Simulation of Resistive Torque From a Wave-Powered Rotary Crankshaft Pump for Seawater Desalination

1 ABSTRACT

In order to maximize energy absorption from waves, the damping force from a power take-off system must equal the hydrodynamic damping force. Given that the Waves to Water challenge specifies multiple sea states, with differing hydrodynamic damping, it is highly advantageous to have a pump with the ability to vary it's characteristics. As a part of the larger Waves to Water Project, the purpose of this project was to investigate the adjustability that could be built into a rotary pump, in order to maximize energy capture from waves.

2 INTRODUCTION

The Waves to Water competition challenges teams to make small-scale wave-powered desalination systems. Reverse Osmosis is one of the most efficient methods of desalination, but requires extremely high pressures (600-800 psi). To reach pressures this high, there are only a few options. Of these, rotary pumps were chosen for the potential adjustability of multi-piston designs. Pump resistance can be adjusted by shutting off one or more of the pistons, effectively changing the displacement of the pump. Like a modern internal combustion engine deactivating cylinders to save fuel, this is easily accomplished by keeping the valves of a given piston open at all times.

Shutting off pistons, however, presents an interesting dilemma; a pump with "all cylinders firing" could be quite balanced, with the resistive torques of each cylinder adding up to a near-constant torque over all angles of the crankshaft during operation. However, if a single piston is shut off, a certain range of crankshaft angles (during the compression stroke of that piston) will have far lower resistive torque.

This report describes software tools that were developed to find combinations of pistons that could produce a wide range of adjustability for average resistance, while minimizing the variation of pump resistance across crankshaft angles, for each setting of the pump.

3 SYSTEM GEOMETRY

While several geometries are in consideration for the wave energy converter, they are sufficiently similar that this study only evaluates one, depicted in Figure 1. An absorber, with the dimensions of an inflatable queen mattress, floats on the water surface, rising and falling with the heave motion of the waves. It is tethered to a spool driving the rotary pump, which is rotated as the absorber rises. A one way clutch only allows torque to be applied when pulley rpm exceeds pump rpm (positive torque). This allows the pump to continue rotating in the positive direction while the absorber descends.

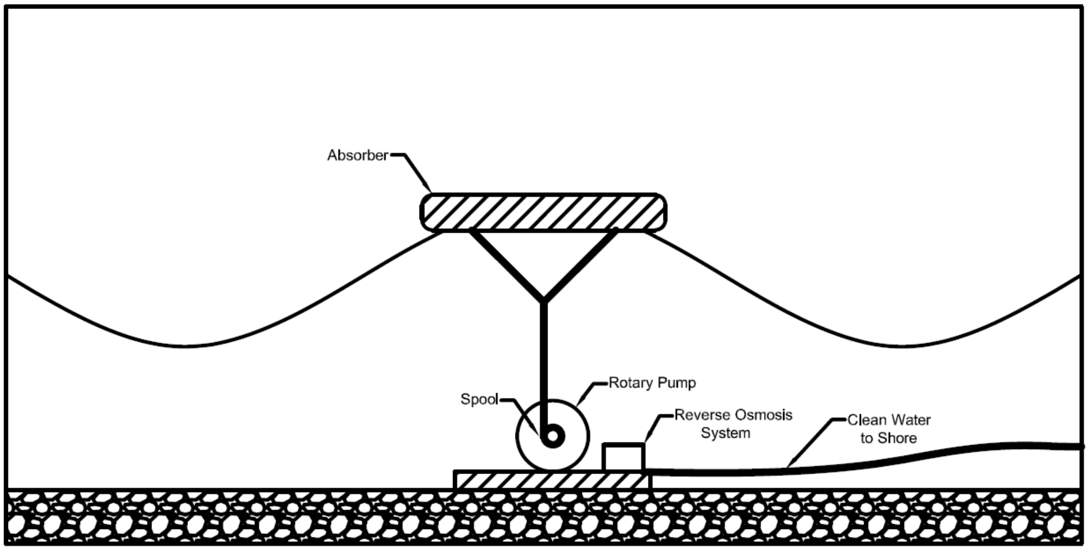

Figure 1: Diagram showing design of desalination system being simulated

4 PUMP CONSTRUCTION

The pump, shown in Figure 2, has a central crankshaft, with each piston mounted to a pivot point a distance away from the crankshaft. For cost reasons, several off-the-shelf hydraulic pistons will be used, as opposed to an "engine block" design.

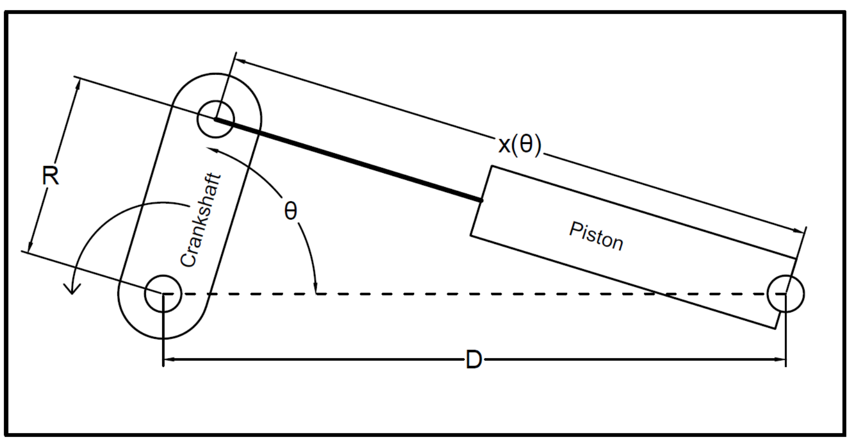

Figure 2: Diagram Showing Piston Pump Geometry

While the piston's motion is periodic with respect to Θ, the exact geometry shown in Figure 2 means that the relation between crankshaft angle and piston length is not exactly a sinusoid. The relation can be shown to be:

$$x(\Theta) = 2 \cdot \sqrt{(R^2 + D^2 - 2 \cdot R \cot D \cdot cos(\Theta)} \tag{4.1}$$

where R is the off axis distance of the crankshaft journals, D is the center-to-center distance from crankshaft to piston pivots, and Θ is the angular position of the crankshaft. Deriving this formula produces the effective moment arm of the system, which is given by:

$$x'(\Theta) = \frac{R \cdot D \cdot sin(\Theta)}{\sqrt{R^2 + D^2 - R \cdot D \cdot cos(\Theta)}} \tag{4.2}$$

Multiplying this formula by piston plunger surface area and water pressure gives the resistive torque from a given piston at a given angle:

$$T_{resistance} = A_{piston} \cdot P_{water} \cdot \frac{R \cdot D \cdot sin(\Theta)}{\sqrt{R^2 + D^2 - R \cdot D \cdot cos(\Theta)}} \tag{4.3}$$

5 SIMULATOR METHODOLOGY

In simulating the resistive torque of a pump, each piston is modeled as a list of one hundred torques, across a sweep of angles between $\Theta = 0$ to $\Theta = 2\pi$. Each piston has a specified bore and angular offset on the crankshaft. For a given set of N pistons, a $N \times 100$ matrix, $M_{pistons}$, is generated, with each row representing torques from each individual piston. Single acting pistons are modeled as a single row, and double acting pistons are modeled by two rows, each equivalent to a single-acting piston, and offset by 180° on the crankshaft.

$$M_{pistons} = \begin{bmatrix} T_{piston1,\Theta=0} & \cdots & T_{piston1,\Theta=2pi} \\ \vdots & \ddots & \vdots \\ T_{pistonN,\Theta=0} & \cdots & T_{pistonN,\Theta=2pi} \end{bmatrix} \tag{5.1}$$

Then, a matrix C is generated, which represents all possible combinations of pistons that can be turned on at a time. Matrix C is $2^N \times N$, with each of the 2^N rows representing a combination of pistons, and each column representing an individual piston. Each element of the matrix can take the value of 1, corresponding to a piston pumping water, or 0, corresponding to a piston that is off.

$$C = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} \tag{5.2}$$

By multiplying $C \times M$, a new $2^N \times 100$ matrix R is produced, which is similar to $M_{pistons}$, however, instead of each row representing torques from a single piston, each row of R represents torques from a given combination of pistons pumping water at once.

$$R = C \times M_{pistons} = \begin{bmatrix} T_{combination1,\Theta=0} & \cdots & T_{combination1,\Theta=2pi} \\ \vdots & \ddots & \vdots \\ T_{combinationN^2,\Theta=0} & \cdots & T_{combinationN^2,\Theta=2pi} \end{bmatrix} \tag{5.3}$$

Plotting each row of R vs. Θ produces a graph of the resistances for every combination possible with a set of input pistons. Figure 3 shows an example plot, for three single acting pistons, equally spaced.

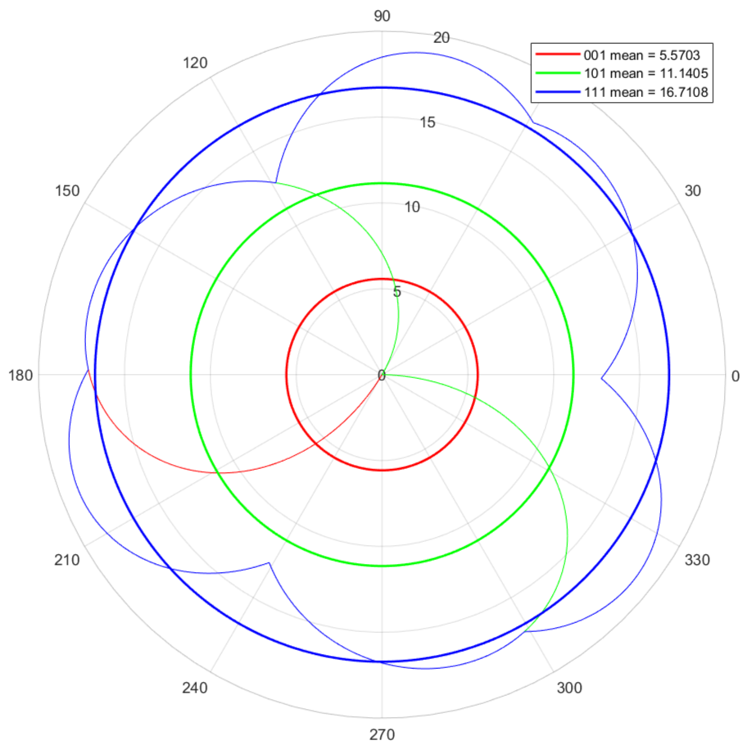

Figure 3: Pump Resistive Torque vs. Crankshaft Angle in Polar Coordinates, for all combinations of three single-acting pistons, at 120° intervals.

6 TARGET MATCHING

The second part of the simulator takes V, a vector of M pump resistance targets, as well as R as inputs. Each target resistance is based on the damping force needed for a given sea state that the desal system is expected to encounter. The simulator seeks to find which combinations of the pistons given meet the target resistances best. Several metrics were considered to evaluate each combination, among them summed error, sum squared error, and summed error to the fourth power. Sum squared error was chosen, as it represented the best compromise between rewarding correct average resistance, and penalizing irregularity across crankshaft angles. For each target input, the simulator calculates sum squared error with the following equation:

$$Error^2_{Combination\ n,\ Target\ m} = \sum_{i=1}^{100} (V_m - R_{n,i})^2 \tag{6.1}$$

For each target, the combination with the lowest $Error^2$ was chosen, and is displayed. Figure 4 shows results for a set of four pistons, each 90° apart on the crankshaft, with a vector of targets $V = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$ Nm.

The targets are shown in black, and the selected combinations to meet those targets are and their averages are displayed as well.

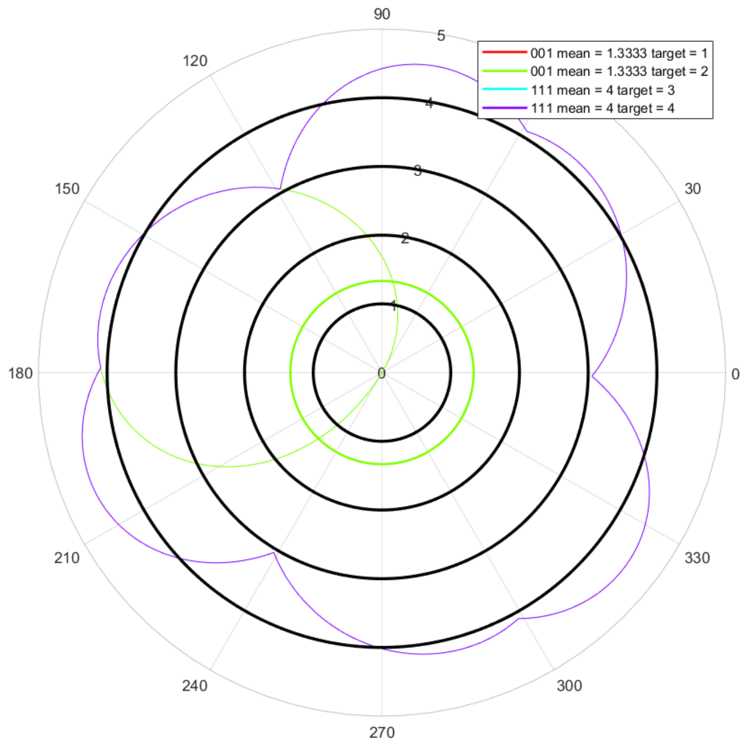

Figure 4: Pump Resistive Torque vs. Crankshaft Angle, Fitting Targets $V = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$ with three single-acting pistons at 120° increments.

7 RESULTS

From Figure 4, it is clear that the pump is not able to meet the targets well. In order to meet target resistances better, many more smaller pistons are required. Figure 5 shows a the same targets being met by six pistons, instead of three. It is clear that the targets are being fit far better. This trend continues as more and more pistons are added.

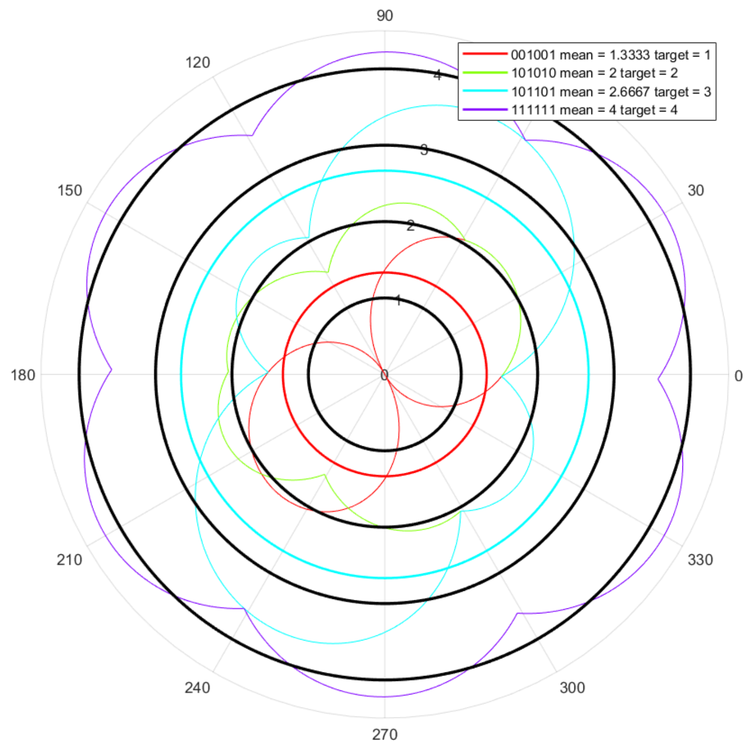

Figure 5: Pump Resistive Torque vs. Crankshaft Angle, Fitting Targets $V = \begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$ with six single-acting pistons at 60° increments.

While it had initially been hoped that target resistances could be met relatively well with only a few pistons, this study made clear that many pistons are required to meet even 3 or 4 target resistances without major irregularity. As a result, it has been decided that the pump needs to spin somewhat faster than initially planned, to decrease the time that the pump spends in low or high resistance regions at a time. This creates another issue, which is high frequency pressure surges on the filtration side of the desalination system, a large issue for RO systems. In order to damp these surges, a large gas charged accumulator between the rotary pump and the RO membrane will be included.