362.79$
$\Rightarrow$ 362 fish per month

5. (APP 2021:CF06)
(a) $A_{n+1} = A_n + 4000$; $A_1 = 14\,000$
(b) $T_n = 32\,000(0.75)^{n-1}$
(c) Program A
From the third week onwards, the number of viewers for A surpasses the number of viewers for B.
(d) 42 000

6. (APP 2021:CA11)

$c_{n+1}=1.025 \cdot c_n$

n	c_n	Σc_n
1	200	200
2	205	405
3	210.13	615.13
4	215.38	830.50
5	220.76	1051.3
6	226.28	1277.5
7	231.94	1509.5
8	237.74	1747.2
9	243.68	1990.9
10	249.77	2240.7
11	256.02	2496.7

(a) (i) 215 units per 100 L (nearest whole)
(ii) $C_{11} = 256.02$
Wednesday 16th December
(b) (i) $C_{n+1} = 1.03C_n$; $C_1 = 200$
(ii) $C_{11} = 269$ units per 100 L (nearest whole)

7. (APP 2021:CA12)
(a) $24\,500 \times 0.87 = \$21\,315.00$
(b) $24\,500 \times 0.87^2 \times 0.905 = \$16\,782.37$
(c) $24\,500 \times 0.89^6 = \$12\,176.04$
$\$12\,176.04 = 24\,500 \times 0.87^2 \times 0.905 \times x^3$
$x = 0.898568036079063$
$\therefore r = 10.14\%$

8. (APP 2022:CA09)
(a) 1200 represents the initial population
$\frac{100-14}{100} = 0.86$ represents the 14% decline.
(b) $T_8 = 1200(0.86)^8 \approx 359$ (nearest whole)
$T_{n+1} = 1.06T_n$; $T_0 = 359$
(c)

$a_{n+1}=1.06 \cdot a_n$

n	a_n
19	1086.2
20	1151.4
21	1220.4
22	1293.7
23	1371.3
24	1453.6

1220.4433325959

21 weeks
(d)

$a_{n+1}=-0.25 \cdot a_n+3000$

n	a_n
483	2400
484	2400
485	2400
486	2400
487	2400
488	2400

2400

The population of penguins will reach a steady state of 2400 penguins.

Chapter 7: Investments

1. (APP 2018:CA08)
(a) (i) \$4573.20; \$4610.24
(ii) $A_{n+1} = \left(1 + \frac{0.0324}{4}\right)A_n$; $A_0 = 45\,000$
(b) $A_{16} = \$5120$
$5120 = 4500\left(1 + \frac{R}{365}\right)^{(4 \times 365)}$
$R = 0.0323$ (4 d.p.) $\Rightarrow$ annual interest rate of 3.23%
Anthony $i_{effective} = \left(1 + \frac{0.0324}{4}\right)^4 = 1.03279579$
Bryan $i_{effective} = \left(1 + \frac{0.0323}{365}\right)^{365} = 1.03282583$
Due to the increase in the compounding period, the required interest rate reduces but Bryan has a higher effective interest rate.

2. (APP 2018:CA16)
(a) Account B $i_{effective} = \left(1 + \frac{0.0430}{365}\right)^{365} = 1.043935251$ $\therefore$ 4.39%
(b) If it is compounded annually.
(c) Option C. Highest effective interest rate.
(d) $\frac{125}{25\,000} \times 100 \times 12 = 6\%$ (e) $c = 1.005$, $d = 250$
(f) (i) $A_{24} = 34\,536.98$
$35\,000 - 34\,536.98 = \$463.02$
(ii) \$268.21 required to reach savings goal
increase of \$18.21 each month

3. (APP 2019:CA09)
(a) $T_{n+1} = T_n\left(1 + \frac{0.026}{12}\right) + 800$; $T_0 = 7000$
$T_{24} = 27\,059.30$
The amount at the end of two years is \$27 059.30
(b) (i) $\frac{20}{100} \times (280\,000 + 22\,000) = 60\,400$
(ii) 62 months
(c) \$868.22

(iii) $T_{141} = 5134.990084$

$5134.990084 \times (1 + \frac{0.061}{12}) = \$5161.09$

(iv) $(141 \times 7000) + 5161.09 - 705\,875.56 = \$286\,285.53$

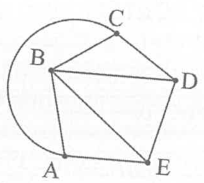
(c) $\frac{0.061}{12} \times \$705\,875.56 = \$3588.20$

Chapter 10: Graphs and Networks

1. (APP 2017:CF03)

(a) (i) $5 + 5 = e + 2 \Rightarrow 8$ edges

(ii)



(iii) B – C – A – E – D – B (other possibilities)

(iv) Two odd vertices (A and D) ⇒ semi-Eulerian

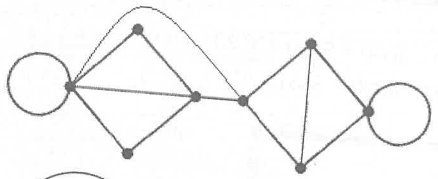
(b) (i) Minimum – 4 Maximum – 10

(ii) $n - 1$ (iii) Complete

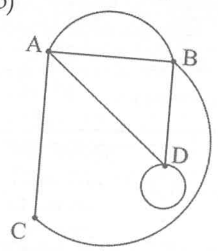
2. (APP 2017:CF07)

(a) (i) More than two odd vertices

(ii)



(b)



3. (APP 2018:CF02)

(a) Row F [0 0 1 0 0 0]

(b) No edges are repeated. Vertex E is repeated.

(c) CDEAB – 23 minutes

(d) D-G-F-C-E-A-B

4. (APP 2018:CF05)

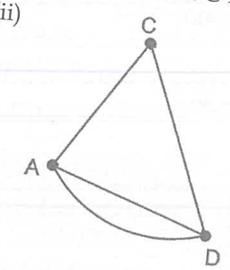
(a) (i) The matrix is not symmetrical over the leading diagonal (C to B has less connections than B to C)

(ii) There are multiple edges between nodes such as B to C.

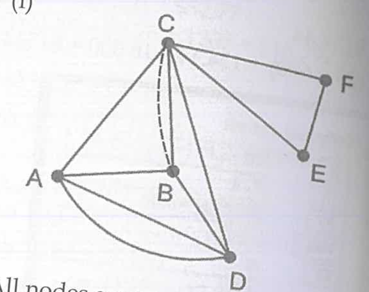
The network contains a loop at node C.

(b) (i) There are three, 2-step routes from node C to node D (C-B-D, C-A-D and C-A-D)

(ii)



(c) (i)



(ii) All nodes are now even

5. (APP 2019:CF01)

(a) No two edges cross

(b) $v + f - e = 2$

$3 + 3 - 4 = 2$

$2 = 2 \Rightarrow$ Euler's Formula Verified

(c) $\begin{array}{c|ccc} & A & B & C \\ \hline A & 1 & 1 & 1 \\ B & 1 & 0 & 1 \\ C & 1 & 1 & 0 \end{array}$

(d) A – A – A

A – B – A

A – C – A

6. (APP 2019:CF04a,b)

(a) (i) 1.4 km (E – T – S – V)

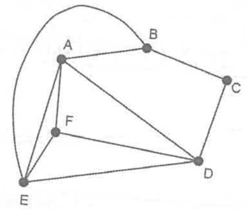
(ii) E – T – U – T – S – V

2 km

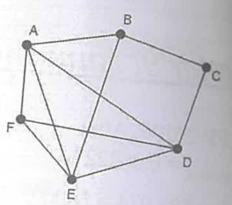
(b) E – P – Q – R – S – V – U – T – E

7. (APP 2019:CA12)

(a) (i)

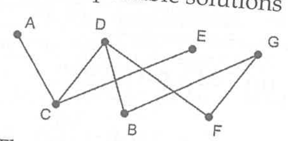


OR



** other possible solutions

(ii)



(b) The number of huts is seven (from diagram)

OR

$21 = \frac{n(n-1)}{2} \Rightarrow n = 7$

8. (APP 2020:CF02)

(a) (i) 2 (ii) 8 (iii) 5

(b) (i) Plan Q (ii) none (iii) Plan P & Plan R

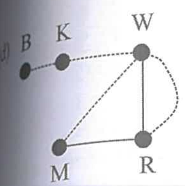
9. (APP 2020:CF06)

(a) There is no doorway between the areas

(b) $X = 1$ $Y = 2$

(c) Find the sum of any row or any column for the corresponding area

(d)



(e) 2 (M – R – W & M – R – W)

10. (APP 2021:CF03)

(a) $F + V - E = 2$

$3 + 7 - 8 = 2$

$2 = 2$

Euler's Formula verified ⇒ graph is planar

(b) T-B-C-D-G-F-E-D-F

(c) Semi Eulerian

(d) (i) TB or BC or CD

(ii) bridge

11. (APP 2022:CF01)

(a) Vertex B contains a loop

(b) Semi-Eulerian. It contains exactly two odd vertices

(c) $F + V - E = 2$

$10 + 8 - 16 = 2$

$2 = 2$

⇒ network is planar

(d) Path is semi-Eulerian, must start at an odd node and finish at the other node. Park at D

12. (APP 2022:CA10a,b)

(a) Yes. All nodes are even.

(b) (i) Hamiltonian Cycle

(ii) A–B–C–D–E–F–H–G–A

$6 + 7 + 9 + 8 + 2 + 8 + 3 + 4 = 47$

4700 m

Chapter 11: Shortest Path

1. (APP 2017:CF06a)

A-Q-S-P-S-R-T-U-B (360 minutes)

2. (FM2 2020:M403)

(a) 3.3 km

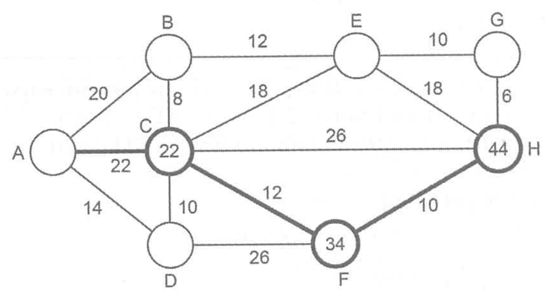
(b) (i) Eulerian Trail (ii) P

3. (APP 2020:CA13)

(a) A – D – C – B – E – G – H – F

70 km

(b)



A – C – F – H.

44 km.

(c) If CH is reduced by 4 km A – C – H will be equal in distance to A – C – F – H.

If it is reduced by more than 4 km, the shortest path will be A – C – H.

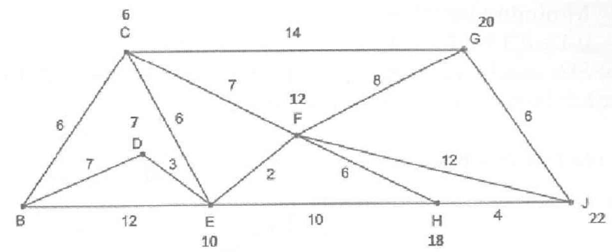
For A – C – H, the road between C – H must be reduced by more than 4 km but the reduction must be less than 26 km.

4. (FM2 2021:M402)

(a) 86 km

(b) G – H – I – K – L – M – N – O – J – G

5. (APP 2021:CF05)



(a) B–D–E–F–H–J

2200 metres

(b) B–C–F–G–J

(c) (i) Hamiltonian Cycle

(ii) 4800 metres

B–D–E–F–H–J–G–C–B

6. (APP 2022:CA10c)

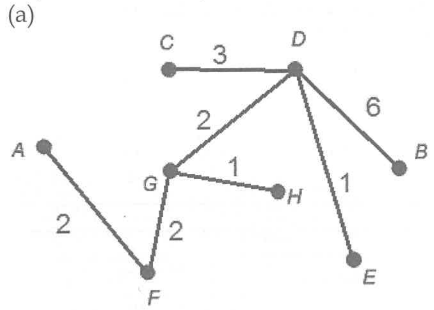
G – F – B – C

1900 m

Chapter 12: Trees and Minimum Connections

1. (APP 2017:CF6b)

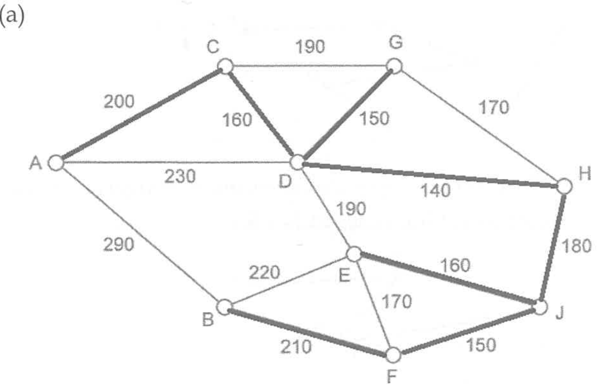
(a)



(b) $2 + 2 + 1 + 2 + 3 + 1 + 6 = 17$ m

2. (APP 2018:CF1)

(a)



(b) 1350 m