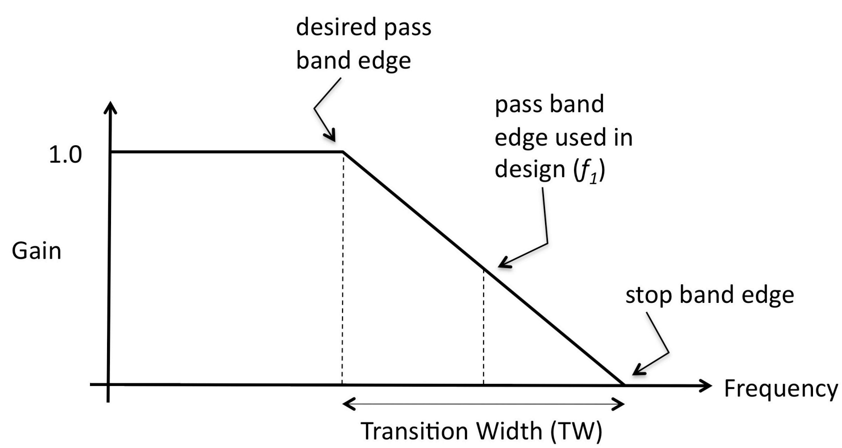
Windowed FIR LPF Filter Design

In this technique, the desired cutoff frequency of the low pass filter is f_c Hz, or $\Omega_c = 2\pi f_c / f_s$ radians/sample. Note: this is also the desired edge of the pass band. We now try to create a filter that approximates this as follows:

1. Choose a pass band edge frequency f_1 in Hz for the filter. This frequency should be in the middle of the filter's transition width, as shown in the following plot:

$$f_1 = \text{(Desired pass band edge frequency)} + \frac{\text{(Transition width)}}{2}$$



2. Calculate the corresponding digital frequency $\Omega_1 = 2\pi f_1 / f_s$ and substitute this into the equation for the impulse response (IIR) of the ideal low pass filter:

$$h_1[n] = \frac{\sin(n\Omega_1)}{n\pi}$$

Note: in terms of the MATLAB *sinc* function, which is sinc(x) = sin(πx)/ πx, this is equal to:

$$= \frac{\Omega_1}{\pi}\,\text{sinc}\left(\frac{n\Omega_1}{\pi}\right)$$

3. Based on other specifications provided in the Window Characteristics table on the next page, for example using the stop band attenuation, choose a suitable window function $w[n]$. Based on the type of window used, use the equation in the table to calculate the number of terms (which should be an odd number so the impulse response is perfectly symmetrical about $n=0$ … this will ensure no phase distortion in the resulting filter). Determine the filter coefficients for $-(N-1)/2 \le n \le (N-1)/2$.

4. Determine the impulse response (FIR) for the filter defined by $h_2[n]=h_1[n]w[n]$ for $-(N-1)/2 \le n \le (N-1)/2$. $h_2[n] = 0$ for all other values of n. Note that this filter is non-causal, since the impulse response is non-zero for some $n < 0$.

5. Shift this impulse response to the right by (N-1)/2 samples so that the first non-zero value occurs at $n = 0$. This makes the filter causal. The final causal FIR low pass filter's impulse response is now

$$h[n]=h_2[n-(N-1)/2].$$

Table: Window Characteristics

Window Type	**Window Equation** $-(N-1)/2 \le n \le (N-1)/2$	**# of Terms (*TW* is transition width, f_s is sample frequency)**	**Stop Band Attenuation (dB)**	**MATLAB**
Rectangular	1	$0.91\frac{f_s}{TW}$	21	w=rectwin(N);
Hanning	$0.5-0.5\cos\left(\frac{2\pi n}{N-1}\right)$	$3.32\frac{f_s}{TW}$	44	w=hanning(N);
Hamming	$0.54-0.46\cos\left(\frac{2\pi n}{N-1}\right)$	$3.44\frac{f_s}{TW}$	55	w=hamming(N);
Blackman	$0.42-0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$	$5.98\frac{f_s}{TW}$	75	w=blackman(N);
Other possible windows: Kaiser, Taylor, Chebyshev, Tukey, Gaussian, Triangle, etc.				