Magic Formula Tire Model

Related terms:

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Hans B. Pacejka, in Tire and Vehicle Dynamics (Third Edition), 2012

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Tyre Characteristics and Modelling

Mike Blundell, Damian Harty, in The Multibody Systems Approach to Vehicle Dynamics (Second Edition), 2015

5.7 Implementation with MBS

General purpose MBS software intended for use in vehicle dynamics will often have specialised modules intended for tyre modelling. Implementation of the Magic Formula tyre model and the Interpolation Method can be achieved using commercial modules, or by writing bespoke subroutines and linking these to provide a customised library function, presuming the software allows such linking (as many do). Current versions of programs such as ADAMS/Car or the Simpack Vehicle Wizard make the incorporation of a tyre model appear seamless and can give the user a false sense of comfort with what is in fact an entirely empirical and quite complex piece of modelling. Example tyre model subroutines developed by the authors are provided in Appendix B, some of which form the basis of a tyre modelling, checking and plotting facility (Blundell, 2000a).

An example interface between MSC ADAMS and a tyre model is through a user-written TYR501[1] subroutine. The subroutine receives tyre states from a road/tyre interface module (usually bundled with the overall tyre modelling module but possible to implement as run time variables totally outside any additional modules) and then defines a set of three forces and three torques acting at the tyre-road surface contact patch and formulated in the TYDEX coordinate system. The equations used to formulate these forces and moments have been programmed into the subroutines to represent the various tyre models. Note that in the example provided in Appendix B the source code was migrated from an earlier TIRSUB that used the SAE co-ordinate frame; just before returning the values to the main solver, the final act is to transform the SAE forces into the TYDEX convention; this is done entirely for reasons of coding convenience and to maintain the availability of legacy support tools – spreadsheets and so on – external to the MBS software. The transformation of the forces and moments from the tyre contact patch to the wheel centre is performed internally by the program.

The TYR501 subroutine is called from within the model data set by a group of statements for each tyre on the vehicle. Appendix B contains a typical group of solver statements used to call the TYR501 model from within MSC ADAMS. It receives the tyre states listed below in the TYDEX ISO co-ordinate frame:

1. Longitudinal Slip Ratio
2. Lateral slip angle (in radians)
3. Camber angle (in radians)
4. Normal deflection of tyre into road surface
5. Normal velocity of penetration of tyre into road surface
6. Longitudinal sliding velocity of contact patch
7. Distance from wheel centre to contact point (loaded radius)
8. Angular velocity about the spin axis of the tyre
9. Longitudinal velocity of tyre tread base
10. Lateral velocity of tyre tread base

5.7.1 Virtual tyre rig model

Because of the complexity of the tyre model and the generic difficulty in assimilating tyre model behaviour as part of a full vehicle model, it is regarded as poor practice not to interrogate tyre models in isolation before they are used in a full vehicle environment. A wide range of failures in data transmission is possible, including but not limited to the use of decaNewtons, kilograms, Newtons or kiloNewtons for forces in the measured data set, the use of radians or degrees in the measured data set, the use of ISO, SAE, modified ISO or any of the six imaginable co-ordinate systems. There is no substitute for an interrogation of the tyre model and a comparison with common sense – if it has a declared friction coefficient around unity, then frictional forces and vertical load should broadly correspond; a road-use tyre is unlikely to develop peak lateral forces much outside the slip angle range of 5 to 20°; and so on.

A functional model of a generic flat bed tyre test machine has been developed in MSC.ADAMS and forms part of the exemplar process described here. It is clearly desirable to use the tyre data parameters or coefficients to generate the sort of plots produced from a tyre test programme and to inspect these plots before using the data files with an actual full vehicle model. Matlab routines for directly plotting

Magic Formula tyre model forces are available, as are built in test rig routines for both ADAMS/Car and the Simpack Vehicle Wizard.

The tyre rig model is also useful where test data has been used to extract mathematical model parameters. The plots obtained from the mathematical model can be compared with test data to ensure the mathematical parameters are usefully accurate, represent the actual tyre and are read by modelling software in the same manner as the parameter-fitting software (these are often quite separate software tools). The process that this involves is shown conceptually in Figure 5.87. Note that this system has also been used to manage the data and model used to develop a low parameter tyre model for use in aircraft ground dynamics (Wood et al., 2012).

FIGURE 5.87. Overview of the tyre modelling system.

Also possible is the use of a dedicated external piece of software for handling, such as Optimum Tire from Optimum G, that can interrogate and render a tire data file with ease as well as handling a large amount of the fitting process described in Section 5.6.5.

Attention must be given to the orientation as well as the location of tyre attachment frames within the model, particularly when suspension adjustments such as static toe and static camber are intended to be made parametrically. Not all tyre models run symmetrically in both forward and backward directions and so it is often good practice to ensure that all tyres are rotating the same way, as shown in Figure 5.88. One possible approach with full vehicle modelling is to set up a global coordinate system or GRF where the x-axis points back along the vehicle, the y-axis points to the right of the vehicle and the z-axis is up. The local z-axis of each tyre part is orientated to point towards the left side of the vehicle so that the wheel spin vector is positive when the vehicle moves forward during normal motion. Note that this is the coordinate system as set up at the wheel centre and should not be confused with the SAE coordinate system that is used at the tyre contact patch in order to describe the forces and moments occurring there. There are many traps for the inattentive; it is clear that the tyre test rig should emulate the vehicle model attachments to be useful.

FIGURE 5.88. Orientation of tyre coordinate systems on the full vehicle model.

The model of the tyre test machine presented here contains a tyre part that rolls forward on a flat uniform road surface in the same way that the tyre interacts with a moving belt in the actual machine. In this model the road is considered fixed as opposed to the machine where the belt represents a moving road surface and the tyre is stationary; modelling a moving belt is surprisingly awkward in a MBS environment. Considering the system schematic of the model shown in Figure 5.89, the tyre part 02 is connected to a carrier part 03 by a revolute joint aligned with the spin axis of the wheel. The carrier part 03 is connected to another carrier part 04 by a revolute joint that is aligned with the direction of travel of the vehicle. A motion input applied at this joint is used to set the required camber angle during the simulation of the test process. The carrier part 04 is connected to a sliding carrier part 05 by a cylindrical joint that is aligned in a vertical direction. A rotational motion is applied at this joint that will set the slip angle of the tyre during the tyre test simulation. The cylindrical joint allows the carrier part 04 to slide up or down relative to part 05, which is important as a vertical force is applied downwards on the carrier part 04 at this joint and effectively forces the tyre down on to the surface of the road. The model has been set up to ignore gravitational forces so that this load can be varied and set equal to the required wheel vertical load that would be set during the tyre test process. The sliding carrier part 05 is connected to the ground part 01 by a translational joint aligned with the direction of travel of the wheel. A motion input applied at this joint will control the forward velocity of the tyre during the test.

FIGURE 5.89. Mechanism sketch for a flat bed tyre test machine model.

The joint controlling camber angle can be located at the tyre contact patch rather than at the wheel centre. This will avoid introducing lateral velocity and hence slip angle for the change in camber angle during a dynamic simulation.

The model of the tyre test machine has two rigid body degrees of freedom as demonstrated by the calculation of the DOF balance in Table 5.7. One DOF is associated with the spin motion of the tyre, that is dependent on the longitudinal forces generated and the slip ratio. The other DOF is the height of the wheel centre above the road, that is controlled by the applied force representing the wheel load.

Table 5.7. Degree of Freedom (DOF) Balance Equation for the Tyre Rig Model

Model Component	**DOF**	**Number**	**Total DOF**
Parts	6	4	24
Revolutes	–5	2	–10
Translational	–5	1	–5
Cylindrical	–4	1	–4
Motions	–1	3	–3
		ΣDOF = 2	

The tyre test rig model has been used to read the tyre model data files used in a study (Blundell, 2000a) to plot tyre force and moment graphs. The graphics of the tyre rig model are shown in Figure 5.90.

FIGURE 5.90. Computer graphics for the tyre rig model.

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Outlines of Three Advanced Dynamic Tire Models

Hans B. Pacejka, in Tire and Vehicle Dynamics (Third Edition), 2012

13.3.3 MF-Tire/MF-Swift

The Dutch organization for applied research TNO also contributed to the development of the tire model and has turned them into commercial software, which is sold under the name "Delft-Tire". The first software product was the Magic Formula Tire Model implementation in various multi-body software packages, known as *MF-Tire*. In particular, *MF-Tire* version 5.2 achieved the status of an industry standard for modeling passenger-car tires in the late 1990's. The software program *MF-Tool* is used to process measurements and determine the parameters of the tire model. For some years, various separate tire models were available (e.g., motorcycle tires: *MF-MCTire*, early versions of *SWIFT*), but more recently all functionality has been combined into a single tire model known as *MF-Tire/MF-Swift*. Figure 13.8 gives an impression of the TNO *Delft-Tire* tool chain, which supports all steps from tire measurements, parameter identification to multi-body simulations.

FIGURE 13.8. TNO Delft-Tire tool chain.

For more detailed information, we suggest to contact TNO. The website address reads: www.delft-tire.com.

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State and Parameter Estimation of EVs

Brett McAulay, ... Haiping Du, in Modeling, Dynamics and Control of Electrified Vehicles, 2018

11.4.3 Simulation Results of Tire-Friction Force and Tire–Road Friction Estimation

In this section, the numerical simulation is implemented by the MATLAB/Simulink. In the simulations of this section, the vehicle initial velocity is set as 1 m/s and the tire–road friction coefficient is 0.9 to present the cement road. In addition, the vehicle dynamics model in the simulation also considers the rolling resistance, wind-drag force, and road. The simulation parameters have been shown in Table 11.3. In all the simulations of this section, the Magic Formula tire model is used, so the Magic formula tire parameters are shown in Table 11.3. For brevity, the slip-slope method is called Method 1 in the following text and figures. Algorithm 1 of the individual wheel tire-road friction coefficient estimation method is called Method 2 in the following text and figures. Algorithm 2 of the individual wheel tire-road friction coefficient estimation method is called Method 3 in the following text and figures.

Table 11.3. Simulation parameters (Boada et al., 2005)

m	Vehicle mass	1298.9 kg
lf	Distance of CoG from the front axle	1 m
lr	Distance of CoG from the rear axle	1.454 m
Cs	Longitudinal stiffness of vehicle tire	5000 N/unit slip
Iz	Vehicle moment inertial about yaw axis	1627 kg.m2
Rw	Wheel radius	0.35 m
Iw	Wheel moment of inertia	2.1 kg.m2
Da	Wind drag coefficient	0.4176
Cr	Rolling resistance coefficient	0.4176
	Road gradient	0.4176
B	Magic formula tire parameter	20
C	Magic formula tire parameter	1.5
D	Magic formula tire parameter	
E	Magic formula tire parameter	–0.5
a	Proportional constant	0.03
b	Bias constant	0.1

Fig. 11.6 compares the estimated front-tire force and tire-road friction coefficient of traditional friction estimation methods when the tire-road friction coefficient is abruptly changed from 0.9 into 0.5 at 10 s. It shows that all the traditional methods can follow the change of road friction in 10 s. Fig. 11.7 shows the estimation performance of the new proposed method, which is much improved.

Figure 11.6. Comparison of the estimated front-tire force (A) and tire–road friction coefficients (B) when the actual friction coefficient is changed for traditional methods (Li et al., 2014).

Figure 11.7. Comparison of the estimated front-tire force (A) and front-tire friction coefficients (B) when the actual tire–road friction coefficient for the proposed methods (Li et al., 2014).

Fig. 11.8 shows the estimated tire force and friction coefficient when the measurement noise is included in the longitudinal acceleration measurement system (random noise with variance of 0.1 and bias offset value of 0.1). This is really common for the widely used inertia measurement unit or other acceleration measurement system. Fig. 11.8 shows that the estimation performance of Method 1, Method 3, and the EKF method is compromised since the input value of acceleration is required for all three methods. On the other hand, Method 2 only needs the measurement inputs of wheel angular velocity and traction/brake torque without the longitudinal acceleration. Fig. 11.9 shows that the measurement noise also affects the estimation performance of the new proposed estimation method because the estimation method relies on the measured acceleration. In order to get more reliable estimation results, the feedback gain of the measured acceleration can be appropriately tuned so that the estimation relies more on the torque measurement.

Figure 11.8. Comparison of the estimated front-tire force (A) and tire-road friction coefficients of front tire (B) when the measurement noise of longitudinal acceleration is considered for the traditional methods (Li et al., 2014).

Figure 11.9. Comparison of the estimated front-tire force (A) and tire–road friction coefficients of front tire (B) when the measurement noise of longitudinal acceleration is considered for the proposed methods (Li et al., 2014).

In Figs. 11.10 and 11.11, the combined traction torque and brake torque are applied on the front and rear wheel of the vehicle. It can be seen from Fig. 11.10 that only Method 1 can successfully finish the estimation and estimation results of all the other traditional methods are negatively affected by the combined traction and brake motion. However, the proposed estimation method shows very good estimation performance in Fig. 11.11.

Figure 11.10. Comparison of the estimated (A) front-tire force and (B) front tire–road friction coefficients when considering the combined traction and brake motion for the traditional methods (Li et al., 2014).

Figure 11.11. Comparison of the estimated (A) front-tire force and (B) front tire–road friction coefficients when considering the combined traction and brake motion for the proposed methods (Li et al., 2014).

According to the simulation results, it is confirmed that the newly proposed method shows good performance in the friction coefficient variation and in the combined traction and braking condition simulations compared to existing methods. When the measurement noise is considered, although the proposed method shows compromised performance, the estimation performance is still acceptable.

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Single-Contact-Point Transient Tire Models

Hans B. Pacejka, in Tire and Vehicle Dynamics (Third Edition), 2012

7.2.2 Semi-Non-Linear Model

For the extension of the linear theory to cover the non-linear range of the slip characteristics, it may be tempting to employ Eqn (7.20) and use the instantaneous nonlinear force response as input in the differential equation. The input steady-state side force is calculated, e.g. with the *Magic Formula*, using the current wheel slip angle ▯. This method, however, may lead to incorrect results as, due to the phase lag in side force response, the current (varying) wheel load may not correspond to the calculated magnitude attained by the side force. In limit conditions, the tire may then be predicted to be still in adhesion while in reality full sliding occurs. A better approach is to use the original Eqn (7.7) and to calculate the side force afterward by using the resulting transient slip angle as input in the *Magic Formula*.

In general, we have the three Eqns (7.7, 7.9, 7.11) and possibly (7.12) and the first equations of (7.14–7.16) producing , and or which are used as input in the nonlinear force and moment functions (▯ or ▯ directly in the expressions for M_r), e.g. the equations of the *Magic Formula* tire model (Chapter 4):

(7.21)

(7.22)

(7.23)

(7.24)

(4.E71)

where, if required, ▯ may be replaced by ▯ as the spin argument.

This nonlinear model is straightforward and is often used in transient or low-frequency vehicle motion simulation applications. Starting from zero speed or stopping to standstill is possible. However, as has been mentioned before, at V_x equal or close to zero, Eqns (7.7, 7.9) act as integrators of the slip speed components $V_{sx,y}$, which may give rise to possibly very large deflections. The limitation of these deflections may be accomplished by making the derivatives of the deflections u and v equal to zero, if (1) the forward wheel velocity has become very small ($<V_{low}$) and (2) the deflections take values larger than physically possible. This can be seen to correspond with the combined equivalent side slip value exceeding the level □$_{sl}$ where the peak horizontal force occurs. Approximately, we may adopt the following limiting algorithm with the equivalent slip angle according to Eqn (4.E78):

(7.25)

with roughly

Experience in applying the model has indicated that starting from standstill gives rise to oscillations which are practically undamped. Damping increases when speed is built up. To artificially introduce some damping at very low speed, which with the actual tire is established through material damping, one might employ the following expression for the transient slip , as suggested by Besselink, instead of Eqn (7.15):

(7.26)

The damping coefficient k_{Vlow} should be gradually suppressed to zero when the speed of travel V_x approaches a selected low value V_{low}. Beyond that value the model should operate as usual. In Chapter 8, Section 8.6, an application will be given. A similar equation may be employed for the lateral transient slip.

Another extreme situation is the condition at wheel lock. At steady state, Eqn (7.9) reduces to

(7.27)

which indicates that the deflection u according to the semilinear theory becomes as large as the relaxation length □□. Avoiding the deflections from becoming too large, which is of importance at e.g. repetitive braking, calls for an enhanced non-linear model. Another shortcoming of the model that is to be tackled concerns the experimentally observed property of the tire that its relaxation length depends on the level of slip. At higher levels of side slip, the tire shows a quicker response to additional changes in side slip. This indicates that the relaxation lengths decrease with increasing slip.

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Basic Tire Modeling Considerations

Hans B. Pacejka, in Tire and Vehicle Dynamics (Third Edition), 2012

2.5 Tire Models (Introductory Discussion)

Several types of mathematical models of the tire have been developed during the past half-century; each type for a specific purpose. Different levels of accuracy and complexity may be introduced in the various categories of utilization. This often involves entirely different ways of approach. Figure 2.11 roughly illustrates how the intensity of various consequences associated with different ways of attacking the problem tends to vary. From left to right the model is based less on full-scale tire experiments and more on the theory of the behavior of the physical structure of the tire. In the middle, the model will be simpler but possibly less accurate while at the far right the description becomes complex and less suitable for application in the simulation of vehicle motions and may be more appropriate for the analysis of detailed tire performance in relation to its construction.

FIGURE 2.11. Four categories of possible types of approach to develop a tire model.

At the left-hand category, we have mathematical tire models that describe measured tire characteristics through tables or mathematical formulas and certain interpolation schemes. These formulas have a given structure and possess parameters that are usually assessed with the aid of regression procedures to yield a best fit to the measured data. A well-known empirical model is the *Magic Formula* tire model treated in Chapter 4. This model is based on a sin(arctan) formula that not only provides an excellent fit for the F_y, F_x, and M_z curves but in addition features coefficients that have clear relationships with typical shape and magnitude factors of the curves to be fitted.

The similarity approach (second category) is based on the use of a number of basic characteristics typically obtained from measurements. Through distortion, rescaling, and multiplications, new relationships are obtained to describe certain off-nominal conditions. Chapter 4 introduces this method, which is particularly useful for application in vehicle simulation models that require rapid (e.g., real-time) computations.

Depending on the type of the physical model chosen, a simple formulation may already provide sufficient accuracy for limited fields of application. The HSRI model depicted in Figure 2.12 developed by Dugoff, Fancher, and Segel (1970) and later corrected and improved by Bernard, Segel, and Wild (1977) is a good example. The figure illustrates the considerable simplification with respect to a more realistic representation of tire deformation (Figure 2.13) that is needed to keep the resulting mathematical formulation manageable for vehicle dynamics simulation purposes and still include important matters such as the representation of combined slip and a coefficient of friction that may drop with speed of sliding.

FIGURE 2.12. Top: The brush type tire model at combined longitudinal (brake) slip and lateral slip in case of equal longitudinal and lateral stiffnesses. Bottom: The linearly decaying friction coefficient.

FIGURE 2.13. Tire model with flexible carcass at steady-state rolling with slip angle ▯.

The model of Figure 2.13 exhibits carcass flexibility and shows a more realistic parabolic pressure distribution. For such a model, (approximate) analytical solutions are feasible only when pure side slip (possibly including camber) occurs and the friction coefficient is considered constant (e.g., Fiala 1954).

Relatively simple physical models of this third category such as the ‘brush model’ of Figure 2.12 are especially useful to get a better understanding of tire behavior. The brush model with a parabolic pressure distribution will be discussed at length in Chapter 3.

The right-most group of Figure 2.11 is aimed primarily at more detailed analysis of the tire. The complex finite element- or segment-based models belong to this category (e.g., *RMOD-K* and *FTire*, cf. Chap. 13). A simpler representation of carcass compliance that is experienced in the lower part of the tire near the contact patch considerably speeds up the computation. In addition, the way in which the tread elements are handled is crucial. The computer simulation tread-element-following method is attractive and allows considerable freedom to choose pressure distribution and friction coefficient functions of sliding velocity and local contact pressure. The physical model that forms the basis of the latter method has been depicted in Figure 2.14.

FIGURE 2.14. Computer simulation tire model with flexible carcass, arbitrary pressure distribution, and friction coefficient functions. Forces acting on a single tread element mass during one passage through the contact length are integrated to obtain the total forces and moment F_x, F_y, and M_z.

Influence (Green) functions may be used to describe the carcass horizontal compliance in the contact zone and possibly several rows of tread elements may be considered to move through the contact patch. One element per row is followed while it travels through the length of contact (or several elements through respective sub-zones). During such a passage the carcass deflection is kept constant, the motion of the single mass-spring (tread element) system that is dragged over the ground is computed, the frictional forces are integrated, the total forces and moment determined, and the carcass deflection is updated. Instead of using the dynamic way of solving for the deflection of the tread element while it runs through the contact patch, an iteration process may be employed. The model is capable of handling non-steady-state conditions. A relatively simple application of the tread-element-following method will be shown in the subsequent chapter when dealing with the ‘brush model’ subjected to combined slip with camber, a condition that is too difficult to deal with analytically. In addition, the introduction of carcass compliance will be demonstrated. A method based on modal synthesis to model tire deflection has been employed by Guan et al. (1999) and by Shang et al. (2002). For further study we refer to Sec. 3.3 and to the original work of Willumeit (1969), Pacejka, and Fancher (1972a), Sharp and El-Nashar (1986), Gipser et al. (cf. Sec.13.3), Guo and Liu (1997), and Mastinu (1997) and the state-of-the-art paper of Pacejka and Sharp (1991).

Although it is possible to develop a model for non-steady-state conditions by purely empirical means, most relatively simple and more complex transient and dynamic tire models are based on the physical nature of the tire. It is of interest to note that for a proper description of tire behavior at time-varying conditions an essential property must be represented in the physical models belonging to both right-hand categories of Figure 2.11. That is the lateral and sometimes also the fore and aft compliance of the carcass. Less complex non-steady-state tire models feature only carcass compliance without the inclusion of elastic tread elements. In steady-state models, the introduction of such a flexibility is often not required. Only to represent properly the self-aligning torque in case of a braked or driven wheel, is carcass lateral compliance needed. Tire inertia becomes important at higher speeds and frequencies of the wheel motion. The problem of establishing non-steady-state tire models is addressed in Chapters 5, 7, 8, 9, and 10Chapter 5Chapter 7Chapter 8Chapter 9Chapter 10 in successive levels of complexity to meet conditions of increasing difficulty.

Conditions become more demanding when for example: (1) the wheel motion gives rise to larger values of slip, which no longer permits an approximate linear description of the force- and moment-generating properties; (2) combined slip occurs, possibly including wheel camber and turn slip; (3) large camber occurs, which may necessitate the consideration of the dimensions of the tire cross section; (4) the friction coefficient cannot be approximated as a constant quantity but may vary with sliding velocity and speed of travel as occurs on wet or icy surfaces; (5) the wavelength of the path of contact points at non-steady-state conditions can no longer be considered large, which may require the introduction of the lateral and longitudinal compliance of the carcass; (6) the wavelength becomes relatively short, which may necessitate the consideration of a finite contact length (retardation effect) and possibly contact width (at turn slip and camber); (7) the speed of travel is large so that tire inertia becomes of importance, in particular its gyroscopic effect; (8) the frequency of the wheel motion has reached a level that requires the inclusion of the first or even higher modes of vibration of the belt; (9) the vertical profile of the road surface contains very short wavelengths with appreciable amplitudes as would occur in the case of rolling over a short obstacle or cleat, then, among other things, the tire enveloping properties should be accounted for; and (10) motions become severe (large slip and high speed), which may necessitate modeling the effect of the warming up of the tire involving possibly the introduction of the tire temperature as a model parameter. All items mentioned (except the last one) will be accounted for in the remainder of this book.

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