Towfish Cable modelling

Harshada Kale
IIT Gandhinagar
Email: harshada.kale@iitgn.ac.in

***Abstract*— This paper presents a method for estimating the configuration of a towing cable and the position of an attached underwater vehicle, or towfish.**

I. Introduction

Towfish is typically used for underwater data collection and research. It is a critical tool in oceanography, hydrography, and other fields requiring underwater surveys and mapping. It's used for various purposes, from mapping the seafloor and underwater objects to monitoring marine pollution, measuring oceanographic parameters, and inspecting underwater pipelines and cables.

Precise determination of the towfish's position is essential because all collected data must be accurately referenced to real-world coordinates. However, the towfish's position is difficult to determine directly, as it is towed behind a vessel on a long cable that is influenced by water currents, vessel speed, and other environmental factors. Therefore, modeling the cable's shape and estimating the towfish's actual position are critical for improving the accuracy of underwater survey data and ensuring reliable results for scientific research and industrial operations.

While it is common practice to rely on sensors mounted directly on the towfish for position estimation, these measurements are often affected by significant noise, which can compromise the accuracy of the resulting data. Moreover, sensor data alone does not provide information about the shape or behavior of the towing cable under varying operational conditions. Factors such as cable length, towing speed, and the physical properties of different cable materials—each influencing parameters like drag coefficient—can all have a substantial impact on the cable's configuration and, consequently, the towfish's position. Without accounting for these variables, it becomes difficult to accurately predict or control the towfish's location, especially in dynamic marine environments.

To achieve a more reliable and precise estimation of the towfish's position, it is necessary to go beyond sensor readings and incorporate a comprehensive model of the cable's physical dynamics. By integrating sensor data with mathematical models of the tow cable, we can better understand how environmental and operational factors influence the system. This combined approach not only improves the accuracy of position estimates but also enhances our ability to control the towfish's trajectory and stability under changing sea and cable conditions.

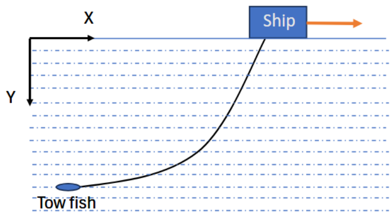


Fig. 1. Towfish and its cable attached to a surface vessel

II. Methodology and Results

Several methods are available for modeling a cable, each with its own advantages. A widely used approach is the Lumped-Mass Model, which works by representing the cable as a series of distinct point masses connected by massless springs and dampers. The primary benefit of this method is its computational efficiency, which makes it very suitable for real-time simulations.

In contrast, the Finite Element Method (FEM), which is used in most advanced physics software, is highly reliable for capturing more complex, non-linear system behaviors, such as a cable's stretching (axial elasticity) and bending stiffness. While other techniques also exist—including the Finite Difference Method, modal representation, and various analytical approaches—the best choice depends on the specific goals of a project.

For our purposes, we will be using the Lumped-Mass Model. The cable is divided into a series of massless elastic segments and as the name suggests the mass is lumped or concentrated at the connecting nodes. This dynamic model accounts for the key forces acting on the system, including hydrodynamics and gravity whilst neglecting the effect of torsional and bending stiffness and internal damping in the cable for simplicity. All forces are therefore treated as acting directly on the nodes.

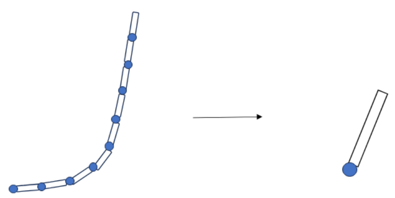


Fig. 2. Lumped mass model

To start, we simplify how we calculate the drag force. Instead of dealing with separate drag components along and perpendicular to the cable (tangential and normal drag), we use a single, modified drag coefficient. This allows us to treat the entire drag force as acting in one simple direction: directly opposite to the water flow. In this model, we describe the position of each node using the length of the cable segments and the angle it makes with the horizontal axis. By differentiating these positions, we can derive expressions for the velocity and acceleration of each node.

$$\vec{r}_i = \left(x_m - l\sum\cos\theta_i\right)\hat{\imath} + \left(l\sum\sin\theta_i\right)\hat{\jmath}$$

$$\vec{a}_i = \left(A_x + l\sum\cos\theta_i\,\dot{\theta}_i^2 + l\sum\sin\theta_i\,\ddot{\theta}_i\right)\hat{\imath} + \left(l\sum\cos\theta_i\,\ddot{\theta}_i - l\sum\sin\theta_i\,\dot{\theta}_i^2\right)\hat{\jmath}$$

- $\vec{r}_i$: Position vector of i^{th} node
- $\vec{a}_i$: Acceleration vector of i^{th} node

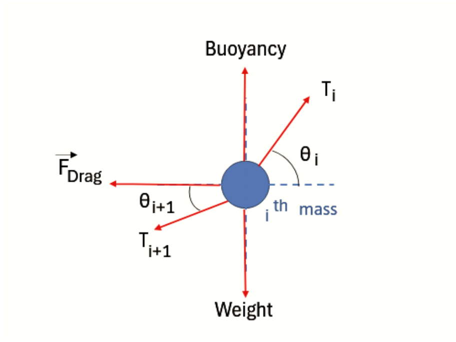


Fig. 3. Free body diagram of the i^{th} node

The force balance for each node is then set up using Newton's second law.

X force balance on ith node:

$$T_i\cos\theta_i - T_{i+1}\cos\theta_{i+1} - k_i V x_i = m a_{ix}$$

Y force balance on ith node:

$$mg + T_{i+1}\sin\theta_{i+1} - F_B - T_i\sin\theta_i - k_i V y_i = m a_{iy}$$

X force balance on towfish:

$$T_N\cos\theta_N - k_N V x_N = M a_{Nx}$$

Y force balance on towfish:

$$Mg - F_{BN} - T_N\sin\theta_N - k_N V y_N = M a_{Ny}$$

- T_i : Tension in i^{th} segment
- F_B : Buoyancy
- m : mass of each node
- $V_x i$: Velocity component of segment in x direction
- $V_y i$: Velocity component of segment in y direction

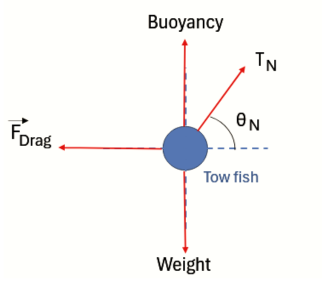


Fig. 4. Free body diagram of the Towfish

A. *Method 1*

For simplicity we assume the massless links are rigid so we can ignore axial elasticity in this initial model. We simplify the problem further by assuming a steady-state condition. This means we are analyzing the system at a point where the cable's shape is stable and no longer changing over time. Using the known boundary conditions at the towfish, we can work our way back up the cable recursively, solving for each node one by one. To determine the steady state configuration of the system, we begin by assuming zero acceleration for both the ship and each node, implying constant velocity or static conditions. The solution process starts from the last node, the towfish, where initial guesses for the angles of each node are made. Using these assumed angles, the system is solved recursively, moving from the towfish back towards the ship and at each step, the forces and positions are calculated based on the current angle estimates. Once the first node (the ship) is reached, the assumed angles are updated, and the process is repeated iteratively until the angles converge to stable values.

$$k_i = 0.5\,\rho\,C_d\,A\,|V_i|$$
$$F_B = \rho\,V\,g$$

X force balance:

$$T_i\cos\theta_i - T_{i+1}\cos\theta_{i+1} - k_i V_{x_i} = 0$$

Y force balance:

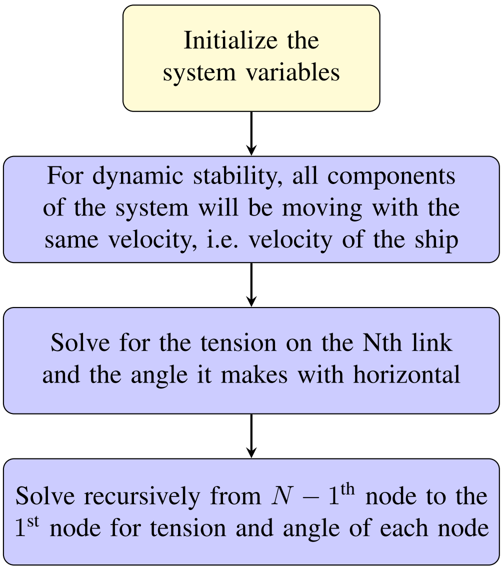
$$mg + T_{i+1}\sin\theta_{i+1} - F_B - T_i\sin\theta_i = 0$$

X force balance:

$$T_N\cos\theta_N - k_N V x_N = 0$$

Y force balance:

$$Mg - F_{BN} - T_N\sin\theta_N = 0$$



B. Result 1

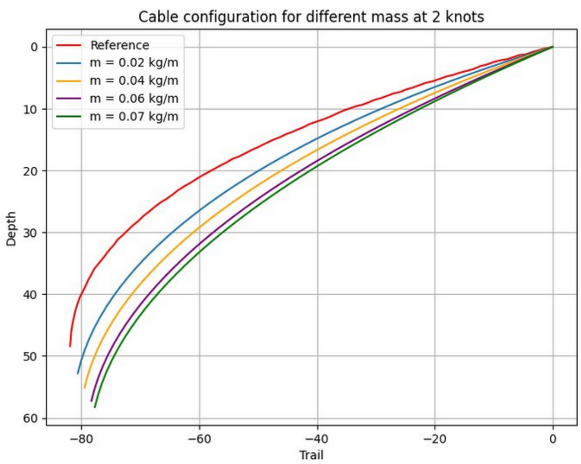


Fig. 5. Comparison of obtained results for varying mass per unit length

Since the exact value of the mass per unit length was not provided in the reference paper, the cable configuration for different mass per unit length is plotted. We can observe as we decrease the value of 'm' the obtained results get more closer to the reference.

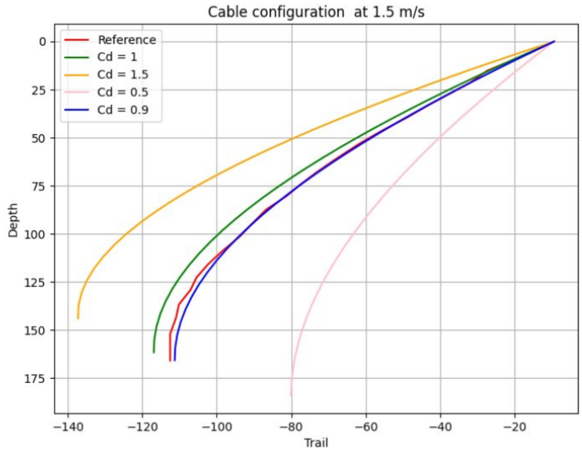


Fig. 6. Comparison of obtained results for varying drag coefficient

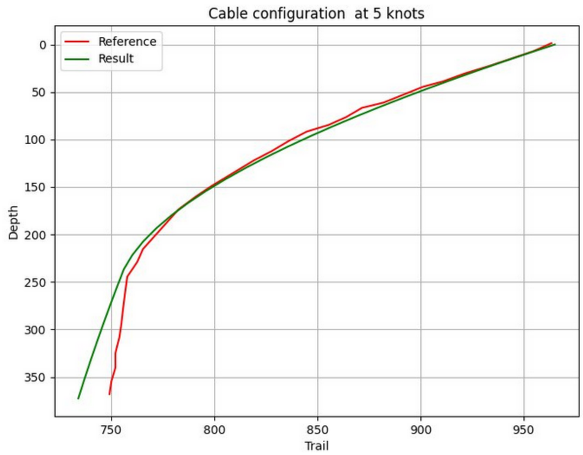


Fig. 7. Comparison of obtained results with reference

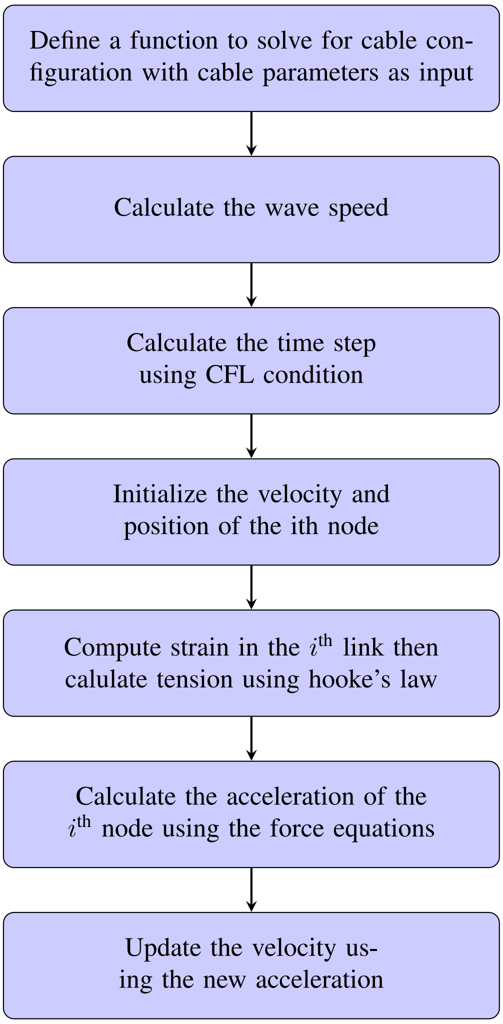
Here, we could observe that our current method is not exactly able to capture the behavior of the cable, especially the curve as we approach the rear end of the cable. This could happen because of several reasons; using a generalized drag coefficient may lead to inaccurate prediction of the cable shape near the towfish. The bend near the towfish could also be the effect of axial stiffness that was ignored in this model. The most likely reason could be the assumption of the towfish as a point mass rather than fully accounting for its rigid body dynamics.

However, this method proves inadequate when solving dynamic conditions where velocity varies with time, and the complex expressions of velocity and acceleration lead to the accumulation of errors in the recursive calculations. The solution does not converge to any realistic values of angles. Whilst this method was computationally efficient for steady state conditions, we need a more robust method for capturing the behavior of the cable under dynamic conditions.

C. Method 2

To solve for the system's dynamic configuration, we implemented a numerical solver based on the force balance equations for each node. In this model we are accounting for the axial elasticity. We are also accounting the added mass force that was neglected in the previous models.

In each time step, the solver calculates the net forces, including elastic tension derived from Hooke's law, to determine the acceleration of each mass. The system's state is then advanced using the Verlet integration method, a time-stepping algorithm where the integration time step (dt) is optimized using the Courant–Friedrichs–Lewy (CFL) condition to ensure numerical stability. Although this method yields accurate results, we found that the solution is sensitive to discretization parameters, such as segment length, which require careful tuning to guarantee a stable and physically accurate simulation.



Then we used existing and more reliable solvers to solve the sytem of ODE's. BDF (Backward Differentiation Formula) is an implicit method to solve for the numerical solution of Ordinary differential equations. Unlike explicit methods like Velocity Verlet, where the new state is calculated directly from the previous state, implicit methods define the new state through an equation that includes the new state itself. This means a system of equations, often non-linear, must be solved at each time step, making BDF methods computationally more intensive per step. BDF methods are renowned for their ability to solve stiff ODEs. Stiff systems have components that vary on widely different time scales, which forces explicit methods to use extremely small time steps to maintain stability.

D. Result 2

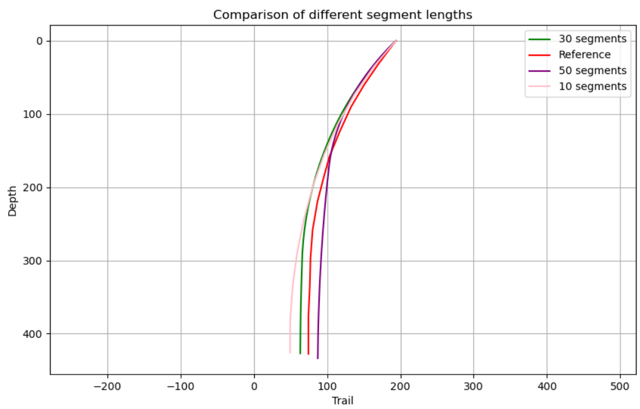


Fig. 8. Comparison for cable configuration for different segment length

We can observe even BDF is sensitive to the segment length but less sensitve compared to verlet method.

III. Conclusion

The verlet integration and BDF provides similar results whilst the BDF is more time and computationally expensive. A better numerical method or improvement in the existing methods is needed for a better accuracy. In the formulated methods some parameter is causing dependency on the segment length, knowing the exact reason will give us more insight and help us improve the mathematical model further.

References

[1] EdgeTech, *4125i/4125 Side Scan Sonar System User Hardware Manual*, Doc. 0004823_REV_K, Sep. 2024. [Online]. Available: https://www.edgetech.com/wp-content/uploads/2023/04/0004823_Rev_K.pdf

[2] Apogee Components, *Kevlar Cord 750#*, [Online]. Available: https://www.apogeerockets.com/Building-Supplies/Shock-Cord/Kevlar-Cord-750.

[3] M.-K. Kim, D.-J. Park, Y.-S. Oh, J.-H. Kim, and J.-K. Choi, "Towfish Attitude Control: A Consideration of Towing Point, Center of Gravity, and Towing Speed," *J. Mar. Sci. Eng.*, vol. 9, no. 6, p. 641, Jun. 2021.

[4] C. M. Ablow and S. Schechter, "Numerical simulation of undersea cable dynamics," *Ocean Engineering*, vol. 10, no. 6, pp. 443–457, 1983.